This question concerns several analyses of a set of data from a field trial involving 5 Varieties of a crop and 5 Fertilizer regimens. The response variable is

$$y = \text{yield (in bushels per acre)}$$

and the 25 different combinations of Variety and Yield were each run once on a part (1/25th) of a single rectangular field that had been divided evenly into 5 Rows and 5 Columns. The table below gives the yield data, where "Rows" are labeled 1-5 North to South and "Columns" are labeled 1-5 West to East.

Case	Row	Column	Variety	Fertilizer	y, Yield
1	1	1	1	1	163
2	1	2	4	4	216
3	1	3	2	2	173
4	1	4	5	5	222
5	1	5	3	3	198
6	2	1	5	2	183
7	2	2	3	5	224
8	2	3	1	3	203
9	2	4	4	1	135
10	2	5	2	4	203
11	3	1	4	3	208
12	3	2	2	1	141
13	3	3	5	4	210
14	3	4	3	2	167
15	3	5	1	5	216
16	4	1	2	5	220
17	4	2	5	3	198
18	4	3	3	1	148
19	4	4	1	4	205
20	4	5	4	2	165
21	5	1	3	4	210
22	5	2	1	2	177
23	5	3	4	5	217
24	5	4	2	3	199
25	5	5	5	1	144

Define 25×1 column vectors $\mathbf{X}^{\text{V2}},\mathbf{X}^{\text{V3}},\mathbf{X}^{\text{V4}},\mathbf{X}^{\text{V5}},\mathbf{X}^{\text{F2}},\mathbf{X}^{\text{F3}},\mathbf{X}^{\text{F4}},\mathbf{X}^{\text{F5}}$ whose entries are all 0's and 1's by

$$X_i^{\text{V}j} = \begin{cases} 1 & \text{if case } i \text{ is of Variety } j \\ 0 & \text{otherwise} \end{cases}$$

and

$$X_i^{\text{F}j} = \begin{cases} 1 & \text{if case } i \text{ uses Fertilizer regimen } j \\ 0 & \text{otherwise} \end{cases}$$

Let $\mathbf{Y}$ be the 25×1 column vector of responses in last column of the data table above, and $\mathbf{1}$ be a 25×1 column of 1's.

To begin, we will ignore the possibility of any spatial effects on yield (fertility variations in the field) and suppose that only "Variety" and "Fertilizer" are important in determining response.

a) Use the notation above and write out particular $\mathbf{X}$ matrices and parameter vectors $\boldsymbol{\beta}$ for the linear model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ describing the possibilities that

- i) only "Variety" effects on yield are important,
- ii) only "Variety" effects are important and varieties may be separated into the groups $\{1,2\},\{3,4\}$, and $\{5\}$ with equivalent yields within a given group,
- iii) there are "Variety" and "Fertilizer" main effects on yield, but no interaction effects.

b) In your model from a) iii) above, what matrix $\mathbf{C}$ could you use to write a hypothesis $\mathrm{H}_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$ expressing the possibility that Varieties 1 and 2 have the same main effects on yield and (simultaneously) Varieties 3 and 4 have the same main effects on yield?

For

y_{ij} = the yield for Variety i and Fertilizer regimen j

the first part of the R printout for this problem concerns analysis of the yield data under the model

$$y_{ij} = \mu + v_i + f_j + \varepsilon_{ij} \qquad (*)$$

for constants $\mu, v_1, v_2, v_3, v_4, v_5, f_1, f_2, f_3, f_4, f_5$ and iid $\mathrm{N}\left(0,\sigma^2\right)$ random variables ε_{ij}.

c) Under model (*), what are 90% confidence limits for the difference in Variety 1 and Variety 2 main effects? (Plug correct numbers into a correct formula, but you need not evaluate the limits.)

d) In model (*), what are the value of an F statistic and the degrees of freedom for that F statistic for testing $\mathrm{H}_0 : f_1 = f_2 = f_3 = f_4 = f_5$?

