4.73. (a) Y: Time between accidents

$E(Y) = \beta = 44$ days

$P(Y \le 31) = \int_0^{31} \frac{1}{44} e^{-y/44}\, dy = 1 - e^{-31/44} = 0.5057$

(b) $V(Y) = \beta^2 = 44^2 = 1936$

4.74 (a) Y: one-hour carbon monoxide concentration

$E(Y) = \beta = 3.6$ ppm

$P(Y > 9) = \int_9^{\infty} \frac{1}{3.6} e^{-y/3.6}\, dy = -e^{-y/3.6} \Big|_9^{\infty} = e^{-9/3.6} = 0.0821$

(b) $E(Y) = \beta = 2.5$ ppm

$P(Y > 9) = \int_9^{\infty} \frac{1}{2.5} e^{-y/2.5}\, dy = -e^{-y/2.5} \Big|_9^{\infty} = e^{-9/2.5} = 0.0273$

4.83 Y = rainfall total

$E(Y) = \alpha\beta = 1.6 \times 2.0 = 3.2$

$V(Y) = \alpha\beta^2 = 1.6 \times 2.0^2 = 6.4$

4.84 Y: response time

$$\left.\begin{array}{l} E(Y) = \alpha\beta = 4 \\ V(Y) = \alpha\beta^2 = 8 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \alpha = 2 \\ \beta = 2 \end{array}\right.$$

$$f(y) = \begin{cases} \frac{1}{\Gamma(2) 2^2} y e^{-y/2}, & y > 0 \\ 0 & \text{o.w} \end{cases}$$

4.92 $P(Y \ge 0.4) = \int_{0.4}^{1} 12 y^2 (1-y)\, dy = [4y^3 - 3y^4]\Big|_{0.4}^{1} = 0.8208$

4.100 (a) $E(Y) = \frac{\alpha}{\alpha+\beta} = \frac{3}{3+3} = \frac{1}{2}$

$V(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{3 \times 3}{(3+3)^2 \times (3+3+1)} = \frac{1}{28}$

(b) $E(Y) = \frac{2}{2+2} = \frac{1}{2}$

$V(Y) = \frac{2 \times 2}{(2+2)^2 \times (2+2+1)} = \frac{1}{20}$

(c) $E(Y) = \frac{1}{1+1} = \frac{1}{2}$

$V(Y) = \frac{1 \times 1}{(1+1)^2 \times (1+1+1)} = \frac{1}{12}$

(d) case (a) yields the most homogeneous blending since $V(Y)$ is the smallest

4.115. $P(|Y-\mu| \le 1) \ge 0.75$

$$\Rightarrow \begin{cases} K\sigma = 1 \\ 1-\frac{1}{K^2} = 0.75 \end{cases} \quad \left(\text{since } P(|Y-\mu| < K\sigma) \ge 1 - \frac{1}{K^2}\right)$$

$$\Rightarrow \begin{cases} K = 2 \\ \sigma = \frac{1}{2} \end{cases}$$

4.117 $Y \sim$ uniform (θ_1, θ_2)

$$E(Y) = \frac{\theta_1+\theta_2}{2} \qquad V(Y) = \frac{(\theta_2-\theta_1)^2}{12}$$

$$2\sigma = 2\sqrt{V(Y)} = \frac{\theta_2-\theta_1}{\sqrt{3}}$$

$$P(|Y-\mu| \le 2\sigma) = P(\mu - 2\sigma \le Y \le \mu + 2\sigma)$$
$$= P\left(\frac{\theta_1+\theta_2}{2} - \frac{\theta_2-\theta_1}{\sqrt{3}} \le Y \le \frac{\theta_1+\theta_2}{2} + \frac{\theta_2-\theta_1}{\sqrt{3}}\right)$$

Since $\frac{\theta_1+\theta_2}{2} - \frac{\theta_2-\theta_1}{\sqrt{3}} < \frac{\theta_1+\theta_2}{2} - \frac{\theta_2-\theta_1}{2} = \theta_1$

$$\frac{\theta_1+\theta_2}{2} + \frac{\theta_2-\theta_1}{\sqrt{3}} > \frac{\theta_1+\theta_2}{2} + \frac{\theta_2-\theta_1}{2} = \theta_2$$

$$P(|Y-\mu| \le 2\sigma = P(\theta_1 \le Y \le \theta_2) = 1$$

By Tchebysheff's Theorem

$$P(|Y-\mu| \le 2\sigma) \ge 1 - \frac{1}{2^2} = 0.75$$

The theorem is satisfied but the empirical rule is inaccurate.

