

Stat/IE 531
Exam I

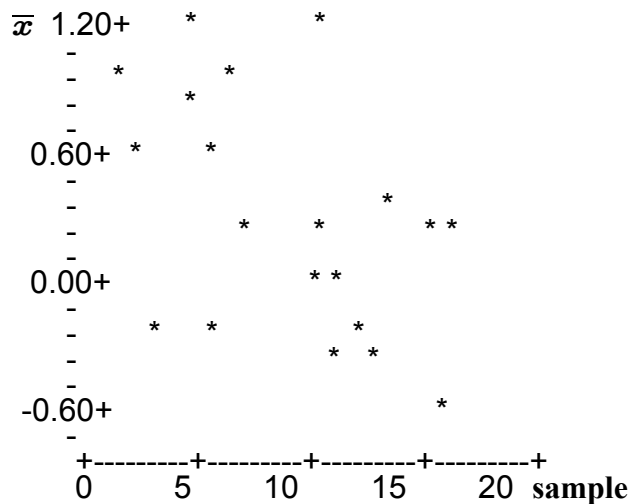
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1. IE 361 students Grilliot, Chang and Wong worked with a company on the manufacture of some rubber hoses. In one part of their study, they monitored the lengths of hoses cut on an automatic saw. Below are some summary statistics taken from 20 samples of $n = 5$ hoses coming off a particular saw. (Units are 1/16 inch above the target length.)

| Sample | \bar{x} | R | Sample | \bar{x} | R |
|--------|-----------|-----|--------|-----------|-----|
| 1 | .6 | 1 | 11 | 0.0 | 2 |
| 2 | 1.0 | 6 | 12 | 0.0 | 2 |
| 3 | 1.2 | 1 | 13 | .2 | 1 |
| 4 | - 0.2 | 3 | 14 | - 0.4 | 1 |
| 5 | .6 | 3 | 15 | - 0.2 | 3 |
| 6 | .8 | 3 | 16 | - 0.4 | 4 |
| 7 | - 0.2 | 5 | 17 | .4 | 1 |
| 8 | 1.0 | 2 | 18 | - 0.6 | 4 |
| 9 | .2 | 1 | 19 | .2 | 2 |
| 10 | 1.2 | 2 | 20 | .2 | 1 |

For these hoses, $\bar{\bar{x}} = .28$ and $\bar{R} = 2.4$. A time series plot of the \bar{x} values follows.



- (a) Consider first the matter of assessing the behavior of the process short term standard deviation, σ . Find retrospective control limits for R , and say what they indicate about the behavior of the sawing process over the period studied.
- (b) Consider now assessing the behavior of the process mean, μ . Find retrospective control limits for \bar{x} and say what they indicate about the behavior of the hose cutting process over the period of the study.
- (c) Describe the plot of \bar{x} values. Do you see any indications of lack of process stability on the plot (that may or may not be detected by your limits from part (b))?

(d) Find an estimate of the process short term standard deviation based on \bar{R} . If one simply takes the 100 measurements that lead to the summary statistics given on the previous page and computes their sample standard deviation, the value obtained is 1.13 (1/16 inch). How does this compare to your estimate? Based on your answers to parts (a) through (c), argue that the relationship between these figures is about what one should expect.

2. Suppose that in a context like that of Problem 1, one is going to CUSUM sample means \bar{x} . The ideal or target value for $\mu_{\bar{x}} = \mu$ is 0.

(a) Apply a two-sided CUSUM scheme with $k_1 = .5$, $k_2 = -.5$, $u = v = 0$ and $h_1 = h_2 = 4$ to the first 10 values \bar{x} given in Problem 1. Are any alarms signaled in the first 10 periods? Explain.

Suppose for the rest of this problem that the "all-OK" process standard deviation σ is .9 (in the units of Problem 1).

(b) Among two-sided CUSUM schemes with 0 head starts and the same on-target ARL as the scheme from (a), the one from (a) is optimal for detecting what kind of a change in μ ?

(c) Approximately what is the "all-OK" ARL here? (Give parameters needed to enter a V&J table and say which table. Don't waste time interpolating.)

3. Consider the problem of monitoring integer-valued variables Q_1, Q_2, Q_3, \dots (we'll suppose that Q can take any integer value, positive, 0 or negative). Define

$$h(x) = 4 - |x|$$

and consider the following definition of an alarm scheme:

- 1) alarm at time $i = 1$ if $|Q_1| \geq 4$, and
- 2) for $i \geq 2$ alarm at time i if $|Q_i| \geq h(Q_{i-1})$.

For integer j , let $q_j = P[Q_1 = j]$ and suppose the Q_i are iid. Carefully describe how to find the ARL for this situation. (You don't need to produce a formula, but you do need to set up an appropriate MC and tell me exactly/completely what to do with it in order to get the ARL.)