Now consider including spatial effects in an analysis of yield. Define

$r(i,j)$ = the row number in which Variety i appears with Ferilizer regimen j

and

$c(i,j)$ = the column number in which Variety i appears with Ferilizer regimen j

A model that allows for linear North-South and West-East fertility gradients in the test field is

$$y_{ij} = \mu + \gamma^{\text{row}} r(i,j) + \gamma^{\text{col}} c(i,j) + v_i + f_j + \varepsilon_{ij} \qquad (**)$$

for constants γ^{row} and γ^{col}. And for constants $\beta_1^{\text{row}}, \beta_2^{\text{row}}, \beta_3^{\text{row}}, \beta_4^{\text{row}}, \beta_5^{\text{row}}, \beta_1^{\text{col}}, \beta_2^{\text{col}}, \beta_3^{\text{col}}, \beta_4^{\text{col}}$, and β_5^{col}, a model that allows arbitrary row and column fertility effects in the test field is

$$y_{ij} = \mu + \beta_{r(i,j)}^{\text{row}} + \beta_{c(i,j)}^{\text{col}} + v_i + f_j + \varepsilon_{ij} \qquad (***)$$

Notice that model equation (**) is the special case of model equation (***) where

$$\beta_l^{\text{row}} = l\gamma^{\text{row}} \text{ and } \beta_l^{\text{col}} = l\gamma^{\text{col}}$$

The 2nd and 3rd parts of the R printout concern analyses of the yield data corresponding to model equations (**) and (***). Use them to answer the following questions.

e) Considering fixed effects versions of models (*), (**), and (***), give values of F statistics and degrees of freedom for judging whether

i) there are detectable "Row" or "Column" spatial effects on yield,

ii) any "Row" or "Column" spatial effects on yield are adequately described as *linear* fertility gradients.

f) Continuing to use fixed effects versions of models (**) and (***), say why model (**) will support prediction of Variety 1 with Fertilizer 1 yield in a plot located in Row 6 and Column 6 (adjacent to the test field) while model (***) will not. Then make 90% prediction limits for Variety 1 and Fertilizer regimen 1 in this location the season of the study. (Again, plug correct numbers into a correct formula, but you need not evaluate the limits.)

Henceforth consider a version of model (***) where the β_l^{row} are iid $\text{N}\left(0,\sigma_{\text{row}}^2\right)$, the β_l^{col} are iid $\text{N}\left(0,\sigma_{\text{col}}^2\right)$, and the $\varepsilon_{ij}, \beta_l^{\text{row}}$, and β_l^{col} are all independent. (Only μ, the v_i, and the f_j are fixed effects.) It is possible to verify (you need not do so) that the field trial was laid out so that each Variety appears once in each row and in each column, as does each Fertilizer regimen.

g) Write the lth row average yield as a function of the terms appearing in (***). Do the same for the lth column average yield. What is the expected value of sample variance of the row average yields? Of the column average yields? Explain.

h) Notice that the row mean square corresponding to equation (***) is in fact 5 times the sample variance of row average yields and the column mean square is in fact 5 times the sample variance of column average yields. How does it seem that σ_{row}^2 compares to σ_{col}^2? Explain.

i) Let

$$\bar{y}_{1\cdot} = \frac{1}{5}\sum_{j=1}^{5} y_{1j},\ \bar{y}_{\cdot 1} = \frac{1}{5}\sum_{i=1}^{5} y_{i1},\ \text{and}\ \bar{y}_{\cdot\cdot} = \frac{1}{25}\sum_{i,j} y_{ij}$$

Under this model it is possible to show that

$$\text{E}\left(\bar{y}_{1\cdot} + \bar{y}_{\cdot 1} - \bar{y}_{\cdot\cdot}\right) = \mu + v_1 + f_1 \quad \text{and} \quad \text{Var}\left(\bar{y}_{1\cdot} + \bar{y}_{\cdot 1} - \bar{y}_{\cdot\cdot}\right) = \frac{1}{5}\sigma_{\text{row}}^2 + \frac{1}{5}\sigma_{\text{col}}^2 + \frac{9}{25}\sigma^2$$