5.2 (a)

outcomes	(y_1, y_2)	Probability
HHH	(3, 1)	$\frac{1}{8}$
HHT	(2, 1)	$\frac{1}{8}$
HTH	(2, 1)	$\frac{1}{8}$
HTT	(1, 1)	$\frac{1}{8}$
THH	(2, 2)	$\frac{1}{8}$
THT	(1, 2)	$\frac{1}{8}$
TTH	(1, 3)	$\frac{1}{8}$
TTT	(0, −1)	$\frac{1}{8}$

		y_1			
		0	1	2	3
y_2	−1	$\frac{1}{8}$	0	0	0
	1	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
	2	0	$\frac{1}{8}$	$\frac{1}{8}$	0
	3	0	$\frac{1}{8}$	0	0

(b) $F(2,1) = P(Y_1 \le 2, Y_2 \le 1) = P(0,-1) + P(1,1) + P(2,1)$
$= \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$

5-3 4 married, 3 never married, 2 divorced

$y_1 = 0, 1, 2, 3$. $y_2 = 0, 1, 2, 3$

$P(Y_1 = 0, Y_2 = 0) = P(3 \text{ divorced}) = 0$

$P(Y_1 = 1, Y_2 = 0) = P(1 \text{ married}, 2 \text{ divorced}) = \frac{\binom{4}{1}\binom{3}{0}\binom{2}{2}}{\binom{9}{3}} = \frac{4}{84}$

$P(2,0) = P(2 \text{ married}, 1 \text{ divorced}) = \frac{\binom{4}{2}\binom{3}{0}\binom{2}{1}}{\binom{9}{3}} = \frac{12}{84}$

$P(3,0) = P(3 \text{ married}) = \frac{\binom{4}{3}\binom{3}{0}\binom{2}{0}}{\binom{9}{3}} = \frac{4}{84}$

$P(0,1) = P(1 \text{ never married}, 2 \text{ divorced}) = \frac{\binom{4}{0}\binom{3}{1}\binom{2}{2}}{\binom{9}{3}} = \frac{3}{84}$

$P(1,1) = P(1 \text{ married}, 1 \text{ never married}, 1 \text{ divorced}) = \frac{\binom{4}{1}\binom{3}{1}\binom{2}{1}}{\binom{9}{3}} = \frac{24}{84}$

$P(2,1) = P(2 \text{ married}, 1 \text{ never married}) = \frac{\binom{4}{2}\binom{3}{1}\binom{2}{0}}{\binom{9}{3}} = \frac{18}{84}$

$P(0,2) = P(2 \text{ never married}, 1 \text{ divorced}) = \frac{\binom{4}{0}\binom{3}{2}\binom{2}{1}}{\binom{9}{3}} = \frac{6}{84}$

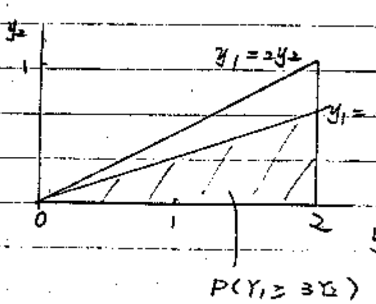
$P(1,2) = P(1 \text{ married}, 2 \text{ never married}) = \frac{\binom{4}{1}\binom{3}{2}\binom{2}{0}}{\binom{9}{3}} = \frac{12}{84}$

