STAT 511 HW#6 SPRING 2009

PROBLEM 1:

```
y<-c(6.0,6.1,8.6,7.1,6.5,7.4,9.4,9.9,9.5,7.5,6.4,9.1,8.7)
A<-c(1,1,1,1,1,1,2,2,2,2,2,2,2)
B<-c(1,1,2,2,2,2,1,1,2,2,3,3,3)
options(contrasts=c("contr.sum","contr.sum"))
a)
```

The pooled variance from the 5 samples (of sizes 2,4,2,2, and 3) is 1.089(standard error =1.044). An exact 95% confidence interval for σ is $\left(\sqrt{\frac{1.089*8}{\chi^2_{0.975,60}}}, \sqrt{\frac{1.089*8}{\chi^2_{0.025,60}}}\right)$=(0.705,2)

```
##Using lme
A<-as.factor(A)
B<-as.factor(B)
lme.out<-lme(y~1,random=~1|A/B)
summary(lme.out)
      Linear mixed-effects model fit by REML
       Data: NULL
             AIC      BIC    logLik
        49.46496 51.40458 -20.73248
      Random effects:
       Formula: ~1 | A
              (Intercept)
      StdDev:    1.165728
       Formula: ~1 | B %in% A
              (Intercept) Residual
      StdDev:   0.4955114 1.056588
      Fixed effects: y ~ 1
                    Value Std.Error DF  t-value p-value
      (Intercept) 7.79261 0.9051327  8 8.609356       0
      Standardized Within-Group Residuals:
             Min          Q1         Med          Q3         Max
      -1.8490691 -0.6716102  0.1801882  0.7063261  1.3159194
      Number of Observations: 13
      Number of Groups:
             A B %in% A
             2        5
intervals(lme.out)
      Approximate 95% confidence intervals
       Fixed effects:
                    lower    est.   upper
      (Intercept) 5.70537 7.79261 9.87985
      Random Effects:
        Level: A
                          lower     est.    upper
      sd((Intercept)) 0.2184694 1.165728 6.220191
        Level: B
                           lower      est.    upper
      sd((Intercept)) 0.03991159 0.4955114 6.151885
       Within-group standard error:
          lower      est.     upper
      0.6391959 1.0565880 1.7465355
```

An approximate 95% confidence interval for σ is (0.6391959 , 1.7465355)

```
##Using lmer
lmer.out<- lmer(y ~ (1|A) + (1|A:B))
summary(lmer.out)
      Linear mixed model fit by REML
      Formula: y ~ (1 | A) + (1 | A:B)
         AIC   BIC logLik deviance REMLdev
       49.46 51.72 -20.73    43.06   41.46
      Random effects:
       Groups   Name        Variance Std.Dev.
       A:B      (Intercept) 0.24553  0.49551
       A        (Intercept) 1.35892  1.16573
       Residual             1.11638  1.05659
      Number of obs: 13, groups: A:B, 5; A, 2

      Fixed effects:
                  Estimate Std. Error t value
      (Intercept)   7.7926     0.9051    8.61
sims <- mcmcsamp(lmer.out, 50000)
HPDinterval(sims)
      $fixef
                     lower    upper
      (Intercept) 5.006724 10.53065
      $ST
           lower    upper
      [1,]     0 0.992304
      [2,]     0 3.311172
      $sigma
               lower    upper
      [1,] 0.7268979 1.814918
```

b)

```
AA<-as.factor(A)
BB<-as.factor(B)
lm.out<-lm(y~1+AA/BB)
summary(lm.out)
      Call:
      lm(formula = y ~ 1 + AA/BB)

      Residuals:
             Min         1Q     Median         3Q        Max
      -1.667e+00 -3.000e-01  4.612e-16  6.333e-01  1.200e+00

      Coefficients: (1 not defined because of singularities)
                  Estimate Std. Error t value Pr(>|t|)
      (Intercept)   8.0694     0.3293  24.503 8.22e-09 ***
      AA1          -0.6694     0.3293  -2.033   0.0765 .
      AA1:BB1      -1.3500     0.9040  -1.493   0.1737
      AA2:BB1       0.9111     0.5857   1.556   0.1584
      AA1:BB2           NA         NA      NA       NA
      AA2:BB2      -0.2389     0.5857  -0.408   0.6941
      Residual standard error: 1.044 on 8 degrees of freedom
      Multiple R-squared: 0.6286,     Adjusted R-squared: 0.4429
      F-statistic: 3.385 on 4 and 8 DF,  p-value: 0.06686
```