(You may take this as true without proof.) Suppose that y^* is a 26th yield not in the original data set, corresponding to Variety 1 and Fertilizer 1 in Row 6 and Column 6 the season of the study. Give a standard error for $\left(\bar{y}_{1\cdot} + \bar{y}_{\cdot 1} - \bar{y}_{\cdot\cdot}\right)$ as a predictor of y^*, say SE_{pred}. Then briefly indicate how you would find a multiplier τ so that

$$\left(\bar{y}_{1\cdot} + \bar{y}_{\cdot 1} - \bar{y}_{\cdot\cdot}\right) \pm \tau\,\text{SE}_{\text{pred}}$$

will serve as prediction limits for y^*. (You need not carry out the details.)

```
> row
 [1] 1 1 1 1 1 2 2 2 2 2 3 3 3 3 3 4 4 4 4 4 5 5 5 5 5
> col
 [1] 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5
> variety
 [1] 1 4 2 5 3 5 3 1 4 2 4 2 5 3 1 2 5 3 1 4 3 1 4 2 5
> fertilizer
 [1] 1 4 2 5 3 2 5 3 1 4 3 1 4 2 5 5 3 1 4 2 4 2 5 3 1
> yield
 [1] 163 216 173 222 198 183 224 203 135 203 208 141 210 167 216 220 198 148 205
[20] 165 210 177 217 199 144
> Row<-as.factor(row)
> Col<-as.factor(col)
> Vari<-as.factor(variety)
> Fert<-as.factor(fertilizer)
> options(contrasts=c("contr.sum","contr.sum"))
```

Part #1

```
> analysis.1<-lm(yield~1+Vari+Fert,model=TRUE)
> summary(analysis.1)

Call:
lm(formula = yield ~ 1 + Vari + Fert, model = TRUE)

Residuals:
   Min     1Q Median     3Q    Max
  -9.6   -3.8   -0.4    2.6   13.8

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  189.800      1.393 136.268  < 2e-16 ***
Vari1          3.000      2.786   1.077  0.29748
Vari2         -2.600      2.786  -0.933  0.36451
Vari3         -0.400      2.786  -0.144  0.88762
Vari4         -1.600      2.786  -0.574  0.57371
Fert1        -43.600      2.786 -15.651 4.03e-11 ***
Fert2        -16.800      2.786  -6.031 1.75e-05 ***
Fert3         11.400      2.786   4.092  0.00085 ***
Fert4         19.000      2.786   6.821 4.11e-06 ***
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 6.964 on 16 degrees of freedom
Multiple R-Squared: 0.9586,     Adjusted R-squared: 0.9379
F-statistic: 46.33 on 8 and 16 DF,  p-value: 1.268e-09

> anova(analysis.1)
Analysis of Variance Table

Response: yield
          Df  Sum Sq Mean Sq F value    Pr(>F)
Vari       4   105.2    26.3  0.5423     0.707
Fert       4 17870.8  4467.7 92.1175 7.797e-11 ***
Residuals 16   776.0    48.5
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
```

```
> vcov(analysis.1)
            (Intercept)         Vari1         Vari2         Vari3         Vari4
(Intercept)        1.94  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
Vari1              0.00  7.760000e+00 -1.940000e+00 -1.940000e+00 -1.940000e+00
Vari2              0.00 -1.940000e+00  7.760000e+00 -1.940000e+00 -1.940000e+00
Vari3              0.00 -1.940000e+00 -1.940000e+00  7.760000e+00 -1.940000e+00
Vari4              0.00 -1.940000e+00 -1.940000e+00 -1.940000e+00  7.760000e+00
Fert1              0.00 -1.129698e-15 -1.019216e-15  1.800144e-15  4.738432e-16
Fert2              0.00  1.129535e-15  1.781187e-15 -2.303211e-15 -1.464606e-15
Fert3              0.00  6.740780e-16 -9.841124e-17 -5.946922e-16  4.307665e-17
Fert4              0.00 -1.167187e-15 -5.862107e-16  1.692452e-15  9.046097e-16
                    Fert1         Fert2         Fert3         Fert4
(Intercept)  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
Vari1       -1.129698e-15  1.129535e-15  6.740780e-16 -1.167187e-15
Vari2       -1.019216e-15  1.781187e-15 -9.841124e-17 -5.862107e-16
Vari3        1.800144e-15 -2.303211e-15 -5.946922e-16  1.692452e-15
Vari4        4.738432e-16 -1.464606e-15  4.307665e-17  9.046097e-16
Fert1        7.760000e+00 -1.940000e+00 -1.940000e+00 -1.940000e+00
Fert2       -1.940000e+00  7.760000e+00 -1.940000e+00 -1.940000e+00
Fert3       -1.940000e+00 -1.940000e+00  7.760000e+00 -1.940000e+00
Fert4       -1.940000e+00 -1.940000e+00 -1.940000e+00  7.760000e+00
```

