4.4. a. In order to make $f(y)$ a probability density function,

$$1 = \int_0^1 f(y)\,dy = \int_0^1 ky(1-y)\,dy = k\left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^1 = \frac{k}{6}$$

$$\Rightarrow k = 6$$

b. $P(0.4 \le Y \le 1) = \int_{0.4}^1 6y(1-y)\,dy = 6\left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_{0.4}^1 = 0.648$

c. $f(y)$ is continuous $\Rightarrow$ $P(0.4 \le Y < 1) = P(0.4 \le Y \le 1) = 0.648$

d. $$P(Y \le 0.4 \mid Y \le 0.8) = \frac{P(Y \le 0.4)}{P(Y \le 0.8)} = \frac{\int_0^{0.4} 6y(1-y)\,dy}{\int_0^{0.8} 6y(1-y)\,dy}$$

$$= \frac{6\left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^{0.4}}{6\left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^{0.8}} = 0.393$$

e. $f(y)$ is continuous $\Rightarrow$ $$P(Y < 0.4 \mid Y < 0.8) = \frac{P(Y<0.4)}{P(Y<0.8)} = \frac{P(Y \le 0.4)}{P(Y \le 0.8)}$$

$$= 0.393$$

4.13. a. $$f(y) = \frac{dF(y)}{dy}$$

$$\Rightarrow f(y) = \begin{cases} 0 & y \le 0 \\ \frac{1}{8} & 0 < y < 2 \\ \frac{y}{8} & 2 \le y < 4 \\ 0 & y \ge 4 \end{cases}$$

b. $P(1 \le Y \le 3) = F(3) - F(1) = \frac{9}{16} - \frac{1}{8} = \frac{7}{16}$

c. $P(Y \ge 1.5) = 1 - F(1.5) = 1 - \frac{1.5}{8} = \frac{13}{16}$

d. $$P(Y \ge 1 \mid Y \le 3) = \frac{P(1 \le Y \le 3)}{P(Y \le 3)} = \frac{\frac{7}{16}}{\frac{9}{16}} = \frac{7}{9}$$

4.19 $$E(Y) = \int_{-\infty}^{\infty} y f(y)\,dy = \int_0^2 y \cdot \frac{1}{8}\,dy + \int_2^4 y \cdot \frac{y}{8}\,dy$$

$$= \frac{y^2}{16}\Big|_0^2 + \frac{y^3}{24}\Big|_2^4 = \frac{31}{12}$$

$E(Y^2) = \int_{-\infty}^{\infty} y^2 f(y)\,dy = \int_0^2 y^2 \frac{1}{8}\,dy + \int_2^4 y^2 \cdot \frac{y}{8}\,dy$

$= \frac{y^3}{24}\Big|_0^2 + \frac{y^4}{32}\Big|_2^4 = \frac{47}{6}$

$Var(Y) = E(Y^2) - [E(Y)]^2 = \frac{167}{144} = 1.16$

4.25. $E(Y) = \int_2^6 y \cdot \frac{3}{32}(y-2)(6-y)\,dy$

$= \int_2^6 \frac{3}{32}(-y^3 + 8y^2 - 12y)\,dy$

$= -\frac{3}{128} y^4 \Big|_2^6 + \frac{1}{4} y^3 \Big|_2^6 - \frac{9}{16} y^2 \Big|_2^6 = -30 + 52 - 18 = 4$

4.39 $Y$ = cycle time

$f(y) = \frac{1}{70-50} = \frac{1}{20}$ for $50 \le y \le 70$

$P(Y > 65 \mid Y > 55) = \dfrac{P(Y>65)}{P(Y>55)} = \dfrac{\int_{65}^{70} \frac{1}{20}\,dy}{\int_{55}^{70} \frac{1}{20}\,dy} = \frac{1}{3}$