$P(0,3) = P(3 \text{ never married}) = \frac{\binom{4}{0}\binom{3}{3}\binom{2}{0}}{\binom{9}{3}} = \frac{1}{84}$

$P(3,1) = P(2,2) = P(3,2) = P(1,3) = P(2,3) = P(3,3) = 0$

y_1 \ y_2	0	1	2	3
0	0	$\frac{3}{84}$	$\frac{6}{84}$	$\frac{1}{84}$
1	$\frac{4}{84}$	$\frac{24}{84}$	$\frac{12}{84}$	0
2	$\frac{12}{84}$	$\frac{18}{84}$	0	0
3	$\frac{4}{84}$	0	0	0

5-8 a.



To make $f(y_1, y_2)$ a probability density function

$\int_0^1 \int_{2y_2}^{2} f(y_1, y_2)\, dy_1\, dy_2 = 1$

Hence $\int_0^1 \int_{2y_2}^{2} k\, dy_1\, dy_2 = \int_0^1 k(2 - 2y_2)\, dy_2$
$= k(2y_2 - y_2^2)\Big|_0^1 = k = 1$

b. $P(Y_1 \ge 3Y_2) = \int_0^{2/3}\int_{3y_2}^{2} dy_1\, dy_2 = \int_0^{2/3} (2 - 3y_2)\, dy_2 = (2y_2 - \frac{3}{2}y_2^2)_0^{2/3} = \frac{2}{3}$

5.13 a. $P(Y_1 < 2, Y_2 > 1) = \int_1^2 \int_{y_2}^2 e^{-y_1}\, dy_1\, dy_2 = \int_1^2 [-e^{-y_1}]_{y_2}^2\, dy_2$

$= \int_1^2 (e^{-y_2} - e^{-2})\, dy_2 = [-e^{-y_2} - y_2 e^{-2}]_1^2$

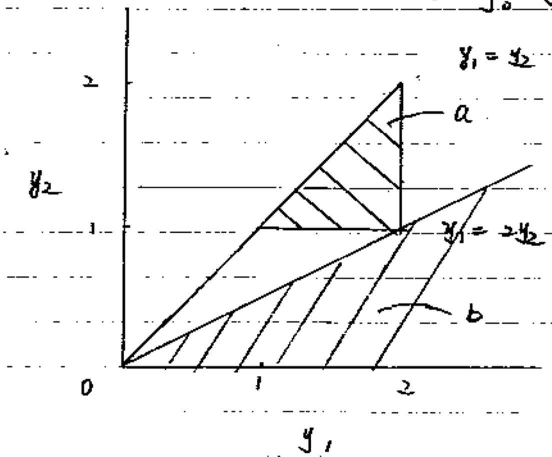
$= -e^{-2} - 2e^{-2} + e^{-1} + e^{-2} = e^{-1} - 2e^{-2}$

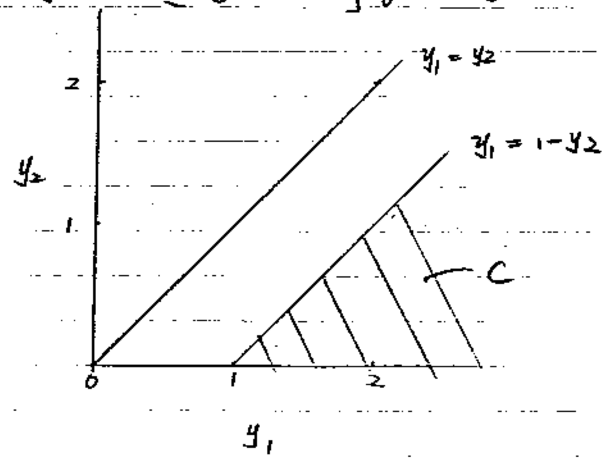
b. $P(Y_1 \ge 2Y_2) = \int_0^\infty \int_{2y_2}^\infty e^{-y_1}\, dy_1\, dy_2 = \int_0^\infty [-e^{-y_1}]_{2y_2}^\infty\, dy_2$