Part #2

```
> analysis.2<-lm(yield~1+row+col+Vari+Fert,model=TRUE)
> summary(analysis.2)

Call:
lm(formula = yield ~ 1 + row + col + Vari + Fert, model = TRUE)

Residuals:
   Min     1Q Median     3Q    Max
 -7.96  -1.72   0.48   1.40   5.76

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 202.1600     2.8506  70.919  < 2e-16 ***
row          -1.2400     0.6374  -1.945 0.072091 .
col          -2.8800     0.6374  -4.518 0.000482 ***
Vari1         3.0000     1.8029   1.664 0.118320
Vari2        -2.6000     1.8029  -1.442 0.171254
Vari3        -0.4000     1.8029  -0.222 0.827619
Vari4        -1.6000     1.8029  -0.887 0.389812
Fert1       -43.6000     1.8029 -24.184 8.08e-13 ***
Fert2       -16.8000     1.8029  -9.319 2.22e-07 ***
Fert3        11.4000     1.8029   6.323 1.88e-05 ***
Fert4        19.0000     1.8029  10.539 4.86e-08 ***
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 4.507 on 14 degrees of freedom
Multiple R-Squared: 0.9848,     Adjusted R-squared: 0.974
F-statistic: 90.91 on 10 and 14 DF,  p-value: 5.774e-11

> anova(analysis.2)
Analysis of Variance Table
```

```
Response: yield
          Df  Sum Sq Mean Sq  F value    Pr(>F)
row        1    76.9    76.9   3.7845  0.072091 .
col        1   414.7   414.7  20.4152  0.000482 ***
Vari       4   105.2    26.3   1.2947  0.319223
Fert       4 17870.8  4467.7 219.9290 1.826e-12 ***
Residuals 14   284.4    20.3
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
> vcov(analysis.2)
              (Intercept)           row           col         Vari1         Vari2
(Intercept)  8.125714e+00 -1.218857e+00 -1.218857e+00  6.410957e-16 -1.722347e-16
row         -1.218857e+00  4.062857e-01  5.741155e-17 -1.530975e-16  3.827437e-17
col         -1.218857e+00  5.741155e-17  4.062857e-01 -6.060108e-17  1.913718e-17
Vari1        6.410957e-16 -1.530975e-16 -6.060108e-17  3.250286e+00 -8.125714e-01
Vari2       -1.722347e-16  3.827437e-17  1.913718e-17 -8.125714e-01  3.250286e+00
Vari3       -1.722347e-16  3.827437e-17  1.913718e-17 -8.125714e-01 -8.125714e-01
Vari4       -1.243917e-16  3.827437e-17  3.189531e-18 -8.125714e-01 -8.125714e-01
Fert1        3.444693e-16 -8.930686e-17 -2.551625e-17 -2.547257e-16 -2.582733e-16
Fert2       -1.339603e-16  2.232671e-17  2.232671e-17  2.295515e-16 -2.210267e-16
Fert3       -8.611733e-17  2.232671e-17  6.379061e-18 -1.692677e-16  5.301444e-16
Fert4        9.568592e-18  2.232671e-17 -2.551625e-17  1.072755e-16 -5.802244e-17
                    Vari3         Vari4         Fert1         Fert2         Fert3
(Intercept) -1.722347e-16 -1.243917e-16  3.444693e-16 -1.339603e-16 -8.611733e-17
row          3.827437e-17  3.827437e-17 -8.930686e-17  2.232671e-17  2.232671e-17
col          1.913718e-17  3.189531e-18 -2.551625e-17  2.232671e-17  6.379061e-18
Vari1       -8.125714e-01 -8.125714e-01 -2.547257e-16  2.295515e-16 -1.692677e-16
Vari2       -8.125714e-01 -8.125714e-01 -2.582733e-16 -2.210267e-16  5.301444e-16
Vari3        3.250286e+00 -8.125714e-01  2.634790e-16  2.183722e-16 -3.057211e-16
Vari4       -8.125714e-01  3.250286e+00 -1.804271e-16  2.034442e-31 -3.129911e-32
Fert1        2.634790e-16 -1.804271e-16  3.250286e+00 -8.125714e-01 -8.125714e-01
Fert2        2.183722e-16  2.034442e-31 -8.125714e-01  3.250286e+00 -8.125714e-01
Fert3       -3.057211e-16 -3.129911e-32 -8.125714e-01 -8.125714e-01  3.250286e+00
Fert4        9.021355e-17 -3.608542e-16 -8.125714e-01 -8.125714e-01 -8.125714e-01
                    Fert4
(Intercept)  9.568592e-18
row          2.232671e-17
col         -2.551625e-17
Vari1        1.072755e-16
Vari2       -5.802244e-17
Vari3        9.021355e-17
Vari4       -3.608542e-16
Fert1       -8.125714e-01
Fert2       -8.125714e-01
Fert3       -8.125714e-01
Fert4        3.250286e+00
```

