1. There is no signal if

$$\frac{1}{\lambda}\left(\text{target} - h - (1-\lambda)u\right) < X < \frac{1}{\lambda}\left(\text{target} + h - (1-\lambda)u\right)$$

and $\quad \text{target} - k < X < \text{target} + k$

This is that there is no signal if

$$\underbrace{\max\left(\frac{1}{\lambda}(\text{target}-h-(1-\lambda)u), \text{target}-k\right)}_{*} < X < \underbrace{\min\left(\frac{1}{\lambda}(\text{target}+h-(1-\lambda)u), \text{target}+k\right)}_{**}$$

So as in class

$$L(u) = 1 + \int_{*}^{**} L(\lambda t + (1-\lambda)u) f(t)\,dt$$

$y = \lambda t + (1-\lambda)u$

$$= 1 + \frac{1}{\lambda}\int_{***}^{****} L(y) f\left(\frac{y-(1-\lambda)u}{\lambda}\right) dy$$

for $\quad *** = \max\left(\text{target} - h, \lambda(\text{target}-k) + (1-\lambda)u\right)$

$\quad\quad **** = \min\left(\text{target} + h, \lambda(\text{target}+k) + (1-\lambda)u\right)$

2. y_{ijk} = observation at level i of A, j of B within level i of A, k of C within j of B within i of A

Use the same notation as in class,

$$s_{22}^2 = \frac{1}{2-1}\left((4-5)^2 + (6-5)^2\right) = 2 \quad \text{estimates } \sigma^2$$

$$\bar{y}_{21} = 7 \quad \bar{y}_{22} = 5 \quad s_2^2 = \frac{1}{2-1}\left((7-6)^2 + (5-6)^2\right)$$
$$= 2$$

$$\bar{y}_{21} = \mu + \alpha_2 + \beta_{21} + \epsilon_{211}$$
$$\bar{y}_{22} = \mu + \alpha_2 + \beta_{22} + \bar{\epsilon}_{22}$$

<u>So</u> $\quad E s_2^2 = \frac{1}{2}\left(\sigma_\beta^2 + \sigma^2 + \sigma_\beta^2 + \frac{\sigma^2}{2}\right) = \sigma_\beta^2 + \frac{3\sigma^2}{4}$

and $\therefore \quad E\left(s_2^2 - \frac{3}{4}s_{22}^2\right) = \sigma_\beta^2$

So a sensible estimate of σ_β^2 is $s_2^2 - \frac{3}{4}s_{22}^2 = \frac{1}{2}$ here

Finally, $\quad \bar{y}_1 = 10$
$$\bar{y}_2 = \frac{1}{2}\left(\bar{y}_{21} + \bar{y}_{22}\right) = \frac{1}{2}(7+5) = 6$$

and $s^2 = \frac{1}{2-1}\left((10-8)^2 + (6-8)^2\right) = 8$

$$\bar{y}_1 = \mu + \alpha_1 + \beta_{11} + \epsilon_{111}$$
$$\bar{y}_2 = \mu + \alpha_2 + \underbrace{\bar{\beta}_2}_{\text{average of 2}} + \frac{1}{2}\big(\epsilon_{212} + \underbrace{\bar{\epsilon}_{22}}_{\text{average of 2}}\big)$$

So $\quad Es^2 = \frac{1}{2}\left(\sigma_\alpha + \sigma_\beta^2 + \sigma^2 + \sigma_\alpha^2 + \frac{\sigma_\beta^2}{2} + \frac{1}{4}\left(\sigma^2 + \frac{\sigma^2}{2}\right)\right)$
$$= \sigma_\alpha^2 + \frac{3}{4}\sigma_\beta^2 + \frac{1}{2}\left(1 + \frac{3}{8}\right)\sigma^2$$
$$= \sigma_\alpha^2 + \frac{3}{4}\sigma_\beta^2 + \frac{11}{16}\sigma^2$$

$\therefore \quad E\left(s^2 - \frac{3}{4}\left(s_2^2 - \frac{3}{4}s_{22}^2\right) - \frac{11}{16}s_{22}^2\right) = \sigma_\alpha^2$

So a sensible estimate is
$$8 - \frac{3}{4}\left(\frac{1}{2}\right) - \frac{11}{16}2 = 8 - \frac{28}{16} = 6.25$$

3 a) $\frac{\partial D}{\partial u} = \frac{\frac{1}{2}(2)(5+u)}{\sqrt{(5+u)^2+v^2}}$ and $\frac{\partial D}{\partial v} = \frac{\frac{1}{2}(2)v}{\sqrt{(5+u)^2+v^2}}$

8pts

So at $(u,v) = 0$ $\quad \frac{\partial D}{\partial u} = 1$ and $\frac{\partial D}{\partial v} = 0$

