8.19 a. The point estimate of μ is $\bar{y} = 11.3$

The bound on the error of estimation is $2\sigma_{\bar{y}}$

$n = 467$. $s = 16.6$

The bound is $2\sigma_{\bar{y}} = 2\frac{\sigma}{\sqrt{n}} = 2\frac{16.6}{\sqrt{467}} = 1.54$

b. The point estimate of $\mu_R - \mu_C$ is $\bar{y}_R - \bar{y}_C = 46.4 - 45.1 = 1.3$

The bound on the error of estimation is

$2\sqrt{\frac{\sigma_R^2}{n_R} + \frac{\sigma_C^2}{n_C}} = 2\sqrt{\frac{s_R^2}{n_R} + \frac{s_C^2}{n_C}} = 2\sqrt{\frac{9.8^2}{191} + \frac{10.2^2}{467}} = 1.7$

c. The point estimate of $p_C - p_R$ is $\hat{p}_C - \hat{p}_R = 0.78 - 0.61 = 0.17$

The bound on the error of estimation is

$2\sqrt{\frac{\hat{p}_C\hat{q}_C}{n_C} + \frac{\hat{p}_R\hat{q}_R}{n_R}} = 2\sqrt{\frac{0.78\times 0.22}{467} + \frac{0.61\times 0.39}{191}} = 0.08$

8.33 $Y_1, \cdots, Y_n \sim EXP(\theta) \Rightarrow E(Y_i) = \theta \quad V(Y_i) = \theta^2$

Hence $E(\bar{Y}) = \theta \quad V(\bar{Y}) = \theta^2/n$

An estimate of θ is $\hat{\theta} = \bar{y} = 1020$

The bound on error is

$2\hat{\sigma}_{\bar{Y}} = 2\frac{\bar{y}}{\sqrt{n}} = 2 \times \frac{1020}{\sqrt{10}} = 645.10$

8.40 a. $F_Y(y) = \int_0^y \frac{2(\theta - t)}{\theta^2}\,dt = \left[\frac{2}{\theta}t - \frac{t^2}{\theta^2}\right]_0^y = \frac{2y}{\theta} - \frac{y^2}{\theta^2} \quad 0<y<\theta$

$F_Y(y) = 1$ for $y \geq \theta$

$F_Y(y) = 0$ for $0 \leq y$

b. Let $u = \frac{Y}{\theta}$

$f_U(u) = \frac{2(\theta - \theta u)}{\theta^2}\cdot\theta = 2(1-u)$ for $0 < u < 1$

Hence Y/θ is a pivotal quantity

c. $P(U \leq a) = \int_0^a 2(1-u)\,du = 2a - a^2 = 0.90$

$\Rightarrow a = 1 - \sqrt{0.10} = 0.684$ (we don't use $a = 1 + \sqrt{0.10}$ because when $a \geq 1$, $P(U \leq a) = 1$)

$\Rightarrow P(U \leq 0.684) = P(Y/\theta \leq 0.684) = P(\theta \geq Y/0.684)$

Thus a 90% lower confidence bound for θ is $Y/0.684$.

8.41 a. $P(U \geq b) = 1 - P(U < b) = 1 - \int_0^b 2(1-u)\,du = 1 - 2b + b^2 = 0.90$

$\Rightarrow b = 1 - \sqrt{0.90} = 0.0513$

$\Rightarrow P(U \geq 0.0513) = P(Y/\theta \geq 0.0513) = P(\theta \leq Y/0.0513)$

Thus a 90% upper confidence bound for θ is $Y/0.0513$

b. $P(\theta \geq Y/0.684) = 0.90$
$P(\theta \leq Y/0.0513) = 0.90$
$\Rightarrow P(Y/0.684 \leq \theta \leq Y/0.0513) = 0.80$
The confidence coefficient of the interval $(\hat{\theta}_L, \hat{\theta}_U)$ is 0.80.

8.44 The 95% confidence interval is approximately
$\bar{y} \pm z_{0.025}\frac{s}{\sqrt{n}} = 5.4 \pm 1.96\frac{3.1}{\sqrt{500}} = 5.4 \pm 0.27$ i.e. $(5.13, 5.67)$

8.45 a. $n = 224$ $\hat{p} = \frac{2}{3}$ $\alpha = 0.10$
A 90% confidence interval for p is
$\hat{p} \pm z_{0.05}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{2}{3} \pm 1.645\sqrt{\frac{\frac{2}{3}\times\frac{1}{3}}{224}} = 0.667 \pm 0.052$ i.e $(0.615, 0.719)$

b. Since 0.5 is not included in the 90% CI, it is believable that most of the children in the group thought they would like to experience space travel.