Part #3

```
> analysis.3<-lm(yield~1+Row+Col+Vari+Fert,model=TRUE)
> summary(analysis.3)

Call:
lm(formula = yield ~ 1 + Row + Col + Vari + Fert, model = TRUE)

Residuals:
       Min         1Q     Median         3Q        Max
-5.200e+00 -1.600e+00  1.693e-14  1.600e+00  5.000e+00
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 189.8000     0.9381 202.327 3.99e-16 ***
Row1          4.6000     1.8762   2.452 0.039824 *
Row2         -0.2000     1.8762  -0.107 0.917731
Row3         -1.4000     1.8762  -0.746 0.476898
Row4         -2.6000     1.8762  -1.386 0.203212
Col1          7.0000     1.8762   3.731 0.005779 **
Col2          1.4000     1.8762   0.746 0.476898
Col3          0.4000     1.8762   0.213 0.836503
Col4         -4.2000     1.8762  -2.239 0.055546 .
Vari1         3.0000     1.8762   1.599 0.148487
Vari2        -2.6000     1.8762  -1.386 0.203212
Vari3        -0.4000     1.8762  -0.213 0.836503
Vari4        -1.6000     1.8762  -0.853 0.418567
Fert1       -43.6000     1.8762 -23.239 1.25e-08 ***
Fert2       -16.8000     1.8762  -8.954 1.92e-05 ***
Fert3        11.4000     1.8762   6.076 0.000297 ***
Fert4        19.0000     1.8762  10.127 7.72e-06 ***
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 4.69 on 8 degrees of freedom
Multiple R-Squared: 0.9906,     Adjusted R-squared: 0.9718
F-statistic: 52.77 on 16 and 8 DF,  p-value: 2.429e-06

> anova(analysis.3)
Analysis of Variance Table

Response: yield
          Df  Sum Sq Mean Sq  F value    Pr(>F)
Row        4   150.4    37.6   1.7091   0.24035
Col        4   449.6   112.4   5.1091   0.02427 *
Vari       4   105.2    26.3   1.1955   0.38310
Fert       4 17870.8  4467.7 203.0773 4.488e-08 ***
Residuals  8   176.0    22.0
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
> vcov(analysis.3)
              (Intercept)          Row1          Row2          Row3          Row4
(Intercept)  8.800000e-01  2.930989e-17 -1.953993e-17 -1.953993e-17 -1.953993e-17
Row1         2.930989e-17  3.520000e+00 -8.800000e-01 -8.800000e-01 -8.800000e-01
Row2        -1.953993e-17 -8.800000e-01  3.520000e+00 -8.800000e-01 -8.800000e-01
Row3        -1.953993e-17 -8.800000e-01 -8.800000e-01  3.520000e+00 -8.800000e-01
Row4        -1.953993e-17 -8.800000e-01 -8.800000e-01 -8.800000e-01  3.520000e+00
Col1        -7.777510e-33 -4.120586e-16  2.431809e-17  2.854193e-16  3.907985e-16
Col2         3.527643e-33  1.743186e-16  6.522442e-17  7.832233e-18 -3.907985e-16
Col3         4.402844e-33  2.137341e-16  1.046399e-16 -1.104143e-16 -3.907985e-16
Col4         1.708356e-33  9.238518e-17 -1.670900e-17 -1.371660e-16 -8.827970e-32
Vari1       -7.688388e-33 -7.169982e-16  1.495373e-17  3.795709e-16  2.979839e-16
Vari2        1.133455e-33  6.649394e-17  1.091205e-17  1.385719e-16 -2.515765e-16
Vari3       -8.681429e-35  1.153790e-17 -4.404398e-17  8.361590e-17 -3.175238e-17
Vari4        3.927921e-33  5.012988e-16 -6.390749e-17 -3.069856e-16  1.709743e-17
Fert1       -5.471125e-33 -6.171416e-16  7.467619e-17  2.642420e-16  1.538769e-16
Fert2        1.093019e-33  6.467288e-17  3.584715e-17 -5.613788e-17 -7.815970e-17
Fert3        6.184703e-34  4.330109e-17  1.447535e-17 -7.750967e-17  7.327472e-18
Fert4        3.141165e-33  4.658665e-16 -1.394740e-16 -5.308477e-17 -9.037215e-17
```