$= \int_0^\infty e^{-2y_2}\, dy_2 = [-\frac{1}{2} e^{-2y_2}]_0^\infty = \frac{1}{2}$

c. $P(Y_1 - Y_2 \ge 1) = \int_0^\infty \int_{1+y_2}^\infty e^{-y_1}\, dy_1\, dy_2 = \int_0^\infty [-e^{-y_1}]_{1+y_2}^\infty\, dy_2$

$= \int_0^\infty e^{-(1+y_2)}\, dy_2 = [-e^{-(1+y_2)}]_0^\infty = e^{-1}$





5.14 a. $P(Y_1 < 1/2, Y_2 > 1/4) = \int_{1/4}^1 \int_0^{1/2} (y_1 + y_2)\, dy_1\, dy_2 = \int_{1/4}^1 (\frac{1}{8} + \frac{y_2}{2})\, dy_2$

$= [\frac{1}{8} y_2 + \frac{1}{4} y_2^2]_{1/4}^1 = \frac{21}{64}$

b. $P(Y_1 + Y_2 \le 1) = \int_0^1 \int_0^{1-y_2} (y_1 + y_2)\, dy_1\, dy_2 = \int_0^1 [\frac{y_1^2}{2} + y_1 y_2]_0^{1-y_2}\, dy_2$

$= \int_0^1 [\frac{(1-y_2)^2}{2} + y_2(1-y_2)]\, dy_2 = \int_0^1 (\frac{1}{2} - \frac{y_2^2}{2})\, dy_2$

$= [\frac{1}{2} y_2 - \frac{y_2^3}{6}]_0^1 = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

5-17 a.

y_1	0	1	2
$p(y_1)$	1/9 + 2/9 + 1/9 = 4/9	2/9 + 2/9 + 0 = 4/9	1/9 + 0 + 0 = 1/9

b. $f(y) = \binom{2}{y} (\frac{1}{3})^y (\frac{2}{3})^{2-y}$

$f(0) = \binom{2}{0} (\frac{1}{3})^0 (\frac{2}{3})^2 = 4/9$

$f(1) = \binom{2}{1} (\frac{1}{3}) (\frac{2}{3}) = 4/9$

$f(2) = \binom{2}{2} (\frac{1}{3})^2 (\frac{2}{3})^0 = 1/9$

There is no conflict.

5-18 a.

y_2	−1	1	2	3
$p(y_2)$	1/8 + 0 + 0 + 0 = 1/8	0 + 1/8 + 1/4 + 1/8 = 1/2	0 + 1/8 + 1/8 + 0 = 1/4	0 + 1/8 + 0 + 0 = 1/8

b. $P(Y_1 = 3 \mid Y_2 = 1) = \dfrac{P(3, 1)}{P(Y_2 = 1)} = \dfrac{1/8}{1/2} = 1/4$

5.19 a. $P(y_1) = \sum_{y_2=0}^{3-y_1} \dfrac{\binom{4}{y_1}\binom{3}{y_2}\binom{2}{3-y_1-y_2}}{\binom{9}{3}} = \dfrac{\binom{4}{y_1}\binom{5}{3-y_1}}{\binom{9}{3}} \sum_{y_2=0}^{3-y_1} \dfrac{\binom{3}{y_2}\binom{2}{3-y_1-y_2}}{\binom{5}{3-y_1}}$