4. Cylinders of (outside) diameter O must fit in ring bearings of (inside) diameter I , producing clearance $C = I - O$. We would like to have some idea of the variability in actual clearances that will be obtained by "random assembly" of cylinders produced on one production line with ring bearings produced on another.

The gages used to measure I and O are (naturally enough) different.

In a study using a single gage to measure outside diameters of cylinders, $n_O = 10$ different cylinders were measured once each, producing a sample standard deviation $s_O = .001$ inch. In a subsequent

study, this same gage was used to measure the outside diameter of an additional cylinder $m_O = 5$ times, producing a sample standard deviation $s_{O\text{gage}} = .0005$ inch.

In a study using a single gage to measure inside diameters of ring bearings, $n_I = 20$ different inside diameters were measured once each, producing a sample standard deviation $s_I = .003$ inch. In a subsequent study, this same gage was used to measure the inside diameter of another ring bearing $m_I = 10$ times, producing a sample standard deviation $s_{I\text{gage}} = .001$ inch.

- (a) Give a sensible (point) estimate of the standard deviation of C produced under random assembly.
- (b) Find a sensible standard error for your estimate in (a). (No need to do the arithmetic necessary to simplify.)

5. The following is an ANOVA Table from a real Gage R&R study involving 5 parts, 4 operators and 2 measurements for each part \times operator combination.

| Source | SS | df | MS |
|--------------------------|--------|------|-------|
| Parts | 30.850 | 4 | 7.713 |
| Operators | 0.275 | 3 | 0.092 |
| Parts \times Operators | 2.350 | 12 | 0.196 |
| Error | 7.500 | 20 | 0.375 |
| Total | 40.975 | 39 | |

- (a) Give point estimates of both "repeatability" and "reproducibility" standard deviations based on the table above. Also give a standard error for the estimate of repeatability variation.
- (b) As a matter of fact, even a cursory look at the data summarized in the ANOVA table above leads one to question the appropriateness of the standard 2-way random effects analysis in this situation. The "measurements" are in fact (?rounded to?) integers. For example, the data for one of the parts were:

| Operator #1 | Operator #2 | Operator #3 | Operator #4 |
|-------------|-------------|-------------|-------------|
| 101, 101 | 102, 100 | 101, 101 | 100, 101 |

In this part of the problem consider estimating the repeatability standard deviation based only the data above, treating them as rounded values from $r = 4$ different normal distributions with possibly different means μ_i but a common standard deviation σ .

- i) Write out a prescription for $\mathcal{L}^*(\sigma)$, a profile loglikelihood for σ based on the data above. (Indicate appropriate maximization over the μ_i 's but don't try to carry that out.)
- ii) Attached to this exam is a plot of $\mathcal{L}^*(\sigma)$. (The maximum value of $\mathcal{L}^*(\sigma)$ is -12.438 , attained at $\sigma = 1.108$.) Based on this plot give both a point estimate and an approximate 90% confidence interval for σ .

1. On page 243 of V&J there is an ANOVA table for a balanced hierarchical data set. Use it in what follows.

(a) Find standard errors for the usual ANOVA estimates of σ_α^2 and σ^2 (the "casting" and "analysis" variance components).

(b) If you were to later make 100 castings, cut 4 specimens from each of these and make a single lab analysis on each specimen, give a (numerical) prediction of the overall sample variance of these future 400 measurements (based on the hierarchical random effects model and the ANOVA estimates of σ_α^2 , σ_β^2 and σ^2).

2. On page 251 of V&J there are $n = 20$ diameter measurements for some "3-inch saddles." These measurements have $\bar{x} = 3.0377$ inches and $s = .0606$ inch.

(a) Give a two-sided interval (based on a normal distribution assumption) that you are 90% sure will contain the next measured diameter of a saddle of this type.

(b) Suppose that specifications on diameters of these saddles are $3.00 \pm .20$ inches. Give an approximate 90% lower confidence bound for C_{pk} here.

(c) Exposition in class (and formulas on pages 220-221 of V&J) concerned intervals based on minimum and maximum sample values. Consider here using the second smallest sample value as a lower tolerance bound for a fraction p of measured saddle diameters. What confidence level would you associate with such a bound?

3. Consider the problem of variables acceptance sampling.

(a) The upper .002 and .001 points of the standard normal distribution are approximately 2.8782 and 3.0902 respectively. With a lower specification of $L = 0$, find a "known $\sigma = \sqrt{2}$ " variables plan that for normal observations has $Pa(.001) \approx .9$ and $Pa(.002) \approx .1$.

(b) What acceptance probability is associated with your plan from (a) when $p = .001$ if normal observations corrupted by independent (0 mean) measurement error with standard deviation .1 are actually used (in place of the "error-free" measurements implicitly assumed in standard theory).