	Col1	Col2	Col3	Col4	Vari1
(Intercept)	-7.777510e-33	3.527643e-33	4.402844e-33	1.708356e-33	-7.688388e-33
Row1	-4.120586e-16	1.743186e-16	2.137341e-16	9.238518e-17	-7.169982e-16
Row2	2.431809e-17	6.522442e-17	1.046399e-16	-1.670900e-17	1.495373e-17
Row3	2.854193e-16	7.832233e-18	-1.104143e-16	-1.371660e-16	3.795709e-16
Row4	3.907985e-16	-3.907985e-16	-3.907985e-16	-8.827970e-32	2.979839e-16
Col1	3.520000e+00	-8.800000e-01	-8.800000e-01	-8.800000e-01	-7.947924e-16
Col2	-8.800000e-01	3.520000e+00	-8.800000e-01	-8.800000e-01	-5.648310e-17
Col3	-8.800000e-01	-8.800000e-01	3.520000e+00	-8.800000e-01	8.596729e-16
Col4	-8.800000e-01	-8.800000e-01	-8.800000e-01	3.520000e+00	-4.494183e-16
Vari1	-7.947924e-16	-5.648310e-17	8.596729e-16	-4.494183e-16	3.520000e+00
Vari2	-8.717808e-17	2.004505e-16	3.744742e-16	-4.005685e-16	-8.800000e-01
Vari3	-4.979690e-17	-2.972912e-16	-8.184314e-17	4.787282e-16	-8.800000e-01
Vari4	9.305530e-16	-1.355191e-16	-4.800035e-16	-9.769963e-18	-8.800000e-01
Fert1	1.308244e-16	2.914949e-16	3.642979e-16	-7.571721e-16	-7.353355e-17
Fert2	-2.885798e-16	1.079648e-16	-4.739939e-16	-1.709743e-16	1.010717e-17
Fert3	-1.623030e-16	3.355672e-16	6.428545e-16	-9.769963e-17	3.055510e-16
Fert4	2.676668e-16	-4.322757e-16	-2.421543e-16	4.152234e-16	4.377045e-17