Note that the estimate of σ (residual standard error = 1.044) produced by lm.out is exactly the one based on the mean square error referred in part a).

c)
The predictions for the fixed effects model correspond to the cell sample means $\bar{y}_{ij.}$

```
round(predict(lm.out),3)
    1      2      3      4      5      6      7      8      9     10     11     12     13 
6.050  6.050  7.400  7.400  7.400  7.400  9.650  9.650  8.500  8.500  8.067  8.067  8.067 
round(predict(lme.out),3)
  1/1    1/1    1/2    1/2     ½    1/2    2/1    2/1    2/2    2/2    2/3    2/3    2/3 
6.739  6.739  7.210  7.210  7.210  7.210  8.881  8.881  8.530  8.530  8.354  8.354  8.354 
random.effects(lme.out)
      Level: A
        (Intercept)
      1  -0.7504841
      2   0.7504841
      Level: B %in% A
          (Intercept)
      1/1 -0.30308804
      1/2  0.16748966
      2/1  0.33815279
      2/2 -0.01316488
      2/3 -0.18938953
round(fitted(lmer.out),3)
7.446 7.446 7.609 7.609 7.609 7.609 8.632 8.632 8.664 8.664 7.814 7.814 7.814
ranef(lmer.out)
      $`A:B`
          (Intercept)
      1:1 -0.17374831
      1:2  0.01033243
      2:1 -0.16270189
      2:2 -0.12574233
      2:3 -1.08660931
      $A
        (Intercept)
      1  -0.9928770
      2   0.3693292
```

PROBLEM 2

The weights of the (75) pieces of silver were determined and a standard hierarchical (balanced data) ANOVA table was produced as below. (The units of weight were not given in the original article.)

Term	Source	SS	df	E(MS)
A	Machines	1966	2	$\sigma^2 + 5\sigma_\beta^2 + 25\sigma_\alpha^2$
B(A)	Rolls	644	12	$\sigma^2 + 5\sigma_\beta^2$
Error	Pieces	280	60	σ^2
	Total	2890	74	

a)

- $\hat{\sigma}^2 = MSE = 4.667$

  A 95% confidence interval for σ is $\left(\sqrt{\frac{SSE}{\chi^2_{0.975,60}}}, \sqrt{\frac{SSE}{\chi^2_{0.025,60}}}\right) = (1.833, 2.630)$

- $\hat{\sigma}_\beta^2 = \frac{MSB(A) - MSE}{5} = 9.8$

  $$\hat{v} = \frac{\left(\hat{\sigma}_\beta^2\right)^2}{\frac{(MSB(A)/5)^2}{12} + \frac{(MSE/5)^2}{60}} = 9.989$$

A 95% confidence interval for σ_β is $\left(\sqrt{\frac{\hat{\nu}\hat{\sigma}_\beta^2}{\chi^2_{0.975,\hat{\nu}}}}, \sqrt{\frac{\hat{\nu}\hat{\sigma}_\beta^2}{\chi^2_{0.025,\hat{\nu}}}}\right) = (2.186, 5.501)$

- $\hat{\sigma}_\alpha^2 = \frac{MSA - MSB(A)}{5\times 5} = 37.173$

  $$\hat{\nu} = \frac{(\hat{\sigma}_\alpha^2)^2}{\frac{(MSA/25)^2}{2} + \frac{(MSB(A)/25)^2}{12}} = 1.787$$

  A 95% confidence interval for σ_α is $\left(\sqrt{\frac{\hat{\nu}\hat{\sigma}_\alpha^2}{\chi^2_{0.975,\hat{\nu}}}}, \sqrt{\frac{\hat{\nu}\hat{\sigma}_\alpha^2}{\chi^2_{0.025,\hat{\nu}}}}\right) = (3.098, 46.298)$

- The largest part of the variation comes from the difference between machines

b) A 95% confidence interval for μ is $\bar{y}_{\dots} \pm t_{0.975,2}\sqrt{\frac{MSA}{3\times 5\times 5}} = (19.42, 50.58)$

