2.5. $A = \{\text{two males}\} = \{M_1M_2,\ M_1M_3,\ M_2M_3\}$

$B = \{\text{at least one female}\}$

$\bar{B} = \{\text{no females}\} = \{\text{two males}\} = A$

$A \cup B = \{\text{two males or at least one female}\} = S$

$A \cap B = \{\text{two males and at least one female}\} = \emptyset$

$A \cap \bar{B} = A \cap A = A$

2.8 a. $S = \{A, B, AB, O\}$

b. $P(A) = 0.41 \quad P(B) = 0.10 \quad P(AB) = 0.04 \quad P(O) = 0.45$

c. $P(A \text{ or } AB) = P(A) + P(AB) = 0.41 + 0.04 = 0.45$

2.14 a. $P(O^+) = \frac{1}{3}$

b. $P(O) = P(O^+) + P(O^-) = \frac{1}{3} + \frac{1}{15} = \frac{6}{15} = \frac{2}{5}$

c. $P(A) = P(A^+) + P(A^-) = \frac{1}{3} + \frac{1}{16} = \frac{19}{48}$

d. $P(\text{neither } O \text{ nor } A) = 1 - (P(O) + P(A)) = 1 - (\frac{2}{5} + \frac{19}{48}) = \frac{49}{240}$

2.21 a. The experiment is about randomly selecting two jurors from a group of two women and four men. An example of one sample point is W_1M_1 (use W_1, W_2 to denote the women and M_1, M_2, M_3, M_4 to denote the men)

b. The sample space is

$W_1,W_2 \quad W_1,M_1 \quad W_1,M_2 \quad W_1,M_3 \quad W_1,M_4$

$W_2,M_1 \quad W_2,M_2 \quad W_2,M_3 \quad W_2,M_4$

$M_1,M_2 \quad M_1,M_3 \quad M_1,M_4$

$M_2,M_3 \quad M_2,M_4$

M_3,M_4

c $P(W_1, W_2) = \frac{1}{15}$

2.40 Two numbers, 4 and 6, are possible choices for each of the three digits. The number of potential winning numbers is $2^3 = 8$

2.42 Three different temperature levels
Three different pressure levels
Two different catalyst levels
$\Rightarrow$ The total number of experiments is $3 \times 3 \times 2 = 18$

2.45 a. $\binom{90}{10} = \frac{90!}{10!\,80!}$

b. $\frac{\binom{20}{4}\binom{70}{6}}{\binom{90}{10}} = 0.111$

2.59 a. P(at least one R) $= \frac{3}{4}$

b. P(at least one r) $= \frac{3}{4}$

c. P(one r | Red color) $= \frac{P(\text{one r and Red color})}{P(\text{Red color})}$

$= \frac{P(rR + Rr)}{P(\text{Red color})} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$

2.61 a. $P(A) = 0.40$

b. $P(B) = 0.37$

c. $P(AB) = 0.10$

d. $P(A \cup B) = P(A) + P(B) - P(AB) = 0.40 + 0.37 - 0.10 = 0.67$

e. $P(\bar{A}) = 1 - P(A) = 0.60$

f. $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.67 = 0.33$

g. $P(\overline{AB}) = 1 - P(AB) = 1 - 0.10 = 0.90$

h. $P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.10}{0.37} = 0.27$

i. $P(B|A) = \frac{P(AB)}{P(A)} = \frac{0.10}{0.40} = 0.25$

2.70 A : device A detects smoke

B : device B detects smoke

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.95 + 0.90 - 0.88 = 0.97$

b. $P(\text{undetected}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.97 = 0.03$

Problem A rat enters a maze

a) $P(\text{ends at node II})$

$= P(1 \to A \to \text{II}) + P(2 \to A \to \text{II}) + P(2 \to B \to \text{II}) + P(2 \to C \to \text{II})$

$+ P(3 \to B \to \text{II}) + P(3 \to C \to \text{II})$

$= .2 \times 1.0 \times .6 + .4 \times .3 \times .6 + .4 \times .3 \times .5 + .4 \times .4 \times .8$

$+ .4 \times .5 \times .5 + .4 \times .5 \times .8$

b) $P(\text{began at node 2} \mid \text{ends at node II})$

$$= \frac{P(\text{began at node 2} \cap \text{ends at node II})}{P(\text{ends at node II})}$$

$$= \frac{.4 \times .3 \times .6 + .4 \times .3 \times .5 + .4 \times .4 \times .8}{P(\text{ends at node II})}$$

Problem single-elimination tournament

a) $P(\#1, \#2 \text{ in 1st round}) = \frac{1}{7}$

b) To make #1 and #2 meet in a second round game, #1 and #2 have to be in different quarters of the same half and both defeat their opponents.

$$P(\#1, \#2 \text{ in 2nd round}) = \frac{\binom{4}{1}\binom{6}{1}\binom{5}{1}}{\binom{8}{2}\binom{6}{2}} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{14}$$

$P(\#1, \#2 \text{ in 1st round or in 2nd round}) = P(\#1, \#2 \text{ in 1st round}) + P(\#1, \#2 \text{ in 2nd round})$

$= \frac{1}{7} + \frac{1}{14} = \frac{3}{14}$

c) To make #1 and #2 meet in the 3rd round, #1 and #2 have to be in different halves and both defeat two opponents.

$$P(\#1,\ \#2 \text{ in 3rd round}) = \frac{\binom{4}{1}\binom{6}{1}\binom{2}{1}\binom{5}{1}}{\binom{8}{2}\binom{6}{2}} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{28}$$

$$P(\#1,\ \#2 \text{ in some game}) = \frac{1}{7} + \frac{1}{14} + \frac{1}{28} = \frac{1}{4}$$