1. Section 1 Exercises (all) Section 7.1 of Vardeman and Jobe (page 460).

2. Section 2 Exercises 1 and 2, Section 7.2 of Vardeman and Jobe (pages 471 and 472).

3. Section 4 Exercises 1b) and 2b), Section 7.4 of Vardeman and Jobe (page 495).

4. Section 1 Exercises 1, 2, and 3, Section 4.1 of Vardeman and Jobe (pages 139-140).

5. Chapter 4 Exercise 1 of Vardeman and Jobe (page 203).

6. Section 1 Exercises 1a), 1b), 1c), 1d), 1f), 1h), and 2a) through 2f), Section 9.1 of Vardeman and Jobe (page 674).

1. (a) See equation (7-3). The necessary computations are given in the table below.

<table>
<thead>
<tr>
<th>Pressure</th>
<th>( y_{ij} )</th>
<th>( \hat{y}_{ij} = \bar{y}_i )</th>
<th>( e_{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>2.486</td>
<td>2.4790</td>
<td>0.0070</td>
</tr>
<tr>
<td>2000</td>
<td>2.479</td>
<td>2.4790</td>
<td>0.0000</td>
</tr>
<tr>
<td>2000</td>
<td>2.472</td>
<td>2.4790</td>
<td>-0.0070</td>
</tr>
<tr>
<td>4000</td>
<td>2.558</td>
<td>2.5693</td>
<td>-0.0113</td>
</tr>
<tr>
<td>4000</td>
<td>2.570</td>
<td>2.5693</td>
<td>0.0007</td>
</tr>
<tr>
<td>4000</td>
<td>2.580</td>
<td>2.5693</td>
<td>0.0107</td>
</tr>
<tr>
<td>4000</td>
<td>2.560</td>
<td>2.6520</td>
<td>-0.0060</td>
</tr>
<tr>
<td>6000</td>
<td>2.646</td>
<td>2.6520</td>
<td>0.0060</td>
</tr>
<tr>
<td>6000</td>
<td>2.657</td>
<td>2.6520</td>
<td>0.0050</td>
</tr>
<tr>
<td>6000</td>
<td>2.653</td>
<td>2.6520</td>
<td>0.0010</td>
</tr>
<tr>
<td>8000</td>
<td>2.724</td>
<td>2.7687</td>
<td>-0.0447</td>
</tr>
<tr>
<td>8000</td>
<td>2.774</td>
<td>2.7687</td>
<td>0.0053</td>
</tr>
<tr>
<td>8000</td>
<td>2.808</td>
<td>2.7687</td>
<td>0.0393</td>
</tr>
<tr>
<td>10000</td>
<td>2.861</td>
<td>2.8860</td>
<td>-0.0050</td>
</tr>
<tr>
<td>10000</td>
<td>2.879</td>
<td>2.8860</td>
<td>0.0130</td>
</tr>
<tr>
<td>10000</td>
<td>2.858</td>
<td>2.8860</td>
<td>-0.0080</td>
</tr>
</tbody>
</table>
The plot reveals 2 outliers. The assumptions of the one-way normal model appear to be unreasonable for these data. Both of the outliers come from the 8000 psi condition. This is an indication that the common σ part of the one-way normal model assumptions is not reasonable. There seems to be a lot more spread in the 8000 psi sample than in the other samples.

(b) Using equation (7-7), \( s_p = 0.02057 \) g/cc, with \( n - r = 15 - 5 = 10 \) degrees of freedom associated with it. This measures the magnitude of baseline variation within any of the 5 conditions, assuming it is the same for all 5 conditions.

For the confidence interval, use equation (7-10) and Table B-5. For a 90% two-sided interval, \( U = Q_{10}(.95) = 18.307 \) and \( L = Q_{10}(.05) = 3.940 \). The resulting interval for \( \sigma^2 \) is \([.000231132, .001073942]\); taking the square root of each endpoint, the interval for \( \sigma \) is \([.01520, .03277]\) g/cc.

2. (a)
The plot reveals one test from Van 3 (y = 1.01) appears to deviate from a normal distribution assumption (residual = - .0094).

(b) Same as in (a), (standardized residual = -2.9148).

(c) \( s_{pooled} = .0036 \) measures the assumed common standard deviation of tilt angle for repeated tests on any of the four selected vans.

The interval defined as: \[ s_p \sqrt{(13)/5.009}, s_p \sqrt{(13)/24.736} \] becomes \((.0026, .0058)\) and is a 95% two-sided confidence interval for \( \sigma \) based on \( s_p \).

Section 2 Exercises 1 and 2, Section 7.2

1. (a) Use equation (7-14). \( \Delta \) is the same for all five intervals because all five sample sizes are the same. For 95% confidence, the appropriate \( t \) is \( t = Q_{10}(.975) = 2.228 \), from Table B-4.