PROBLEM 3

```
Y<-c(12,13,14,20,8,10,6,7,8,10,11,13,7)
a<-c(1,1,1,1,2,2,2,2,2,3,3,3,3)
b<-c(1,2,2,3,1,2,3,3,3,1,1,2,3)
group<-rep(1,length(Y))
A<-as.factor(a)
B<-as.factor(b)
Group<-as.factor(group)
Fake<-data.frame(Y,A,B,Group)
FakeGrouped<-groupedData(Y~1|group,Fake)
##Using lme
twoway.out<-lme(Y~1,data=FakeGrouped,random=pdBlocked(list(pdIdent(~A-
1),pdIdent(~B-1))))
summary(twoway.out)
      Linear mixed-effects model fit by REML
       Data: FakeGrouped
             AIC       BIC    logLik
        71.49413 73.43375 -31.74706
      Random effects:
       Composite Structure: Blocked
       Block 1: A1, A2, A3
       Formula: ~A - 1 | group
       Structure: Multiple of an Identity
                    A1       A2       A3
      StdDev: 3.304856 3.304856 3.304856
       Block 2: B1, B2, B3
       Formula: ~B - 1 | group
       Structure: Multiple of an Identity
                        B1           B2           B3 Residual
      StdDev: 7.969662e-05 7.969662e-05 7.969662e-05 2.574438
      Fixed effects: Y ~ 1
                     Value Std.Error DF  t-value p-value
      (Intercept) 10.90533   2.03851 12 5.349656   2e-04
      Standardized Within-Group Residuals:
              Min          Q1         Med          Q3         Max
      -1.29594152 -0.48304590 -0.09461158  0.25779573  2.23599429
      Number of Observations: 13
      Number of Groups: 1
```

```
##Using lmer
twoway.out2<-lmer(Y ~ (1 | A) + (1| B))
summary(twoway.out2)
      Linear mixed model fit by REML
      Formula: Y ~ (1 | A) + (1 | B)
        AIC   BIC logLik deviance REMLdev
       71.5 73.75 -31.75    66.72    63.5
      Random effects:
       Groups   Name        Variance   Std.Dev.
       A        (Intercept) 1.0922e+01 3.3049e+00
       B        (Intercept) 4.3566e-20 2.0872e-10
       Residual             6.6277e+00 2.5744e+00
      Number of obs: 13, groups: A, 3; B, 3

      Fixed effects:
                  Estimate Std. Error t value
      (Intercept)   10.905      2.039    5.35
sims<-mcmcsamp(twoway.out2,50000)
HPDinterval(sims,prob=.95)
     $fixef
                    lower    upper
     (Intercept) 6.623834 15.33435
     $ST
          lower    upper
     [1,]     0 1.812893
     [2,]     0 1.235887
     $sigma
             lower    upper
     [1,] 1.793329 4.805657
```

PROBLEM 4

a)

```
Y <-
c(13.5,14.8,10.5,11.7,12.9,12.0,8.8,13.5,12.4,16.0,11.3,12.0,14.0,12.5,13.0,13.1
,12.6,12.7,11.0,10.6)
a <- c(rep(1,10),rep(2,10))
b <- c(rep(c(1,1,2,2,3,3,4,4,5,5),2))
A <- as.factor(a)
B <- as.factor(b)
fitlm.out <- lm(Y~A*B)
anova(fitlm.out)
      Analysis of Variance Table

      Response: Y
                Df  Sum Sq Mean Sq F value  Pr(>F)
      A          1  0.5445  0.5445  0.2598 0.62129
      B          4  2.6920  0.6730  0.3212 0.85751
      A:B        4 24.4980  6.1245  2.9227 0.07695 .
      Residuals 10 20.9550  2.0955
```

The sum squares presented by Vardeman and Jobe are correct.

b) A 95% confidence interval for σ is $\left(\sqrt{\frac{SSE}{\chi^2_{0.975,10}}}, \sqrt{\frac{SSE}{\chi^2_{0.025,10}}}\right) = (1.011, 2.540)$

c) $\hat{\sigma}_{\beta}^2 + \hat{\sigma}_{\alpha\beta}^2 = \frac{MSB+MSAB-2MSE}{4} = 0.652$

$$\hat{v} = \frac{\left(\hat{\sigma}_{\beta}^2 + \hat{\sigma}_{\alpha\beta}^2\right)^2}{\frac{(MSB/4)^2}{4} + \frac{(MSAB/4)^2}{4} + \frac{(MSE/2)^2}{10}} = 0.604$$