8.47 A 95% CI for the difference is
$(\bar{y}_1 - \bar{y}_2) \pm z_{0.025}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = (167.1 - 140.9) \pm 1.96\sqrt{\frac{24.3^2}{30} + \frac{17.6^2}{30}}$
$= 26.2 \pm 10.7$ i.e $(15.5, 36.9)$

8.58 $Y \sim$ binomial (p)
$E(Y) = np$ $V(Y) = npq$
$\Rightarrow \hat{p} = \frac{Y}{n}$ $V(\hat{p}) = \frac{pq}{n}$
$B = z_{0.025}\,\sigma_{(\hat{p})} = 1.96\sqrt{\frac{pq}{n}}$
a. $p = 0.9$ $B = 0.05$
$1.96\sqrt{\frac{0.9 \times 0.1}{n}} = 0.05 \Rightarrow n = 138.3 \approx 139$

b. use $p = 0.5$ $B = 0.05$
$1.96\sqrt{\frac{0.5 \times 0.5}{n}} = 0.05 \Rightarrow n = 384.2 \approx 385$

8.59 $B = 2$ $\sigma = 10$
$n = \frac{4\sigma^2}{B^2} = \frac{4 \times 10^2}{2^2} = 100$

8.69 $\sum y_i = 608$ $\sum y_i^2 = 37,538$ $n = 10$ d.f. $= 9$
$\Rightarrow \bar{Y} = 60.8$ $s^2 = \frac{\sum y_i^2 - \frac{1}{n}(\sum y_i)^2}{n-1} = \frac{37,538 - \frac{1}{10}608^2}{9} = 63.511$
A 95% CI is
$\bar{Y} \pm t_{0.025,9}\frac{s}{\sqrt{n}} = 60.8 \pm 2.262\sqrt{\frac{63.511}{10}} = 60.8 \pm 5.70$ i.e $(55.1, 66.5)$

8.70 a. $n = 20$ $\bar{y} = 419$ $s = 57$ $d.f = 19$

A 90% CI is

$\bar{y} \pm t_{0.05,19} \frac{s}{\sqrt{n}} = 419 \pm 1.729 \frac{57}{\sqrt{20}} = 419 \pm 22.0$ i.e. $(397.0, 441.0)$

b. The interval includes 422. Thus 422 is a believable value for μ at the 90% confidence level. However, numbers such as 397, 441, etc. are also believable values for μ.

c. $n = 20$ $\bar{y} = 455$ $s = 69$ $d.f = 19$

A 90% CI is

$\bar{y} \pm t_{0.05,19} \frac{s}{\sqrt{n}} = 455 \pm 1.729 \frac{69}{\sqrt{20}} = 455 \pm 26.7$ i.e $(428.3, 481.7)$

The interval does include 474. We can conclude that the true mean mathematics SAT score is not different from 474 based on our 90% CI.

8.76. a. A 95% CI for $\mu_1 - \mu_2$ is

$\bar{y}_1 - \bar{y}_2 \pm t_{0.025, 28} \sqrt{s_p^2 (\frac{1}{n_1} + \frac{1}{n_2})}$

where μ_1 = mean verbal for engineering students

μ_2 = mean verbal for language students.

$$s_p^2 = \frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{n_1 + n_2 - 2} = \frac{14 \times 42^2 + 14 \times 45^2}{28} = 1894.5$$

The 95% CI is

$(446 - 534) \pm 2.048 \sqrt{1894.5 (\frac{1}{15} + \frac{1}{15})} = -88 \pm 32.55$ i.e $(-120.55, -55.45)$

b. μ_1 = mean math score for engineering students

μ_2 = mean math score for Language students

$$s_p^2 = \frac{14 \times 57^2 + 14 \times 52^2}{28} = 2976.5$$

The 95% CI is

$(548 - 517) \pm 2.048 \sqrt{2976.5 (\frac{1}{15} + \frac{1}{15})} = 31 \pm 40.80$ i.e. $(-9.80, 71.80)$

c. From (a), the 95% CI indicates that a significant difference exists in the mean verbal score for engineering and language students (language students have higher verbal scores).