(c) The double exponential distribution with density

$$f(x|\mu) = \frac{1}{2} \exp - |x - \mu| \text{ for all } x \in \mathcal{R}$$

has mean μ and standard deviation $\sigma = \sqrt{2}$. If, in fact, basic measurements are not normal but rather double exponential, approximately what acceptance probability should be associated with your plan from (a) when $p = .001$? (Do not include measurement error in this calculation.)

4. Here's a prescription for a possible fraction nonconforming attributes acceptance sampling plan:

- stop and reject the lot the first time that $X_n \geq 2 + \frac{n}{4}$
- stop and accept the lot the first time that $n - X_n \geq 2 + \frac{n}{4}$

(a) Find a formula for the OC for this "symmetric wedge-shaped plan." (One never samples more than 7 items and there are exactly 8 stop sampling points prescribed by the rules above.)

(b) Consider the use of this plan where lots of size $N = 100$ are subjected to rectifying inspection and inspection error is possible. (Assume that any item inspected and classified as defective is replaced with one drawn from a population that is in fact a fraction p defective and has been inspected and classified as good.) Use the parameters w_G and w_D defined in §5.2 of the notes and give a formula for the real AOQ of this plan as a function of p , w_G and w_D .

5. Consider a "perspective A" economic analysis of some fraction defective "fixed n inspection plans." (Don't simply try to use the type B calculations made in class. They aren't relevant. Work this out from first principles.)

Suppose that $N = 10$, $k_1 = 1$ and $k_2 = 10$ in a "Deming Inspection Problem" cost structure. Suppose further that a "prior" distribution for p (the actual lot fraction defective) places equal probabilities on $p = 0, .1$ and $.2$. Here we will consider only plans with $n = 0, 1$ or 2 .

(a) For $n = 1$, find the conditional distributions of p given $X =$ the number of defectives in a simple random sample from the lot.

For $n = 2$, it turns out that

the joint distribution of X and p is: and the conditionals of p given $X = x$ are:

| | | x | | | |
|-----|----|------|------|------|------|
| | | 0 | 1 | 2 | |
| p | 0 | .333 | 0 | 0 | .333 |
| | .1 | .267 | .067 | 0 | .333 |
| | .2 | .207 | .119 | .007 | .333 |
| | | .807 | .185 | .007 | |

| | | x | | |
|-----|----|------|------|------|
| | | 0 | 1 | 2 |
| p | 0 | .413 | 0 | 0 |
| | .1 | .330 | .360 | 0 |
| | .2 | .257 | .640 | 1.00 |

(b) Use your answer to (a) and show that the best $n = 1$ plan REJECTS if $X = 0$ and ACCEPTS if $X = 1$. (Yes, this is correct!) Then use the conditionals above for $n = 2$ and show that the best $n = 2$ plan REJECTS if $X = 0$ and ACCEPTS if $X = 1$ or 2 .

(c) Standard acceptance sampling plans REJECT FOR LARGE X . Explain in qualitative terms why the best plans from (b) are not of this form.

(d) Which sample size ($n = 0, 1$ or 2) is best here? (Show calculations to support your answer.)

6. A process has a Good state and a Bad state. Every morning a gremlin tosses a coin with $P(\text{Heads}) = u > .5$ that governs how states evolve day to day. Let

$$C_i = P[\text{change state on day } i \text{ from that on day } i - 1] .$$

Each C_i is either u or $1 - u$.

(a) Before the gremlin tosses the coin on day i , you get to choose whether

$$C_i = u \text{ (so that Heads } \Rightarrow \text{ change)}$$

or

$$C_i = 1 - u \text{ (so that Heads } \Rightarrow \text{ no change) .}$$

(You either apply some counter-measures or let the process evolve naturally.) Your object is to see that the process is in the Good state as often as possible. What is your optimal strategy? (What should you do on any morning i ? This needs to depend upon the state of the process from day $(i - 1)$.)

(b) If all is as described here, the evolution of the states under your optimal strategy from (a) is easily described in probabilistic terms. Do so. Then describe in rough/qualitative terms how you might monitor the sequence of states to detect the possibility that the gremlin has somehow changed the rules of process evolution on you.

(c) Now suppose that there is a one-day time delay in your counter-measures. Before the gremlin tosses his coin on day i you get to choose only whether

$$C_{i+1} = u$$

or

$$C_{i+1} = 1 - u .$$

(You do not get to choose C_i on the morning of day i .) Now what is your optimal strategy? (What you should choose on the morning of day i depends upon what you already chose on the morning of day $(i - 1)$ and whether the process was in the Good state or in the Bad state on day $(i - 1)$.) Show appropriate calculations to support your answer.