A 95% confidence interval for $\sqrt{\hat{\sigma}_{\beta}^2 + \hat{\sigma}_{\alpha\beta}^2}$ is $\left(\sqrt{\frac{\hat{v}\left(\hat{\sigma}_{\beta}^2+\hat{\sigma}_{\alpha\beta}^2\right)}{\chi^2_{0.975,\hat{v}}}}, \sqrt{\frac{\hat{v}\left(\hat{\sigma}_{\beta}^2+\hat{\sigma}_{\alpha\beta}^2\right)}{\chi^2_{0.025,\hat{v}}}}\right) = (0.321, 238.470)$

d)

```
##Using lme
ab<-c(1,1,2,2,3,3,4,4,5,5,6,6,7,7,8,8,9,9,10,10)
group<-c(rep(1,20))
AB<-as.factor(ab)
Group<-as.factor(group)
Gauge<-data.frame(Y,A,B,AB,Group)
GaugeGrouped<-groupedData(Y~1|Group,Gauge)
fitlme.out<-
lme(Y~1,data=GaugeGrouped,random=list(Group=pdBlocked(list(pdIdent(~A-
1),pdIdent(~B-1),pdIdent(~AB-1)))))
Linear mixed-effects model fit by REML
       Data: GaugeGrouped
             AIC      BIC    logLik
        84.44239 89.16459 -37.22120
      Random effects:
       Composite Structure: Blocked
       Block 1: A1, A2
       Formula: ~A - 1 | Group
       Structure: Multiple of an Identity
                        A1           A2
      StdDev: 0.0001327714 0.0001327714
       Block 2: B1, B2, B3, B4, B5
       Formula: ~B - 1 | Group
       Structure: Multiple of an Identity
                        B1           B2           B3           B4           B5
      StdDev: 9.246162e-08 9.246162e-08 9.246162e-08 9.246162e-08 9.246162e-08
       Block 3: AB1, AB2, AB3, AB4, AB5, AB6, AB7, AB8, AB9, AB10
       Formula: ~AB - 1 | Group
       Structure: Multiple of an Identity
                    AB1       AB2       AB3       AB4       AB5       AB6
      AB7       AB8       AB9      AB10 Residual
      StdDev: 0.7021792 0.7021792 0.7021792 0.7021792 0.7021792 0.7021792
      0.7021792 0.7021792 0.7021792 0.7021792 1.447584
      Fixed effects: Y ~ 1
                   Value Std.Error DF t-value p-value
      (Intercept) 12.445  0.392531 19 31.7045       0
      Standardized Within-Group Residuals:
              Min          Q1          Med          Q3          Max
      -2.23171950 -0.46808911 -0.03495508  0.32702849  2.06786074
      Number of Observations: 20
      Number of Groups: 1

##Using lmer
fitlmer.out<-lmer(Y ~(1|A) + (1| B) + (1| A:B))
      Linear mixed model fit by REML
      Formula: Y ~ (1 | A) + (1 | B) + (1 | A:B)
```

```
AIC   BIC logLik deviance REMLdev
 84.44 89.42 -37.22    74.38   74.44
Random effects:
 Groups   Name        Variance Std.Dev.
 A:B      (Intercept) 0.49306  0.70218
 B        (Intercept) 0.00000  0.00000
 A        (Intercept) 0.00000  0.00000
 Residual             2.09550  1.44758
Number of obs: 20, groups: A:B, 10; B, 5; A, 2
Fixed effects:
            Estimate Std. Error t value
(Intercept)  12.4450     0.3925    31.7
```

The point estimate of the repeatability standard deviation σ is 1.447. The point estimate of the reproducibility standard deviation $\sqrt{\hat{\sigma}_\beta^2 + \hat{\sigma}_{\alpha\beta}^2} =$0.702 based on REML estimates. Note that the confidence interval for σ_β is extremely wide. Then, the point estimate of reproducibility standard deviation is not reliable. Our approximate interval from (c) does not do justice to the uncertainty here.