Thus $\sigma_D^2 \approx (1)^2(.02)^2 + (0)^2(.02)^2$ and $\sigma_D \approx .02$

6pts b) Use $\bar{x} \pm \tau_2 s$ i.e. $5.0017 \pm 3.621(.0437)$

6pts c) $\hat{C}_{pk} = \min\left(\frac{5.1-5.0017}{3(.0437)}, \frac{5.0017-4.9}{3(.0437)}\right) = .7498$

and the lower confidence bound is thus

$$.7498 - 1.282\sqrt{\frac{1}{9(20)} + \frac{(.7498)^2}{2(20)-2}} = .567$$

4 a) $a(0) = -\hat{z}(1|0) = -\phi z(0)$

7pts $Y(1) = z(1) + A(a(0),1) = z(1) - \phi z(0) = \epsilon(1)$

$z(1) = Y(1) - (-\phi z(0)) = Y(1) + \phi z(0)$

$$\begin{aligned} a(1) &= -\left[\hat{z}(2|1) + A(a(0),2)\right] \\ &= -\left[\phi z(1) - \tfrac{1}{2}\phi z(0)\right] \\ &= -\left[\phi Y(1) + \phi^2 z(0) - \tfrac{1}{2}\phi z(0)\right] \\ &= -\phi Y(1) - \left(\phi^2 - \tfrac{\phi}{2}\right) z(0) \end{aligned}$$

b) $Y(1)$ is above

7pts $$\begin{aligned} Y(2) &= z(2) + A(a(0),2) + A(a(1),1) \\ &= z(2) - \phi Y(1) - \left(\phi^2 - \tfrac{\phi}{2}\right)z(0) - \tfrac{1}{2}\phi z(0) \\ &= z(2) - \phi Y(1) - \phi^2 z(0) \\ &= \phi z(1) + \epsilon(2) - \phi Y(1) - \phi^2 z(0) \\ &= \phi\left(z(1) - Y(1)\right) + \epsilon(2) - \phi^2 z(0) \\ &= \phi^2 z(0) + \epsilon(2) - \phi^2 z(0) = \epsilon(2) \end{aligned}$$

c) $E(Z(t))^2 = (E\,Z(t))^2 + \mathrm{Var}\,Z(t) \rightarrow \frac{\sigma^2}{1-\phi^2}$

3pts And $E(Y(t))^2 = \mathrm{Var}\,\epsilon(t) = \sigma^2 < \frac{\sigma^2}{1-\phi^2}$

$Y(t)$ is then preferable to $Z(t)$ (as hitting the target of $T=0$).

d) Use control limits of $\pm 3\sigma$ for $Y(t)$ in standard "Shewhart monitoring" fashion.
3pts

5. a) $p=.1$

		Judged G	Judged D	
Actual	G	.72	.18	.9
	D	.01	.09	.1
		.73	.27	

$P[D \mid \text{judged } G] = \frac{1}{73}$

$p=.2$

		Judged G	Judged D	
Actual	G	.64	.16	.8
	D	.02	.18	.2
		.66	.34	

$P[D \mid \text{judged } G] = \frac{2}{66}$

b) $p=.1 \Rightarrow p^* = .27$ $\quad p=.2 \Rightarrow p^* = .34$

p \ x	0	1	
.1	.5(.73) = .365	(.5)(.27) = .135	.5
.2	.5(.66) = .330	.5(.34) = .170	.5
	.695	.305	

← joint dsn of p and x

so conditionals of p given X are

p	$f(p\|0)$
.1	$\frac{365}{695} = .5252$
.2	$\frac{330}{695} = .4748$

p	$f(p\|1)$
.1	$\frac{135}{305} = .4426$
.2	$\frac{170}{305} = .5574$

c) $\underline{x=0}$

with acceptance, the conditional expected total cost is

$$1.5 + 10\left(.5252\left(\tfrac{1}{73}\right) + .4748\left(\tfrac{2}{66}\right)\right)$$
$$+ \ 2(10)\left(.1(.5252) + .2(.4748)\right) = 4.6654$$

with rejection, the conditional expected total cost is

$$3\left(1.5 + 10\left(.5252\left(\tfrac{1}{73}\right) + .4748\left(\tfrac{2}{66}\right)\right)\right) = 5.1475$$

so for $x=0$ the optimal decision is acceptance

$\underline{x=1}$

with acceptance the conditional expected total cost is

$$1.5 + 10\left(.4426\left(\tfrac{1}{73}\right) + .5774\left(\tfrac{2}{66}\right)\right)$$
$$+ \ 2(10)\left(.1(.4426) + .2(.5574)\right) = 4.8443$$

with rejection the conditional expected total cost is

$$3\left(1.5 + 10\left(.4426\left(\tfrac{1}{73}\right) + .5574\left(\tfrac{2}{66}\right)\right)\right) = 5.1886$$

so for $x=1$ the optimal decision is acceptance

d) $n=0$ has expected total cost

$$.5(10)(3)(.1) + .5(10)(3)(.2) = 4.5$$

$n=1$ (with acceptance number 1) has expected total cost

$$.5\left(1.5 + 10\left(\tfrac{1}{73}\right) + 10(2)(.1)\right)$$
$$+ .5\left(1.5 + 10\left(\tfrac{2}{66}\right) + 10(2)(.2)\right) = 4.72$$

$n=2$ (with acceptance number 2) has expected total cost

$$.5\left(3.0 + 2(10)\left(\tfrac{1}{73}\right) + 10(.1)\right)$$
$$+ .5\left(3.0 + 2(10)\left(\tfrac{2}{66}\right) + 10(.2)\right) = 4.94$$

<u>$n=3$</u> has expected total cost

$$.5\left(4.5+3(10)\left(\frac{1}{73}\right)\right)+.5\left(4.5+3(10)\left(\frac{2}{66}\right)\right)=5.1600$$

$n=0$ is best. Without inspection error, $n=1$ is best. The optimal expected cost here exceeds the perfect inspection optimal cost. Inspection error both increases costs (as information degrades) and makes it less attractive to do any inspection in the first place.