From (b), the 95% CI indicates that there is no significant difference between mean math score for engineering and language students.

d. we assume the scores for the two groups are randomly and independently selected from two normal distributions with equal variance.

8.81 A 90% CI for σ^2 is

$$\left(\frac{(n-1)s^2}{\chi^2_{0.05,5}}, \frac{(n-1)s^2}{\chi^2_{0.95,5}} \right)$$

$n = 6$. $\chi^2_{.95,5} = 1.145476$ $\chi^2_{0.05,5} = 11.0705$

$\Sigma y_i = 514.4$ $\Sigma y_i^2 = 44,103.74$

$$s^2 = \frac{44,103.74 - \frac{(514.4)^2}{5}}{5} = 0.50267$$

The 90% CI for σ^2 is

$$\frac{5 \times 0.50267}{11.0705} < \sigma^2 < \frac{5 \times 0.50267}{1.145476}$$

$$0.227 < \sigma^2 < 2.194$$

8.82 As calculated in 8.69.

$s^2 = 63.511$ $n = 10$

The 90% CI for σ^2 is

$$\left(\frac{(n-1)s^2}{\chi^2_{0.05,9}}, \frac{(n-1)s^2}{\chi^2_{0.95,9}} \right)$$

$$= \left(\frac{9 \times 63.511}{16.9190}, \frac{9 \times 63.511}{3.32511} \right)$$

$$= (33.785, 171.90)$$

8.110

$$F = \frac{\frac{(n_1-1)s_1^2}{\sigma_1^2} / (n_1-1)}{\frac{(n_2-1)s_2^2}{\sigma_2^2} / (n_2-1)} = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} = \frac{s_1^2}{s_2^2} \times \frac{\sigma_2^2}{\sigma_1^2}$$

A 95% CI for σ_2^2/σ_1^2 is

$$\left(\frac{1}{F_{\nu_2,\nu_1,0.025}} \frac{s_2^2}{s_1^2}, F_{\nu_1,\nu_2,0.025} \frac{s_2^2}{s_1^2} \right)$$

$\nu_1 = 10-1 = 9$ $\nu_2 = 10-1 = 9$ $F_{9,9,0.025} = 4.03$

The 95% CI is

$$\left(\frac{1}{4.03} \times \frac{0.094}{0.273}, 4.03 \times \frac{0.094}{0.273} \right)$$

$$= (0.085, 1.388)$$

9.76 a. $Y_1, \cdots, Y_n \sim \text{gamma}(2, \theta)$

$$L = \prod_{i=1}^{n} \left(\frac{1}{\theta^2} y_i e^{-y_i/\theta} \right) = \frac{1}{\theta^{2n}} e^{-\Sigma y_i/\theta} \prod_{i=1}^{n} y_i$$

$$\log L = -2n \log\theta - \Sigma y_i/\theta + \Sigma \log y_i$$

$\frac{d \log L}{d\theta} = -\frac{2n}{\theta} + \frac{\Sigma y_i}{\theta^2}$

set at 0, $\hat{\theta} = \frac{\Sigma y_i}{2n} = \frac{120 + 130 + 128}{2 \times 3} = 63$

b. $E(Y) = 2\theta$ $V(Y) = 2\theta^2$

$E(\hat{\theta}) = E\left(\frac{\Sigma y_i}{2n}\right) = \frac{1}{2n} \cdot n \cdot 2\theta = \theta$

$V(\hat{\theta}) = V\left(\frac{\Sigma y_i}{2n}\right) = \frac{1}{4n^2} \cdot n \cdot 2\theta^2 = \frac{\theta^2}{2n} = \frac{\theta^2}{6}$

c. The bound on the error of estimation is

$2\sqrt{V(\hat{\theta})} = 2\sqrt{\frac{\theta^2}{6}} = 2\sqrt{\frac{130^2}{6}} = 106.14$

9.80 $L = \prod_{i=1}^{n} (\theta+1) \, y_i^{\theta} = (\theta+1)^n \prod_{i=1}^{n} y_i^{\theta}$

$\log L = n \log(\theta+1) + \theta \Sigma \log y_i$

$\frac{d \log L}{d\theta} = \frac{n}{\theta+1} + \Sigma \log y_i$

set at 0,

$\frac{n}{\hat{\theta}+1} = -\Sigma \log y_i$

$\Rightarrow \hat{\theta} = -\frac{n}{\Sigma \log y_i} - 1$