1. This sample has range 1 in coded units. (the coded sample is \{1, 2, 2, 1, 2, 1, 2, 1\}) So we'll use Tables 1.3 and 1.6 of the notes.

\( n_1 = 3, n_{21} = 5, \; m = 5. \) For \( n = 8 \) and \( \sigma = 5 \)

Table 1.3 shows that \( \Delta_1 = .527 \) and \( \Delta_2 = .251 \).

Then \( \Delta_1 = .281 \) and \( \Delta_2 = .527 \)

and in coded units the interval is

\[(1.5 - .281, 1.5 + .527)\]

\[(1.219, 2.027)\]

In original units this is

\[(1.001219, 1.002027)\] inches

2. \( P[ |\bar{X} - \mu| > 2.88 \sigma_{\bar{X}}] = P[ |\bar{X}| > 2.88] = 2 (1 - .9580) = .004 \)

So the Showhart all-ok ARL is \( \frac{1}{.004} = 250 \)
13. (a) \[ k_1 = \frac{1}{2} (10 + 10.5208) = 10.2604 \]
\[ k_2 = \frac{1}{2} (10 + 9.7392) = 9.7396 \]

So \[ g = \frac{10.2604 - 10}{(5)/2.88} = 1.5 \]
and consulting Table 4.6, we see we want \[ g = 1.47 \] and then
\[ h = 1.47 \left( \frac{5}{2.88} \right) = 2.552 \]
with \( U_0 = L_0 = 0 \)

13. (b) If the current process mean is 10.5208 (and
\( \sigma = 0.5/2.88 \))

\[
P[ \text{\textit{X} plots outside Shewhart limits}] = P[Z > \frac{10.5 - 10.1302}{0.5/2.88}] + P[Z < \frac{9.5 - 10.1302}{0.5/2.88}] = P[Z > 2.13] + P[Z < -3.63] \approx 0.0166
\]

So Shewhart ARL is about \( \frac{1}{0.0166} \approx 60 \)

For the CUSUM, (using 4.15), (4.19) and (4.18)

\[ k^* = \frac{A}{\sigma} = 1.47 \left( \frac{5}{2.88} \right) = 1.47 \]

\[ c^* = \frac{10.2604 - 9.7396}{2 \left( \frac{5}{2.88} \right)} = 1.5 \]

\[ \bar{c}^* = \frac{10.1302 - 10}{5/2.88} = 0.75 \]

ARL is around (less than) 42. From Table 4.5
b) i) \[ UCL_x = p + 3\sqrt{2p} = 2 + 3\sqrt{4} = 8 \]

Roughly, we expect \( \bar{e} = 0 \) and

\[ \text{Var } e = \left( \sigma_e \right)^2 = \text{from SLR model} \]

(This ignores the fact that \( b_i \)'s aren't \( \beta_i \)'s). The MSE from SLR, estimates \( \sigma_e^2 \). So sensible 3-sigma limits for \( e_i \)'s are

\[ UCL_e = 0 + 3\sqrt{\text{MSE}} = 3\sqrt{106.6} 
\]

\[ LCL_e = -3\sqrt{\text{MSE}} = -3 \]

ii) I'd probably monitor \( e_i \). \( e_i \)'s will be large (in magnitude), only if the relationship between \( x \) and \( y \) changes. \( \chi^2 \) could also be large by virtue of \( e_i \)s also producing a change in the plan of \( x_i \)'s).

4. a) Using the basic measurement model, the "right" measurement is repeatability and

\[ \sigma_y^2 = \sigma_x^2 + \sigma_{\text{measurement}}^2 \]

so

\[ \sigma_x = \sqrt{\sigma_y^2 - \sigma_{\text{measurement}}^2} = \text{part-to-part standard deviation} \]

Further, an obvious estimate is

\[ \hat{\sigma}_x = \sqrt{\left( \frac{\bar{R}}{d_2(s)} \right)^2 - \left( \hat{\sigma}_{\text{repeatability}} \right)^2} \]

\[ = \sqrt{\left( \frac{.010}{2.326} \right)^2 - (.002)^2} = .0038 \text{ inch} \]
b) \( \hat{\sigma}_y = \frac{\bar{R}}{2.326} \) and \( \hat{\sigma}_{\text{Repeatability}} \) are plausibly modeled as independent. So the delta method gives

\[
\text{Var} \left( \frac{\bar{R}}{J} \right) = \left( \frac{\hat{\sigma}_y}{J \sigma_y} \right)^2 \text{Var} \left( \frac{\bar{R}}{J} \right) + \left( \frac{\hat{\sigma}_{\text{Repeatability}}}{J \sigma_y} \right)^2 \text{Var} \left( \frac{\bar{R}}{J} \right)
\]

\[
= \left( \frac{\hat{\sigma}_y}{J \sigma_y} \right)^2 \left( \frac{1}{2.326} \right)^2 \left( \frac{1}{40} \right) \left( \frac{1.693}{30} \right) \text{Var} \left( \frac{\bar{R}}{J} \right)
\]

From one sample

\[
= \left[ \frac{\hat{\sigma}_y^4}{J \sigma_y^2} \left( \frac{.864}{2.326} \right)^2 \frac{1}{40} + \hat{\sigma}_{\text{Repeatability}}^4 \left( \frac{1.693}{30} \right) \right]
\]

So

\[
\sqrt{\text{Var} \left( \frac{\bar{R}}{J} \right)} = \frac{1}{J \sigma_y} \sqrt{\hat{\sigma}_y^4 \left( \frac{.864}{2.326} \right)^2 \frac{1}{40} + \hat{\sigma}_{\text{Repeatability}}^4 \left( \frac{1.693}{30} \right) ^2 \frac{1}{30} }
\]

To produce a standard error, replace the \( \hat{\sigma} \)'s with estimates, namely

\( \hat{\sigma}_y = .0038 \)

\( \hat{\sigma}_{\text{Repeatability}} = .002 \)

and

\( \hat{\sigma}_y = \frac{.010}{2.326} = .0043 \)

Arithmetic then gives a standard error of .0003 inch.
5. As suggested, consider half-hour periods and the MC with transition diagram below:

Then the mean number of steps to absorption from $S_1$ (or $S_4$) is the mean number of half hours required to produce an alarm. That is, with

$$P = \begin{pmatrix}
S_1 & S_2 & S_3 & S_4 & S_5 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & q_2 & 1-q_2 & q_{12} \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}$$

Then with the usual $L = (I - R)^{-1} 1$, the mean clock time to alarm is

$$\frac{1}{2} L_1 = \frac{1}{2} L_4 \left( \frac{2 + q_2 q_{12}}{2(q_1 + q_2 q_{12})} \right)$$