

Stat 531 Exam II

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Prof. Vardeman

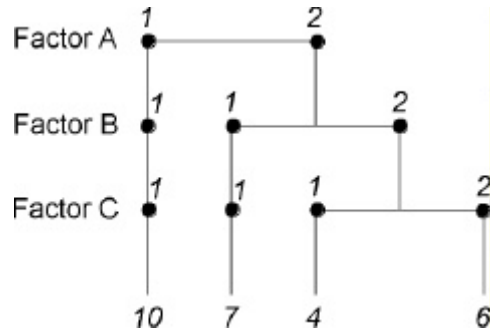
These problems will be scored at 20 points each. Start each new problem on a separate sheet of paper.

1. In the typed notes and in class, Vardeman argued that if X_1, X_2, X_3, \dots are iid with marginal density $f(x)$, then for action limits on EWMA's at $target \pm h$ and $L(u) = ARL$ beginning from u ,

$$L(u) = 1 + \frac{1}{\lambda} \int_{target-h}^{target+h} L(y) f\left(\frac{y - (1-\lambda)u}{\lambda}\right) dy .$$

Suppose now that Shewhart limits on each individual X at $target \pm k$ are added to the action limits on EWMA's. Write (in as simple a form as you can) an integral equation for $L(u)$ that could be solved to find ARLs for a combined Shewhart/EWMA process monitoring scheme.

2. Below is a schematic representation of a small (4 point) data set with unbalanced hierarchical structure.

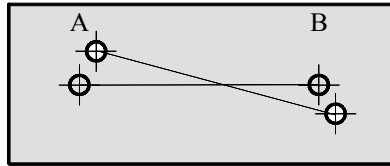


In tabular form the data are:

A	B	C	y
1	1	1	10
2	1	1	7
2	2	1	4
2	2	2	6

Assume that these data were generated as observations from the usual (normal theory) random effects model, with variance components σ_α^2 , σ_β^2 and σ^2 . Using sample variances in appropriate ways, give sensible estimates for each of these three variance components.

3. Below is a two-dimensional sketch of a mass-produced part that has 2 holes drilled in it. The ideal positions of the holes are indicated by the grey-filled circles and the actual positions (as drilled) are indicated by the empty circles.



On a coordinate system with origin at the ideal position of hole A, ideal A is at $(0, 0)$ and ideal B is at $(5, 0)$. Suppose hole A is actually positioned at (x_1, y_1) and that hole B is at actually at $(5 + x_2, y_2)$. (The numbers x_1 and x_2 are horizontal positioning errors and the numbers y_1 and y_2 are vertical positioning errors.) With

$$u = x_2 - x_1 \quad \text{and} \quad v = y_2 - y_1$$

the actual distance between the positions of holes A and B is

$$D = \sqrt{(5 + u)^2 + v^2}$$

(a) Suppose that u and v can be described as independent variables (with 0 means and) standard deviations $\sigma_u = \sigma_v = .02$. Find an approximate standard deviation for D .

The above part of this problem is based on theoretical/"what if" calculations. The rest of the problem deals with empirical data from actual manufactured parts. $n = 20$ actual parts are made and values of D determined using highly precise and accurate gauging. These have sample mean $\bar{D} = 5.0017$ and sample standard deviation $s = .0437$ and a fairly linear normal plot.

(b) Give endpoints of an interval that you are "95% sure" will contain the distances between hole positions for 99% of all such parts. (No need to simplify.)

(c) Suppose that engineering specifications on D are set at $5.0 \pm .1$. Give an approximate 90% lower confidence bound for the process capability index C_{pk} . (No need to simplify.)

4. Consider a simple MV control problem with the AR(1) process model of Problem 3.8, namely

$$Z(t) = \phi Z(t - 1) + \epsilon(t)$$

for a known constant ϕ (with absolute value less than 1), where the $\epsilon(t)$ are iid Normal $(0, \sigma^2)$ random variables. We will consider a version of the problem where the effect of a process adjustment dies out quickly. Take

$$A(a, s) = \begin{cases} a & \text{if } s = 1 \\ \frac{a}{2} & \text{if } s = 2 \\ 0 & \text{if } s \geq 3 \end{cases} .$$

(A control action is fully effective at the next period, drops by half in the ensuing one and is gone in one more period.) Assume that $T(t) = 0$ for all t .

- (a)** Find the optimal control actions $a(0)$ and $a(1)$ as functions of $Z(0)$ and $Y(1)$.
- (b)** Argue carefully that under the MV control policy, $Y(1) = \epsilon(1)$ and $Y(2) = \epsilon(2)$. (As a matter of fact, it is true in general that under MV control here, $Y(t) = \epsilon(t)$. But don't try to prove the general result.)
- (c)** Treating $Z(0)$ as fixed, the limiting mean of $Z(t)$ is 0 and the limiting variance of $Z(t)$ is fairly easily seen to be $\frac{\sigma^2}{1-\phi^2}$. In light of the general result quoted in (b), argue that the mathematics here clearly promises that MV control will improve process performance.
- (d)** In light of the general result quoted in (b), how do you suggest doing process monitoring to detect changes in this process (assuming one is doing MV control)? (Tell me what to monitor and exactly what should cause an "out of control" signal.)

5. Consider an "inspection error" modification of problem 5.16. (Continue with $N = 3$, $k_1 = 1.5$, $k_2 = 10$ and the prior distribution G assigning $P[p = .1] = .5$ and $P[p = .2] = .5$.) Suppose that in each inspection of a good item there is a probability $w_G = .2$ of inspection error and in each inspection of a defective item there is a probability $w_D = .1$ of inspection error.

Find an optimal fixed n inspection plan based on

$X =$ the number of sampled items judged to be defective

with the understanding that every item inspected and judged to be defective will be replaced (by the supplier at no additional cost) with another item from the same production run that has been inspected and judged to be good (by a similar inspection system at the supplier's facility).

To do this:

(a) First if $p = .1$ and then if $p = .2$ evaluate $P[\text{an item is actually defective} \mid \text{it is judged to be good}]$. (Hint: first for $p = .1$ and then for $p = .2$ make a two-way table giving joint probabilities of actual item condition and inspection result. In both cases, use this to find the desired conditional probability.)

(b) Next determine the two possible values of p^* governing the distribution of X . Use these to find the joint distribution for p and X for $n = 1$. For $n = 1$, use this joint distribution to find conditional distributions for p given $X = 0$ and 1 .

(c) For $n = 1$, find the optimal accept/reject decision after seeing x apparent defectives in the sample. (Note that $X = x$ gives you a conditional distribution over p . If you accept there will be $n = 1$ inspected and apparently good items and $N - n = 3 - 1 = 2$ uninspected items. If you reject there will be $N = 3$ inspected and apparently good items. Each of those possibilities has an expected cost for fixed p that should be averaged according to your conditional distribution for p given $X = x$ found in b) in order to compare accepting and rejecting.)

As it turns out, calculations like those in part (c) show that the best $n = 2$ plan always accepts (regardless of the value of X).

(d) For sample sizes $n = 0, 1, 2$ and 3 find total expected costs if the corresponding best fixed n plans are used. Which n is best? How does the overall best total expected cost here compare to the best value you found in problem 5.16? Why does this make sense?