

Exam 1 Ground Rules:

- 1) Open book, open notes, you may use any references you like (including your own homework).
- 2) Two hours may be used for the exam.
- 3) Write on 4 questions (all of 1 through 3 and either #4 or #5). (If you write on both #4 and #5 I will score your best answer of the two.)

Stat/IE 531 Exam 1

March 22, 2001

Prof. Vardeman

Do problems 1-3 and then **one of problems 4 and 5**. All problems will be scored on a 25 point basis.

1. A manufacturer believes that real widget diameters produced in individual production runs are approximately normally distributed. However, the gage used to measure these is fairly "crude" relative to part-to-part variation. A sample of $n = 8$ widgets from one production period have diameters measured to the nearest .001 inch

1.001, 1.002, 1.002, 1.002, 1.001, 1.002, 1.001, 1.002

- a) Give approximate 95% confidence limits for the mean diameter during this period of production.
- b) Give approximate 95% confidence limits for the standard deviation of diameters during this period of production.

2. A company uses a 2.88-sigma Shewhart \bar{x} chart with $UCL_{\bar{x}} = 10.5$ and $LCL_{\bar{x}} = 9.5$.

- a) Design a two-sided CUSUM scheme for \bar{x} s to have "all-OK" behavior comparable to the current \bar{x} chart, while providing quickest possible detection of a shift in process mean from 10 to 10.5208 or 9.4792. (Give values of k_1 , k_2 , U_0 , L_0 and h .)
- b) What ARLs do the current \bar{x} chart and your new CUSUM chart provide if in fact the process mean is 10.1302? (Assume that the process standard deviation remains at its "standard" value.)

3. A chemical producer can make both fast/cheap/imprecise in-plant measurements and slow/expensive/precise laboratory measurements. In an effort to monitor the performance of a particular in-plant sensor, periodically a sample of product is split in two and measured both in the plant and in the laboratory, producing

$$x = \text{the laboratory measurement}$$

and

$$y = \text{the in-plant measurement} .$$

Attached to this exam is a JMP printout of a SLR analysis of $n = 15$ data pairs (x, y) . (In addition to the least squares line through the data, 95% prediction limits for a single additional y at x 's ranging from about 500 to about 700 are shown.) Suppose that these data were obtained during a period when "all was well" both with the chemical production process and with the measurement system.

a) If a future in-plant measurement is $y = 500$, what would you report for a 95% confidence interval for the corresponding lab measurement?

b) One might monitor future (x, y) pairs on the basis of

$$X^2 = (x - 605.2, y - 584.6) \begin{pmatrix} (45.72)^2 & .975(45.72)(45.81) \\ .975(45.72)(45.81) & (45.81)^2 \end{pmatrix}^{-1} \begin{pmatrix} x - 605.2 \\ y - 584.6 \end{pmatrix}$$

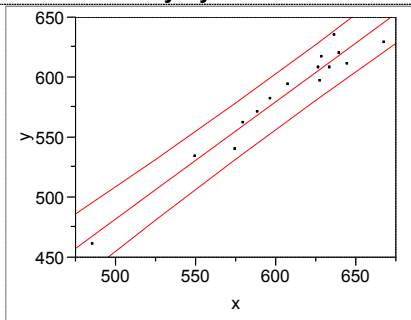
or on the basis of

$$e = y - (-7.279 + .978x) .$$

(The constants in the formula for X^2 are appropriate sample means, standard deviations and correlation between x and y .)

- i) What control limits do you suggest for these two possible monitoring schemes?
- ii) If the object is monitoring of the plant measuring system (not the production system) would you monitor X^2 or would you monitor e ? Choose one and defend your choice.

Bivariate Fit of y By x



Linear Fit

$$y = -7.278586 + 0.9779884 x$$

Summary of Fit

RSquare	0.952841
RSquare Adj	0.949213
Root Mean Square Error	10.32331
Mean of Response	584.6
Observations (or Sum Wgts)	15

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	27992.180	27992.2	262.6628
Error	13	1385.420	106.6	Prob > F
C. Total	14	29377.600		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	-7.278586	36.61736	-0.20	0.8455
x	0.9779884	0.060344	16.21	<.0001

4. A company (sadly) uses sample ranges instead of sample standard deviations in both its gage studies and its process monitoring efforts. You are furnished with the following information by this company:

- i) A gage R&R study for measuring certain widget diameters used $I = 10$ parts, $J = 3$ operators and $m = 3$ measurements of each part by each operator. From this study, $\hat{\sigma}_{\text{repeatability}} = .002$ inch.
 - ii) Process monitoring records based on measurements from the gage referred to in i) are available. Over $r = 40$ periods, there was no evidence of process instability in samples of size $n = 5$. Those 40 samples produced ranges R with average $\bar{R} = .010$ inch. (All process monitoring measurements were made by the same operator.)
- a)** Give an estimate of a "part-to-part" standard deviation for actual widget diameters produced during the period of process monitoring. (This figure should not include measurement variability.)
- b)** Give a standard error (an estimated standard deviation) for your value from a). (Plug in numbers, but you don't need to do arithmetic to simplify.)

5. An ARL is a "mean number of 'periods' to alarm." The "clock time" lengths of those periods are unspecified. Where the periods are of equal length, an ARL is proportional to "mean clock time to alarm." In this question we consider a scheme (like the so-called variable sampling interval charts of Reynolds) where "periods" need not be of equal length. For sake of discussion, consider monitoring based on a sample mean, \bar{x} .

At times $t = 1, 2, 3, \dots$ hours after start of production, one observes \bar{x} and

- i) signals an alarm if \bar{x} is outside k -sigma limits
- ii) goes on "alert status" if \bar{x} is inside k -sigma limits but outside k' -sigma limits (where $k' < k$).

When placed on "alert status" at time t hours after start of production, at time $t + .5$ hours after start of production, a new \bar{x} is observed and one

- i) signals an alarm if \bar{x} is outside k'' -sigma limits
- ii) goes off "alert status" and back to "regular monitoring" if \bar{x} is inside k'' -sigma limits.

For iid sample means \bar{x} use notation:

$$q_1 = P[\bar{x} \text{ is outside } k\text{-sigma limits}]$$

$$q_2 = P[\bar{x} \text{ is inside } k\text{-sigma limits but outside } k'\text{-sigma limits}]$$

and

$$q_3 = P[\bar{x} \text{ is outside } k''\text{-sigma limits}] \ .$$

Carefully describe how to find the *mean clock time to alarm*. (If you refer to some MC, show me the transition matrix and be explicit about what you are going to do with anything calculated using it.)

(Hint: Consider modeling where "one period" is "one half hour.")