The resulting \( \Delta \) is

\[
2.228 \frac{.02057}{\sqrt{3}} = .02646 \text{ g/cc.}
\]

Using equation (7-26), the minimum overall (simultaneous) confidence is

\[ 1 - (.05 + .05 + .05 + .05 + .05) = .75 \]

or 75%.

(b) Use equation (7-15). \( \Delta \) is the same for all 10 intervals because all five sample sizes are the same. \( t \) is the same as in part (c). The resulting \( \Delta \) is

\[
2.228(.02057)\sqrt{\frac{1}{3} + \frac{1}{3}} = .03742 \text{ g/cc.}
\]

(c) The estimate of \( \mu_{6000} \) is 2.652 = \( \bar{y}_{6000} \), the estimate of \( \mu_{4000} \) is 2.5693 = \( \bar{y}_{4000} \) and the estimate of \( \mu_{2000} \) is \( \bar{y}_{2000} = 2.479 \). The estimated \( L \) is 2.652 - 2(2.5693) + 2.479 = -.0076. The 95% confidence interval for \( L \) is:

\[-.0076 \pm t_{10}(.02057)\sqrt{(1/3 + 4/3 + 1/3)} \text{ or } -.0076 \pm .0648. \]

The interval \((- .0724, .0572)\) indicates mean density is a linear function of pressure from 2000 to 6000 psi because the interval for \( L \) includes 0.

2. (a) The 99% confidence interval for each of the Vans is of the form \( \bar{x} \pm t_{10} s_p / \sqrt{4} \), where the \( \bar{x} \) is the sample average for the selected van.

Van 1: 1.093 ± 3.012 (.0036/2) or 1.093 ± .00542

becomes (1.0875, 1.0984).

Van 2: .96625 ± (3.012)(.0036/2) or .96625 ± .00542

becomes (.9608, .9716).

Van 3: 1.0194 ± (3.012)(.0036/\sqrt{3}) or 1.0194 ± .004849

becomes (1.0145, 1.0242).
Van 4: $1.00225 \pm .00542$ becomes (.9968, 1.0076).

The Bonferroni Inequality guarantees $\gamma \geq 1 - [.01 + .01 + .01 + .01]$, i.e., at least 96% joint confidence.

(b) Let $n_1 = n_2 = 4$. $\Delta = t_{13} \cdot s_p (2/4)^{1/2} = 3.012(.0036)(1/2)^{1/2} = .0076$.
Let $n_1 = 4, n_2 = 5$. $\Delta = t_{13} \cdot s_p (1/4 + 1/5)^{1/2} = 3.012(.45)^{1/2}(.0036) = .0072$.

(c) The estimate of $1/2(\mu_1 + \mu_2) - 1/2(\mu_3 + \mu_4)$ is $1/2(1.093 + .96625) - 1/2(1.0194 + 1.00225) = .0188$. The 99% two-sided confidence limit for $1/2(\mu_1 + \mu_2) - 1/2(\mu_3 + \mu_4)$ is:

$.0188 \pm t_{13}s_p(1/2)(1/4 + 1/4 + 1/5 + 1/4)^{1/2}$ or
$.0188 \pm .005284$.

Thus, the 99% two-sided confidence limits are [.013516, .02408].

Section 4 Exercises 1b) and 2b), Section 7.4

(b) Using the general form given in Table 7-12, the calculations yield the following table.

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>.285135</td>
<td>4</td>
<td>.071284</td>
<td>168.47</td>
</tr>
<tr>
<td>Error</td>
<td>.004231</td>
<td>10</td>
<td>.000423</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>.289366</td>
<td>14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using equation (7-53),

$$R^2 = \frac{.285135}{.289366} = .985.$$ 

The p-value for an $F$ test of the null hypothesis given in part (1) is

$$P(\text{an } F_{4,10} \text{ random variable } > 168.47).$$

Using Tables B-6, this is less than .001.

2b)

(b) $\text{SSTR} = .034134$, $\text{MSTR} = .011378$, df = 3; $\text{SSE} = .000175$, $\text{MSE} = .000013$, df = 13; $\text{SSTot} = .034308$, df = 16; $f = 846.67$ on 3,13 df; p-value $< .001$; $R^2 = .935$
Section 1 Exercises 1, 2, and 3, Section 4.1

1. (a) The following table shows the necessary computations.