PROBLEM 5

```
y<-c(9.5,11.3,8.4,8.5,6.7,9.4,6.1,12,12.2,13.1,10.7,8.9,10.8,7.4,8.6)
A<-c(rep(1,4),rep(2,3),rep(1,4),rep(2,4))
B<-c(1,2,3,3,1,2,3,1,1,2,3,1,2,3,3)
Block<-c(rep(1,7),rep(2,8))
A<-as.factor(A)
B<-as.factor(B)
FBlock<-as.factor(Block)
options(contrasts=c("contr.sum","contr.sum"))
lmf.out<-lm(y~A*B+FBlock)
summary(lmf.out)
      Residuals:
           Min       1Q   Median       3Q      Max
      -0.65238 -0.12143  0.07143  0.12857  0.54762
      Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
      (Intercept)   9.5929     0.1040  92.245 2.13e-13 ***
      A1            1.2849     0.1040  12.356 1.72e-06 ***
      B1           -0.2476     0.1473  -1.681    0.131
      B2            1.5571     0.1541  10.104 7.86e-06 ***
      FBlock1      -1.0286     0.1053  -9.768 1.01e-05 ***
      A1:B1         0.2603     0.1487   1.751    0.118
      A1:B2        -0.2349     0.1541  -1.524    0.166
      Residual standard error: 0.394 on 8 degrees of freedom
      Multiple R-squared: 0.9793,     Adjusted R-squared: 0.9638
      F-statistic: 63.07 on 6 and 8 DF,  p-value: 2.665e-06
```

An estimate of σ is the residual standard error (= 0.394).

b)

```
##Using lme
lmef.out<-lme(y~A*B,random=~1|Block)
summary(lmef.out)
      Linear mixed-effects model fit by REML
       Data: NULL
             AIC      BIC    logLik
        43.26254 44.84034 -13.63127
```

```
      Random effects:
       Formula: ~1 | Block
              (Intercept)  Residual
      StdDev:    1.446977 0.3940026
      Fixed effects: y ~ A * B
                      Value Std.Error DF   t-value p-value
      (Intercept)  9.593456 1.0284387  8  9.328175  0.0000
      A1           1.284322 0.1039918  8 12.350226  0.0000
      B1          -0.246421 0.1472967  8 -1.672959  0.1329
      B2           1.556544 0.1541129  8 10.100022  0.0000
      A1:B1        0.262713 0.1486697  8  1.767092  0.1152
      A1:B2       -0.234322 0.1541129  8 -1.520455  0.1669
intervals(lmef.out)
      Approximate 95% confidence intervals
      Fixed effects:
                        lower       est.       upper
      (Intercept)  7.22187218  9.5934561 11.96503993
      A1           1.04451629  1.2843217  1.52412716
      B1          -0.58608791 -0.2464212  0.09324546
      B2           1.20115891  1.5565439  1.91192898
      A1:B1       -0.08011985  0.2627131  0.60554606
      A1:B2       -0.58970676 -0.2343217  0.12106331
      Random Effects:
        Level: Block
                          lower     est.    upper
      sd((Intercept)) 0.3566116 1.446977 5.871215
       Within-group standard error:
          lower      est.     upper
      0.2413789 0.3940026 0.6431303
##Using lmer
lmerf.out <-lmer(y ~ A*B+(1 |FBlock))
summary(lmerf.out)
      Linear mixed model fit by REML
      Formula: y ~ A * B + (1 | Block)
         AIC   BIC logLik deviance REMLdev
       43.26 48.93 -13.63    16.21   27.26
      Random effects:
       Groups   Name        Variance Std.Dev.
       Block    (Intercept) 2.09374  1.4470
       Residual             0.15524  0.3940
      Number of obs: 15, groups: Block, 2
      Fixed effects:
                  Estimate Std. Error t value
      (Intercept)   9.5935     1.0284   9.329
      A1            1.2843     0.1040  12.350
      B1           -0.2464     0.1473  -1.673
      B2            1.5565     0.1541  10.100
      A1:B1         0.2627     0.1487   1.767
      A1:B2        -0.2343     0.1541  -1.520
      Correlation of Fixed Effects:
            (Intr) A1     B1     B2     A1:B1
      A1    -0.007
      B1     0.000 -0.098
      B2     0.013  0.047 -0.573
      A1:B1 -0.009 -0.009 -0.119  0.057
      A1:B2  0.005  0.132  0.066 -0.032 -0.559
```

The inferences for the fixed effects are the same in both analyses. They produce the same point estimates for the fixed effects and their standard errors. The point estimates for σ are the same in both analyses, but from lmf.out fitted model we obtained $\left(\frac{0.394\sqrt{8}}{\sqrt{\chi^2_{0.975,8}}}, \frac{0.394\sqrt{8}}{\sqrt{\chi^2_{0.025,8}}}\right)$=(0.266, 0.755) as a 95% confidence interval for σ, and for lmef.out fitted model we obtained (0.241, 0.643), a narrower interval.

