2.87. A : buyer sees a magazine ad

B : buyer sees TV ad

C : buyer purchases the product

$P(A) = \frac{1}{50}$, $P(B) = \frac{1}{5}$. $P(A \cap B) = \frac{1}{100} \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{50} + \frac{1}{5} - \frac{1}{100} = 0.21$

$P(C \mid A \cup B) = \frac{1}{3}$ $\qquad P(C \mid \overline{A \cup B}) = \frac{1}{10}$

$$P(C) = P[C \cap (A \cup B)] + P[C \cap (\overline{A \cup B})]$$
$$= P(C \mid A \cup B) \cdot P(A \cup B) + P(C \mid \overline{A \cup B}) \cdot P(\overline{A \cup B})$$
$$= \frac{1}{3} \times 0.21 + \frac{1}{10} \times (1 - 0.21)$$
$$= 0.149$$

2.90 a. P(both positive) = P(positive for truth & positive for lie)

$= 0.10 \times 0.95 = 0.095$

b. P(positive for lie & negative for truth) $= 0.95 \times (1 - 0.10) = 0.855$

c. P(positive for truth & negative for lie) $= 0.10 \times (1 - 0.95) = 0.005$

d. P (positive for either or both suspects)

= 1 − P(negative for both) = 1 − P(negative for truth & negative for lie)

$= 1 - (1 - 0.10) \times (1 - 0.95) = 0.955$

2.100 a. P(red) = P(red | nitrate) · P(nitrate) + P(red | no nitrate) · P(no nitrate)

$= 0.95 \times 0.3 + 0.1 \times 0.7 = 0.355$

b. $$P(\text{nitrate} \mid \text{red}) = \frac{P(\text{nitrate} \cap \text{red})}{P(\text{red})} = \frac{P(\text{red} \mid \text{nitrate}) \cdot P(\text{nitrate})}{P(\text{red})}$$

$$= \frac{0.95 \times 0.3}{0.355} = 0.803$$

2.101 $$P(I_1 \mid H) = \frac{P(H \mid I_1)\, P(I_1)}{P(H \mid I_1)\, P(I_1) + P(H \mid I_2)\, P(I_2) + P(H \mid I_3) \cdot P(I_3)}$$

$$= \frac{0.90 \times 0.01}{0.90 \times 0.01 + 0.95 \times 0.005 + 0.75 \times 0.02} = 0.313$$

2.139 $Y=1$ when choose #1 $P = \binom{7}{3} / \binom{8}{4} = \frac{1}{2}$

$Y=2$ when choose #2 $P = \binom{6}{3} / \binom{8}{4} = \frac{2}{7}$

$Y=3$ when choose #3 $P = \binom{5}{3} / \binom{8}{4} = \frac{1}{7}$

$Y=4$ when choose #4 $P = \binom{4}{3} / \binom{8}{4} = \frac{2}{35}$

$Y=5$ when choose #5 $P = \binom{3}{3} / \binom{8}{4} = \frac{1}{70}$

3.3 D: defective G: good

$P(Y=2) = P(DD) = \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$

$P(Y=3) = P(DGD) + P(GDD)$

$= 2 \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

$P(Y=4) = P(GGDD) + P(GDGD) + P(DGGD)$

$= \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} + \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} + \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2}$

$= \frac{1}{2}$

3.10 $E(Y) = \sum y\,p(y) = 1 \times 0.4 + 2 \times 0.3 + 3 \times 0.2 + 4 \times 0.1 = 2$

$E(\frac{1}{Y}) = \sum \frac{1}{y} p(y) = 1 \times 0.4 + \frac{1}{2} \times 0.3 + \frac{1}{3} \times 0.2 + \frac{1}{4} \times 0.1 = 0.6417$

$E(Y^2 - 1) = E(Y^2) - 1 = \sum y^2 p(y) - 1$

$= 1 \times 0.4 + 4 \times 0.3 + 9 \times 0.2 + 16 \times 0.1 - 1 = 4$

$V(Y) = E(Y^2) - (EY)^2 = 5 - 2^2 = 1$

3.21 To break even on all \$85,000 policies in this area

premium $= E(\text{loss}) = E(Y) = \sum y\,p(y)$

$= 85{,}000 \times 0.001 + \frac{1}{2} \times 85{,}000 \times 0.01 + 0 \times 0.989$

$= 510$

3.24 $E(\text{cost}) = E(10Y) = 10E(Y) = 10 \sum y\,p(y)$

$= 10(0 \times 0.1 + 1 \times 0.5 + 2 \times 0.4) = 13$

$V(\text{cost}) = V(10Y) = 100\,V(Y) = 100(E(Y^2) - (EY)^2)$

$= 100(0 \times 0.1 + 1 \times 0.5 + 4 \times 0.4 - 1.3^2)$

$= 41$

3.25 $P(B) = P(SS) + P(FS)$

$= \frac{2000}{5000} \times \frac{1999}{4999} + \frac{3000}{5000} \times \frac{2000}{4999} = 0.4$

