I have neither given nor received unauthorized assistance on this exam.

________________________________________________________
Name                Date
1. I wish to measure the density of a small rock. My method is to read the volume of water in a graduated cylinder before and after putting the rock in the cylinder, producing

\[ v_1 = \text{volume read without the rock} \]

and

\[ v_2 = \text{volume read with the rock} \]

and then to measure the mass of the rock on a precision balance \( n = 5 \) times producing sample mean \( \bar{m} \) and sample standard deviation \( s \) of measurements. Ultimately, I will use the relationship

\[ d = \frac{m}{v_2 - v_1} \]

to compute density. Suppose that I read \( v_1 = 65 \text{ ml} \) and \( v_2 = 75 \text{ ml} \) and that \( \bar{m} = 27.2 \text{ g} \) and \( s = .1 \text{ g} \).

**a)** Suppose that both the balance and the cylinder are without bias and the cylinder has standard uncertainty .5 ml. What is a sensible measured density and standard uncertainty for the measurement? By the way, partial derivatives of \( d \) are:

\[
\frac{\partial d}{\partial m} = \frac{1}{v_2 - v_1}, \quad \frac{\partial d}{\partial v_2} = \frac{-m}{(v_2 - v_1)^2}, \quad \text{and} \quad \frac{\partial d}{\partial v_1} = \frac{m}{(v_2 - v_1)^2}
\]

\[ d = \underline{\text{___________}} \quad \text{standard uncertainty for } d = \underline{\text{___________}} \]
b) Based on the sample information in a) what are 95% confidence limits for the repeatability standard deviation of the balance? (Plug in completely, but there is no need to simplify.)

c) What would be the effect on the measured density of a positive (but constant) .2 g bias for the balance?

d) If the graduated cylinder is a linear device but has bias 1 ml, how would that affect measurement of the rock's density?
2. Students in IE 361 measure Styrofoam™ packing peanuts with a cheap plastic caliper. Suppose that \( n = 10 \) students each measure the same peanut once apiece, and those measurements produce sample standard deviation .062 in. Subsequently, a single student measures a single peanut \( m = 8 \) times and these measurements have sample standard deviation .036 in. What is a single-number estimate of the reproducibility standard deviation in this context?

3. Suppose that a measuring device is not linear, but rather the mean measurement for measurand \( x \) is

\[
\mu(x) = E[y|x] = x + .001x = 1.001x
\]

Suppose further that precision is not constant, but rather

\[
\sigma(x) = \sqrt{\text{Var}[y|x]} = .01x
\]

If measurands \( x \) from a population with mean \( \mu = 5 \) and standard deviation .3 are measured using this device, what will be the resulting mean and standard deviation of measurements \( y \)? (Remember from probability that \( E[y] = E(E[y|x]) \), \( \text{Var}[y] = \text{Var}(E[y|x]) + E(\text{Var}[y|x]) \), and that \( E(U^2) = \text{Var}U + (EU)^2 \).)

\[
E_y = \phantom{0} \quad \sqrt{\text{Var}y} = \phantom{0}
\]
4. Suppose that a real number (infinite number of decimals) measurement \( y \) is digitalized by rounding to the nearest integer, producing \( \lfloor y \rfloor \). Suppose also that measurand \( x \) produces \( y \) that is uniformly distributed on the interval \( (x-1, x+1) \), i.e. given \( x \), \( y \) has a continuous distribution with the density:

\[
f(y|x) = \begin{cases} 
0.5 & \text{for } x-1 < y < x+1 \\
0 & \text{otherwise}
\end{cases}
\]

pictured below: 

\[
f(y|x)
\]

\[
\begin{array}{cccc}
x-1 & x & x+1 & y \\
\end{array}
\]

\[
0.5 \quad 0.5 \quad 0.5 
\]

10pts a) Find the (discrete) distributions of \( \lfloor y \rfloor \) for measurands \( x = 5.0 \) and \( x = 5.5 \). (Give the possible values of \( \lfloor y \rfloor \) and the corresponding probabilities for those two cases.)

\[
x = 5.0 \\
x = 5.5
\]

5pts b) When the measurand \( x \) changes, so does the mean digital measurement \( E[\lfloor y \rfloor | x] \). Is the standard deviation of the digital measurement also potentially affected? Explain.
The book *Statistics and Chemometrics for Analytical Chemistry* by J. N. Miller and J. C. Miller presents a calibration data set for fluorescence spectrophotometry, where known concentrations $x$ of a chemical (in pg/ml) produce measured fluorescence intensities $y$ (in machine units). The data are below along with a JMP report based on the data (with 95% prediction limits shown on the plot).

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2.1</td>
<td>5.0</td>
<td>9.0</td>
<td>12.6</td>
<td>17.3</td>
<td>21.0</td>
<td>24.7</td>
</tr>
</tbody>
</table>

10pts

a) A read intensity of 10 corresponds to what single-number estimate of concentration?

5pts

b) What are 95% confidence limits for the concentration corresponding to a read intensity of 10?
6. For measurements with mean $\mu$ and standard deviation $\sigma$, the ratio

$$\text{snr} = \frac{\mu}{\sigma}$$

is sometimes called a "signal to noise ratio." There is no elementary "Stat 101" statistical methodology for estimating such a quantity.

Suppose that

$$5.2, 5.5, 6.2, 5.6, 5.0$$

are $n = 5$ measurements of the same measurand. Provide a complete set of BUGS code that could be used to make 95% credible limits for the signal to noise ratio here. (Include everything that needed, not just the model statement.)