<table>
<thead>
<tr>
<th>i</th>
<th>$x_i$</th>
<th>$x_i^2$</th>
<th>$y_i$</th>
<th>$y_i^2$</th>
<th>$x_iy_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>64</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>64</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>36</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>16</td>
<td>6</td>
<td>36</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>25</td>
<td>4</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

15 55 32 216 86

$$b_1 = \frac{\sum x_i y_i - \left( \sum x_i \right) \left( \sum y_i \right)}{\sum x_i^2 - \left( \frac{\sum x_i}{n} \right)^2} = \frac{86 - \frac{(15)(32)}{5}}{55 - \frac{(15)^2}{5}} = -1.0$$

$$b_0 = \bar{y} - b_1 \bar{x} = \frac{32}{5} - (-1.0) \frac{15}{5} = 9.4$$

So the least squares equation is

$$\hat{y} = 9.4 - 1.0x.$$
(b)  
\[ r = \frac{\sum x_i y_i - (\sum x_i)(\sum y_i)}{\sqrt{\left(\sum x_i^2 - (\sum x_i)^2\right)\left(\sum y_i^2 - (\sum y_i)^2\right)}} \]
\[ = \frac{86 - \frac{(15)(32)}{6}}{\sqrt{\left(55 - \frac{(15)^2}{6}\right)\left(216 - \frac{(32)^2}{6}\right)}} = -0.945 \]

(c) The necessary calculations are given in the table below.

<table>
<thead>
<tr>
<th>i</th>
<th>( x_i )</th>
<th>( y_i )</th>
<th>( y_i^2 )</th>
<th>( \hat{y}_i = 9.4 - 1.0x_i )</th>
<th>( \hat{y}_i^2 )</th>
<th>( y_i \hat{y}_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>8</td>
<td>64</td>
<td>8.4</td>
<td>70.56</td>
<td>67.2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>8</td>
<td>64</td>
<td>7.4</td>
<td>54.76</td>
<td>59.2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>36</td>
<td>6.4</td>
<td>40.96</td>
<td>38.4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>6</td>
<td>36</td>
<td>5.4</td>
<td>29.16</td>
<td>32.4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>4</td>
<td>16</td>
<td>4.4</td>
<td>19.36</td>
<td>17.6</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>216</td>
<td>32</td>
<td>214.8</td>
<td>214.8</td>
<td></td>
</tr>
</tbody>
</table>

\[ r = \frac{214.8 - \frac{(32)(32)}{5}}{\sqrt{\left(216 - \frac{(32)^2}{5}\right)\left(214.8 - \frac{(32)^2}{5}\right)}} = .945 \]

This is the negative of the \( r \) in part (b). Since the \( \hat{y} \)'s are on the least squares line, they are perfectly negatively correlated with the \( x \)'s. So the correlation between the \( \hat{y} \)'s and the \( y \)'s is the same as the correlation between the \( x \)'s and the \( y \)'s, except for a difference in the sign.

(d) The calculations are in the table below.

<table>
<thead>
<tr>
<th>i</th>
<th>( y_i )</th>
<th>( \bar{y} )</th>
<th>( \hat{y}_i )</th>
<th>( (y_i - \bar{y}) )</th>
<th>( (y_i - \hat{y}_i)^2 )</th>
<th>( e_i = (y_i - \hat{y}_i) )</th>
<th>( (y_i - \hat{y}_i)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>6.4</td>
<td>8.4</td>
<td>1.6</td>
<td>2.56</td>
<td>-.4</td>
<td>.16</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>6.4</td>
<td>7.4</td>
<td>1.6</td>
<td>2.56</td>
<td>.6</td>
<td>.36</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6.4</td>
<td>6.4</td>
<td>-.4</td>
<td>.16</td>
<td>-.4</td>
<td>.16</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>6.4</td>
<td>5.4</td>
<td>-.4</td>
<td>.16</td>
<td>.6</td>
<td>.36</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>6.4</td>
<td>4.4</td>
<td>-2.4</td>
<td>5.76</td>
<td>-.4</td>
<td>.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.2</td>
<td></td>
<td>1.2</td>
</tr>
</tbody>
</table>

\[ R^2 = \frac{\sum(y_i - \bar{y})^2 - \sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2} = \frac{11.2 - 1.2}{11.2} = .893 \]

This is equal to the square of sample correlation in both (b) and (c).

(e) The residuals \( e_i \) are given in the table in (d). These are the vertical distances from each data point to the least squares line.
2. \( x \leftarrow 1:5 \)
\( y \leftarrow c(8, 8, 6, 6, 4) \)

```r
data <- data.frame(x, y)
data

## x y
## 1 1 8
## 2 2 8
## 3 3 6
## 4 4 6
## 5 5 4

plot(y ~ x, data = data)
```

```r
summary(data)

## x y
## Min. :1 Min. :4.0
## 1st Qu.:2 1st Qu.:6.0
## Median :3 Median :6.0
## Mean :3 Mean :6.4
## 3rd Qu.:4 3rd Qu.:8.0
## Max. :5 Max. :8.0
```

```
#Correlation between x and y
cor(data)
```
## x  y
## x  1.0000000 -0.9449112
## y -0.9449112  1.0000000

#regression coefficients
lm.out<-lm(y~x)
summary(lm.out)

plot(lm.out)
```r
abline(lm(y~x), xlim=c(0,10), ylim=c(0,10))
```
3. (a) $R^2 = .994$.