$P(B \mid \text{the first trial is } S) = \frac{P(SS)}{P(S)} = \frac{\frac{2000}{5000} \times \frac{1999}{4999}}{\frac{2000}{5000}} = \frac{1999}{4999} = 0.3999$

The conditional probability does not differ markedly from P(B)

3.26 a. Since we suppose the two formulas are equally attractive, the probability to choose A is 2/3, and the probability to choose B is 1/3.

$P(0) = (\frac{2}{3})^4 = \frac{16}{81}$

$P(1) = \binom{4}{1}(\frac{1}{3})(\frac{2}{3})^3 = \frac{32}{81}$

$P(2) = \binom{4}{2}(\frac{1}{3})^2(\frac{2}{3})^2 = \frac{24}{81} = \frac{8}{27}$

$P(3) = \binom{4}{3}(\frac{1}{3})^3(\frac{2}{3}) = \frac{8}{81}$

$P(4) = \binom{4}{4}(\frac{1}{3})^4 = \frac{1}{81}$

b. $P(Y \geq 3) = P(3) + P(4) = \frac{8}{81} + \frac{1}{81} = \frac{1}{9}$

c. $E(Y) = \sum y\,P(y) = 0 \times \frac{16}{81} + 1 \times \frac{32}{81} + 2 \times \frac{24}{81} + 3 \times \frac{8}{81} + 4 \times \frac{1}{81}$

$= \frac{108}{81} = \frac{4}{3}$

d. $V(Y) = E(Y^2) - [E(Y)]^2 = 0 \times \frac{16}{81} + 1 \times \frac{32}{81} + 4 \times \frac{24}{81} + 9 \times \frac{8}{81} + 16 \times \frac{1}{81} - \frac{16}{9}$

$= \frac{8}{9}$

3.31 a. $P(Y=5) = \binom{5}{5} \times 0.7^5 = 0.1681$

b. $P(Y \geq 4) = P(Y=4) + P(Y=5) = \binom{5}{4} \times 0.7^4 \times 0.3 + \binom{5}{5} \times 0.7^5$

$= 0.5282$

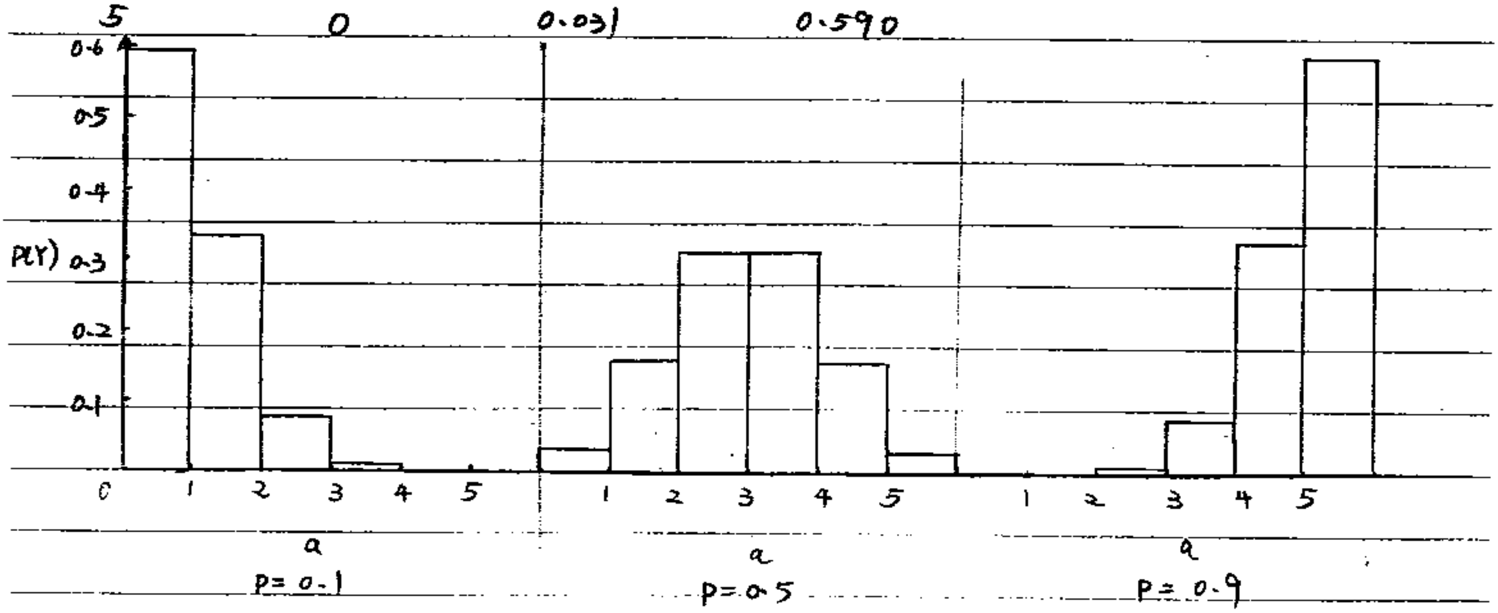
3.34 by Table 1. Appendix III.