	Vari2	Vari3	Vari4	Fert1	Fert2
(Intercept)	1.133455e-33	-8.681429e-35	3.927921e-33	-5.471125e-33	1.093019e-33
Row1	6.649394e-17	1.153790e-17	5.012988e-16	-6.171416e-16	6.467288e-17
Row2	1.091205e-17	-4.404398e-17	-6.390749e-17	7.467619e-17	3.584715e-17
Row3	1.385719e-16	8.361590e-17	-3.069856e-16	2.642420e-16	-5.613788e-17
Row4	-2.515765e-16	-3.175238e-17	1.709743e-17	1.538769e-16	-7.815970e-17
Col1	-8.717808e-17	-4.979690e-17	9.305530e-16	1.308244e-16	-2.885798e-16
Col2	2.004505e-16	-2.972912e-16	-1.355191e-16	2.914949e-16	1.079648e-16
Col3	3.744742e-16	-8.184314e-17	-4.800035e-16	3.642979e-16	-4.739939e-16
Col4	-4.005685e-16	4.787282e-16	-9.769963e-18	-7.571721e-16	-1.709743e-16
Vari1	-8.800000e-01	-8.800000e-01	-8.800000e-01	-7.353355e-17	1.010717e-17
Vari2	3.520000e+00	-8.800000e-01	-8.800000e-01	-6.152712e-16	3.041276e-16
Vari3	-8.800000e-01	3.520000e+00	-8.800000e-01	1.707201e-16	5.766364e-16
Vari4	-8.800000e-01	-8.800000e-01	3.520000e+00	3.907985e-16	-9.769963e-17
Fert1	-6.152712e-16	1.707201e-16	3.907985e-16	3.520000e+00	-8.800000e-01
Fert2	3.041276e-16	5.766364e-16	-9.769963e-17	-8.800000e-01	3.520000e+00
Fert3	3.347827e-16	-1.947763e-16	-4.884981e-16	-8.800000e-01	-8.800000e-01
Fert4	-2.690249e-16	-3.979696e-17	9.769963e-17	-8.800000e-01	-8.800000e-01

	Fert3	Fert4
(Intercept)	6.184703e-34	3.141165e-33
Row1	4.330109e-17	4.658665e-16
Row2	1.447535e-17	-1.394740e-16
Row3	-7.750967e-17	-5.308477e-17
Row4	7.327472e-18	-9.037215e-17
Col1	-1.623030e-16	2.676668e-16
Col2	3.355672e-16	-4.322757e-16
Col3	6.428545e-16	-2.421543e-16
Col4	-9.769963e-17	4.152234e-16
Vari1	3.055510e-16	4.377045e-17
Vari2	3.347827e-16	-2.690249e-16
Vari3	-1.947763e-16	-3.979696e-17
Vari4	-4.884981e-16	9.769963e-17
Fert1	-8.800000e-01	-8.800000e-01
Fert2	-8.800000e-01	-8.800000e-01
Fert3	3.520000e+00	-8.800000e-01
Fert4	-8.800000e-01	3.520000e+00