4.42 a. $P(-0.01 < Y < 0.01) = \int_{-0.01}^{0.01} \frac{1}{0.05-(-0.05)}\,dy = 0.2$

b. $E(Y) = \frac{-0.05+0.05}{2} = 0$

$var(Y) = \frac{[0.05-(-0.05)]^2}{12} = 0.00083$

4.45 $r$ = radius $D$ = diameter $= 2r$ $V = \frac{4}{3}\pi r^3 = \frac{\pi}{6} D^3$

$D \sim \text{Unif}(0.01, 0.05)$

$\Rightarrow E(V) = E(\frac{\pi}{6} D^3) = \int_{0.01}^{0.05} \frac{\pi}{6} D^3 \cdot \frac{1}{0.04}\,dD$

$= \frac{\pi}{6} \cdot \frac{1}{0.04} \frac{D^4}{4}\Big|_{0.01}^{0.05} = 2.04 \times 10^{-5}$

$Var(V) = Var(\frac{\pi}{6} D^3) = \frac{\pi^2}{36} Var(D^3) = \frac{\pi^2}{36}\left[\int_{0.01}^{0.05} D^6 \frac{1}{0.04}\,dD - (E(D^3))^2\right]$

$= \frac{\pi^2}{36}\left[\frac{1}{0.04} \cdot \frac{D^7}{7}\Big|_{0.01}^{0.05} - \left[\frac{1}{0.04} \frac{D^4}{4}\Big|_{0.01}^{0.05}\right]^2\right]$

$= \frac{\pi^2}{36}[2.790 \times 10^{-9} - 1.521 \times 10^{-9}] = 3.479 \times 10^{-10}$

4.60 a. $P(Y > 72) = P(z > \frac{72-78}{6}) = P(z > -1) = 1 - P(z > 1)$

$= 1 - 0.1587 = 0.8413$

b. Need to find $P(Y > c) = 0.1$

$\Rightarrow P(Z > \frac{c-78}{6}) = 0.1$

since $P(Z > 1.28) = 0.1003$, $\frac{c-78}{6} = 1.28 \Rightarrow c = 85.68$

c. Need to find $P(Y > c) = 0.281$

$\Rightarrow P(Z > \frac{c-78}{6}) = 0.281$

since $P(Z > 0.58) = 0.2810$, $\frac{c-78}{6} = 0.58 \Rightarrow c = 81.48$

d. Need to find $P(Y < c) = 0.25$

$\Rightarrow P(Z < \frac{c-78}{6}) = 0.25 \Rightarrow P(Z > -\frac{c-78}{6}) = 0.25$

since $P(Z > 0.67) = 0.2514$, $-\frac{c-78}{6} = 0.67 \Rightarrow c = 73.98$

$P(Y > 73.98+5) = P(Z > \frac{78.98-78}{6}) = P(Z > 0.16) = 0.4364$

e. $P(Y > 84 \mid Y > 72) = \frac{P(Y>84)}{P(Y>72)} = \frac{P(Z > \frac{84-78}{6})}{P(Z > \frac{72-78}{6})} = \frac{P(Z>1)}{P(Z>-1)}$

$= \frac{P(Z>1)}{1-P(Z>1)} = \frac{0.1587}{1-0.1587} = 0.1886$

4.61 $P(Y > 8) = P(Z > \frac{8-\mu}{0.3}) = 0.01$

since $P(Z > 2.33) = 0.0099$, $\frac{8-\mu}{0.3} = 2.33 \Rightarrow \mu = 7.301$

4.62 $P(|X-\mu| < 1) = P(|Z| < \frac{1}{\sigma}) = 0.95$

since $P(Z > 1.96) = 0.0250$, $P(|Z| < 1.96) = 0.95$

$\Rightarrow \frac{1}{\sigma} = 1.96 \Rightarrow \sigma = \frac{1}{1.96} = 0.51$

4.66 $E(A) = E(L \times W) = E(3Y^2) = 3E(Y^2) = 3(Var(Y) + [E(Y)]^2)$

$= 3(\sigma^2 + \mu^2)$