In the fixed effects analysis the estimate of the block effect is -1.0286. In the mixed effect model σ_τ is assumed normally distributed with mean zero and the estimate of σ_τ is 1.447. From this distribution, the value -1.0286 is within one standard deviation from its mean and, therefore consistent with the estimate of σ_τ.

c)

```
random.effects(lmef.out)
        (Intercept)
      1   -1.017791
      2    1.017791
```

The BLUE of $\mu + \alpha_1 + \beta_1 + \alpha\beta_{11} + \tau_1$ is 9.592857 + 1.284920 - 0.2476190 + 0.2603175 - 1.028571 = 9.862.

The BLUP of $\mu + \alpha_1 + \beta_1 + \alpha\beta_{11} + \tau_1$ is 9.593456 + 1.284322 - 0.2464212 + 0.2627131 - 1.017791 = 9.876.

The BLUP of $\mu + \alpha_1 + \beta_1 + \alpha\beta_{11} + \tau_3$ is 9.593456 + 1.284322 - 0.2464212 + 0.2627131 + 0 = 10.894.

In the fixed effects model we require information from the third block in order to estimate its effect and to predict a new observation from this block.

PROBLEM 6

```
time<-c(rep(1,10),rep(2,10),rep(3,10))
ad<-c(rep(c(rep(1,5),rep(2,5)),3))
test<-rep(1:5,6)
loc<-rep(1:10,3)
y<-
c(958,1005,351,549,730,780,229,883,624,375,1047,1122,436,632,784,897,275,964,695
,436,933,986,339,512,707,718,202,817,599,351)
```

a)

```
TIME<-as.factor(time)
AD<-as.factor(ad)
TEST<-as.factor(test)
LOC<-as.factor(loc)
lmf1.out<-lm(y~1+TIME*(AD/TEST))
anova(lmf1.out)
      Analysis of Variance Table
      Response: y
```

	Df	Sum Sq (SS)	Mean Sq (MS)	Class Name
TIME	2	67073	33537	A
AD	1	168151	168151	B
AD:TEST	8	1833681	229210	C(B)
TIME:AD	2	391	196	AB
TIME:AD:TEST	16	5727	358	Error
Residuals	0	0		

```
lmf2.out<-lm(y~1+TIME*AD*TEST)
anova(lmf2.out)
```

```
Analysis of Variance Table

Response: y
                     Df        Sum Sq (SS)    Mean Sq (MS)    Class Name
TIME                  2            67073          33537               A
AD                    1           168151         168151               B
TEST                  4           397490          99373
TIME:AD               2              391            196              AB
TIME:TEST             8             2475            309
AD:TEST               4          1436191         359048
TIME:AD:TEST          8             3252            407
Residuals             0                0
```

SSC(B) = SS(TEST) + SS(AD:TEST) and Df(C(B)) = Df(TEST) + Df(AD:TEST).
SSE = SS(TIME:TEST) + SS(TIME:AD:TEST) and Df(Error) = Df(TIME:TEST) + Df (TIME:AD:TEST).

b)

```
##With lme
lmef.out<-lme(y~1+TIME*AD,random=~1|LOC)
##equivalent to lmef.out<-lme(y~1+TIME*AD,random=~1|AD/TEST)
summary(lmef.out)
      Linear mixed-effects model fit by REML
       Data: NULL
             AIC      BIC    logLik
        295.1452 304.5696 -139.5726
      Random effects:
       Formula: ~1 | LOC
              (Intercept) Residual
      StdDev:    276.1957 18.92001
      Fixed effects: y ~ 1 + TIME * AD
                     Value Std.Error DF   t-value p-value
      (Intercept) 664.5333  87.40902 16  7.602572  0.0000
      TIME1       -16.1333   4.88512 16 -3.302543  0.0045
      TIME2        64.2667   4.88512 16 13.155583  0.0000
      AD1          74.8667  87.40902  8  0.856510  0.4166
      TIME1:AD1    -4.6667   4.88512 16 -0.955281  0.3536
      TIME2:AD1     0.5333   4.88512 16  0.109175  0.9144
       Correlation:
                 (Intr) TIME1 TIME2 AD1  TIME1:
      TIME1       0.0
      TIME2       0.0   -0.5
      AD1         0.0    0.0   0.0
      TIME1:AD1   0.0    0.0   0.0   0.0
      TIME2:AD1   0.0    0.0   0.0   0.0 -0.5
      Standardized Within-Group Residuals:
              Min          Q1         Med          Q3         Max
      -1.46621417 -0.41184117 -0.03544604  0.50875325  1.86359419
      Number of Observations: 30
      Number of Groups: 10
intervals(lmef.out)
      Approximate 95% confidence intervals
       Fixed effects:
                        lower        est.      upper
      (Intercept)  479.234496 664.5333333 849.832170
      TIME1        -26.489336 -16.1333333  -5.777331
      TIME2         53.910664  64.2666667  74.622669
      AD1         -126.698886  74.8666667 276.432220
```

```
      TIME1:AD1    -15.022669  -4.6666667   5.689336
      TIME2:AD1     -9.822669   0.5333333  10.889336
      Random Effects:
        Level: LOC
                           lower      est.     upper
      sd((Intercept)) 169.0765 276.1957 451.1805
       Within-group standard error:
         lower     est.    upper
      13.37976 18.92001 26.75434
predict(lmef.out)
      958.1586 1016.4009  355.1019  543.8067  719.5319  786.5408  224.4200
      876.0674  627.7891  376.1827 1043.7586 1102.0009  440.7019  629.4067
      805.1319  861.7408  299.6200  951.2674  702.9891  451.3827  934.9586
      993.2009  331.9019  520.6067  696.3319  745.7408  183.6200  835.2674
      586.9891  335.3827
```