$= \dfrac{\binom{4}{y_1}\binom{5}{3-y_1}}{\binom{9}{3}}$

y_1	0	1	2	3
$P(y_1)$	10/84	40/84	30/84	4/84

Similarly, $P(y_2) = \dfrac{\binom{3}{y_2}\binom{6}{3-y_2}}{\binom{9}{3}}$

b. $P(Y_1 = 1 \mid Y_2 = 2) = \dfrac{P(Y_1 = 1, Y_2 = 2)}{P(Y_2 = 2)} = \dfrac{12/84}{18/84} = 2/3$

c. $P(Y_3 = 1 \mid Y_2 = 1) = \dfrac{P(Y_1 = 1, Y_2 = 1)}{P(Y_2 = 1)} = \dfrac{24/84}{45/84} = 8/15$

d. The probabilities are identical

5.24 a. $f(y_2) = \int_{2y_2}^{2} dy_1 = 2(1 - y_2) \qquad 0 \le y_2 \le 1$

$P(Y \ge 0.5) = \int_{0.5}^{1} (2 - 2y_2)\, dy_2 = [2y_2 - y_2^2]_{0.5}^{1} = 1/4$

b. $f(y_1 \mid y_2) = \dfrac{f(y_1, y_2)}{f(y_2)} = \dfrac{1}{2(1-y_2)} \qquad 2y_2 \le y_1 \le 2$

given $y_2 = \frac{1}{2} \qquad f(y_1 \mid \frac{1}{2}) = 1 \quad$ for $1 \le y \le 2$

$P(Y_1 \ge 1.5 \mid \frac{1}{2}) = \int_{1.5}^{2} 1\, dy_1 = 0.5$

5.29 $f(y_1) = \int_0^{y_1} e^{-y_1}\, dy_2 = y_1 e^{-y_1} \qquad 0 \le y_1 < \infty$

$f(y_2 \mid y_1) = \dfrac{f(y_1, y_2)}{f(y_1)} = \dfrac{e^{-y_1}}{y_1 e^{-y_1}} = \dfrac{1}{y_1} \qquad 0 \le y_2 \le y_1 < \infty$

$P(Y_2 < 1 \mid Y_1 = 2) = \int_0^1 \frac{1}{y_1}\, dy_2 = \int_0^1 \frac{1}{2}\, dy_2 = \frac{1}{2}$

5.30 a. $f(y_1) = \int_0^1 (y_1 + y_2)\, dy_2 = [y_1 y_2 + \frac{1}{2} y_2^2]_0^1 = y_1 + \frac{1}{2} \qquad 0 \le y_1 \le 1$

$f(y_2) = \int_0^1 (y_1 + y_2)\, dy_1 = [\frac{1}{2} y_1^2 + y_1 y_2]_0^1 = y_2 + \frac{1}{2} \qquad 0 \le y_2 \le 1$

b. $P(Y_2 \ge \frac{1}{2}) = \int_{1/2}^{1} (y_2 + \frac{1}{2})\, dy_2 = [\frac{1}{2} y_2^2 + \frac{1}{2} y_2]_{1/2}^{1} = 5/8$

$P(Y_1 \ge \frac{1}{2}, Y_2 \ge \frac{1}{2}) = \int_{1/2}^{1} \int_{1/2}^{1} (y_1 + y_2)\, dy_1\, dy_2 = \int_{1/2}^{1} [\frac{1}{2} y_1^2 + y_1 y_2]_{1/2}^{1}\, dy_2$

$= \int_{1/2}^{1} (\frac{3}{8} + \frac{1}{2} y_2)\, dy_2 = [\frac{3}{8} y_2 + \frac{1}{4} y_2^2]_{1/2}^{1} = 3/8$

$P(Y_1 \ge \frac{1}{2} \mid Y_2 \ge \frac{1}{2}) = \dfrac{P(Y_1 \ge \frac{1}{2}, Y_2 \ge \frac{1}{2})}{P(Y_2 \ge \frac{1}{2})} = \dfrac{3/8}{5/8} = 3/5$

c. $f(y_1 \mid y_2) = \dfrac{f(y_1, y_2)}{f(y_2)} = \dfrac{y_1 + y_2}{y_2 + \frac{1}{2}}$ $\quad 0 \le y_1 \le 1$

$P(Y_1 > 0.75 \mid Y_2 = 0.5) = \int_{0.75}^{1} \dfrac{y_1 + \frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} \, dy_1 = \left[\frac{1}{2} y_1^2 + \frac{1}{2} y_1\right]_{0.75}^{1} = 0.34375$