$n = 5$. $P(Y \le a)$

a	$p = 0.1$	$p = 0.5$	$p = 0.9$
0	0.590	0.031	0.000
1	0.919	0.188	0.000
2	0.991	0.500	0.009
3	1.000	0.812	0.081
4	1.000	0.969	0.410

$\Rightarrow$ $P(Y)$

a	$p = 0.1$	$p = 0.5$	$p = 0.9$
0	0.590	0.031	0.000
1	0.329	0.157	0.000
2	0.072	0.312	0.009
3	0.009	0.312	0.072
4	0	0.157	0.329
5	0	0.031	0.590



Symmetric

3.44 Y = # of fish that survive. Y is binomial with $n = 20$ and $P = 0.8$

From Table 1, Appendix III

a. $P(Y=14) = P(Y \le 14) - P(Y \le 13) = 0.196 - 0.087 = 0.109$

b. $P(Y \ge 10) = 1 - P(Y \le 9) = 1 - 0.001 = 0.999$

c. $P(Y \le 16) = 0.589$

d. $EY = np = 20 \times 0.8 = 16$

$V(Y) = npq = 20 \times 0.8 \times 0.2 = 3.2$

3.57 Y = the number of the first account containing substantial errors.

a. $P(Y=3) = 0.1^2 \times 0.9 = 0.009$

b. $P(Y \ge 3) = 1 - P(Y \le 2) = 1 - P(Y=1) - P(Y=2)$

$= 1 - 0.9 - 0.1 \times 0.9 = 0.01$

3.58 $EY = \frac{1}{p} = \frac{1}{0.9} = 1.11$

$\sigma = \sqrt{V(Y)} = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{1-0.9}{0.9^2}} = 0.35$

3.60 Y = # of consumers interviewed before a success occurs

a. $P(Y=5) = 0.4^4 \times 0.6 = 0.01536$

b. $P(Y \ge 5) = 1 - P(Y \le 4)$

$= 1 - 0.6 - 0.4 \times 0.6 - 0.4^2 \times 0.6 - 0.4^3 \times 0.6$

$= 0.0256$

3.97 a. $P(Y=4) = \dfrac{2^4 \cdot e^{-2}}{4!} = 0.090$

b. From table 3, appendix III

$P(Y \ge 4) = 1 - P(Y \le 3) = 1 - 0.857 = 0.143$

c. $P(Y < 4) = P(Y \le 3) = 0.857$

d. $P(Y \geq 4 \mid Y \geq 2) = \dfrac{P(Y \geq 4 \,\&\, Y \geq 2)}{P(Y \geq 2)} = \dfrac{P(Y \geq 4)}{P(Y \geq 2)}$

$= \dfrac{1 - P(Y \leq 3)}{1 - P(Y \leq 1)} = \dfrac{1 - 0.857}{1 - 0.406}$

$= \dfrac{0.143}{0.594} = 0.241$

3.98 Y = # of customers arriving $Y \sim \text{Poisson}(7)$

a. $P(Y \leq 3) = 0.082$

b. $P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - 0.007 = 0.993$

c. $P(Y = 5) = \dfrac{7^5 e^{-7}}{5!} = 0.1277$

3.99 $E(\text{service time}) = E(10Y) = 10EY = 10 \cdot \lambda = 10 \times 7 = 70$

$V(\text{service time}) = V(10Y) = 100V(Y) = 100 \cdot \lambda = 100 \times 7 = 700$

$P(\text{time} > 150) = P(10Y > 150) = P(Y > 15) = 1 - P(Y \leq 15)$

$= 1 - 0.998 = 0.002$

It is unlikely the total service time will exceed 2.5 hours

3.111 $EY = \lambda = 2$ $V(Y) = \lambda = 2$

$EY^2 = V(Y) + (EY)^2 = 2 + 2^2 = 6$

$EX = E(50 - 2Y - Y^2) = 50 - 2EY - EY^2$

$= 50 - 2 \times 2 - 6 = 40$