An approximate 95% confidence interval for σ_j is (169.076, 451.181) and an approximate 95% confidence interval for σ is (13.380, 26.754).

```
##With lmer
lmerf.out <-lmer(y ~ TIME*AD +(1 |LOC))
##equivalent to lmerf.out <-lmer(y ~ TIME*AD +(1 |TEST:AD))
summary(lmerf.out)
      Linear mixed model fit by REML
      Formula: y ~ TIME * AD + (1 | LOC)
         AIC   BIC logLik deviance REMLdev
       295.1 306.4 -139.6    319.5   279.1
      Random effects:
       Groups   Name        Variance Std.Dev.
       LOC      (Intercept) 76284.10 276.20
       Residual               357.97  18.92
      Number of obs: 30, groups: LOC, 10
      Fixed effects:
                  Estimate Std. Error t value
      (Intercept) 664.5333    87.4038   7.603
      TIME1       -16.1333     4.8851  -3.303
      TIME2        64.2667     4.8851  13.156
      AD1          74.8667    87.4038   0.857
      TIME1:AD1     -4.6667     4.8851  -0.955
      TIME2:AD1      0.5333     4.8851   0.109
```

c)

- $\hat{\sigma}^2 = 357.94$

  Exact 95% confidence interval for σ is $\left(\sqrt{\frac{5727}{\chi^2_{.975,16}}}, \sqrt{\frac{5727}{\chi^2_{.025,16}}}\right)$ = (14.090, 28.794)

- $\hat{\sigma}_\gamma^2 = \frac{MSC(B)-MSE}{3} = \frac{229210-358}{3} = 7628.06$

  $$\hat{\nu} = \frac{\left(\hat{\sigma}_\gamma^2\right)^2}{\frac{(MSC(B)/3)^2}{8} + \frac{(MSE/3)^2}{16}} = 7.975$$

  A 95% confidence interval for σ_γ is $\left(\sqrt{\frac{\hat{\nu}\hat{\sigma}_\gamma^2}{\chi^2_{0.975,\hat{\nu}}}}, \sqrt{\frac{\widehat{\nu\sigma_\gamma^2}}{\chi^2_{0.025,\hat{\nu}}}}\right) = (186.467, 529.839)$

These intervals are wider than those obtained in 3(b).

d)

- A 95% confidence interval for the difference in Time 1 and Time 2 main effects is $(\bar{y}_{1..} - \bar{y}_{2..}) \pm t_{.975,16}\sqrt{\frac{2}{10}MSE}$ = (−98.4,−62.5)
- A 95% confidence interval for the difference in Ad Campaign 1 and Ad Campaign 2 main effects is $(\bar{y}_{.1.} - \bar{y}_{.2.}) \pm t_{.975,8}\sqrt{\frac{2}{15}MSC(B)}$ = (−253.4, 552.9). This last interval can also be obtained by doubling the limits of the interval for the fixed effect ad1 obtained when using intervals(lmef.out)

e) We can predict $Y_{22k} = 664.5333 + 64.2667 - 74.8667 - 0.5333 + 0 = 653.4$ for *k = new*.