(b) The least squares equation is

$$\hat{y} = -3174.6 + 23.5x.$$ 

$\beta_1$ represents the "true" average change in molecular weight that accompanies a $1^\circ$C increase in pot temperature (assuming that a straight-line model is correct). $b_1 = 23.5$ is a data-based approximation of this value.

(c) The residuals are: 105.36, -21.13, -60.11, -97.58, 16.95, 14.48, 42.00, and .02.
It is difficult to evaluate the appropriateness of the fitted equation based on so little data. The plots of residuals versus $z$ and residuals versus $\hat{y}$ do not contain any obvious patterns, and thus provide no evidence that the equation is inappropriate. The normal plot of residuals is fairly linear, providing no evidence that the residuals are not bell-shaped.

(d) There is no replication (multiple experimental runs at a particular pot temperature). Replication would validate any conclusions drawn from the experiment, and provide more information to confirm the appropriateness of the fitted equation.

(e) For $z = 188^\circ C$,

$$\hat{y} = -3174.6 + 23.5(188) = 1243.1.$$  

For $z = 200^\circ C$,

$$\hat{y} = -3174.6 + 23.5(200) = 1525.1.$$  

It would not be wise to make a similar prediction at $z = 70^\circ C$ because there is no evidence that the fitted relationship is correct for pot temperatures as low as $z = 70^\circ C$. This would be an extrapolation. Some data should be obtained around $z = 70^\circ C$ before making such a prediction.

1. (a) The following table shows the necessary computations.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$x_i$</th>
<th>$x_i^2$</th>
<th>$y_i$</th>
<th>$y_i^2$</th>
<th>$x_i y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.45</td>
<td>.2025</td>
<td>2954</td>
<td>8726116</td>
<td>1329.30</td>
</tr>
<tr>
<td>2</td>
<td>.45</td>
<td>.2025</td>
<td>2913</td>
<td>8485569</td>
<td>1310.85</td>
</tr>
<tr>
<td>3</td>
<td>.45</td>
<td>.2025</td>
<td>2923</td>
<td>8543929</td>
<td>1315.35</td>
</tr>
<tr>
<td>4</td>
<td>.50</td>
<td>.2500</td>
<td>2743</td>
<td>7524949</td>
<td>1371.50</td>
</tr>
<tr>
<td>5</td>
<td>.50</td>
<td>.2500</td>
<td>2779</td>
<td>7722841</td>
<td>1389.50</td>
</tr>
<tr>
<td>6</td>
<td>.50</td>
<td>.2500</td>
<td>2739</td>
<td>7502121</td>
<td>1369.50</td>
</tr>
<tr>
<td>7</td>
<td>.55</td>
<td>.3025</td>
<td>2652</td>
<td>7033104</td>
<td>1458.60</td>
</tr>
<tr>
<td>8</td>
<td>.55</td>
<td>.3025</td>
<td>2607</td>
<td>6786449</td>
<td>1433.85</td>
</tr>
<tr>
<td>9</td>
<td>.55</td>
<td>.3025</td>
<td>2583</td>
<td>6671889</td>
<td>1420.65</td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>2.265</td>
<td>24893</td>
<td>69006067</td>
<td>12399.1</td>
</tr>
</tbody>
</table>

$$b_1 = \frac{\sum x_i y_i - (\sum x_i)(\sum y_i)}{\sum x_i^2 - (\sum x_i)^2} = \frac{12399.1 - \frac{(4.5)(24893)}{9}}{2.265 - \frac{(4.5)^2}{9}} = -3160$$

$$b_0 = \bar{y} - b_1 x = \frac{24893}{9} - (-3160)\frac{4.5}{9} = 4345.889$$
So the least squares equation is

\[ \hat{y} = 4345.889 - 3160x. \]

(b) 

\[
\begin{align*}
\rho &= \frac{\sum x_i y_i - (\sum x_i)(\sum y_i)}{\sqrt{\left(\sum x_i^2 - (\sum x_i)^2\right) \left(\sum y_i^2 - (\sum y_i)^2\right)}} \\
&= \frac{12399.1 - \frac{(4.5)(24893)}{9}}{\sqrt{\left(2.265 - \frac{(4.5)^2}{9}\right) \left(69006067 - \frac{(24893)^2}{9}\right)}} = -0.984
\end{align*}
\]

Since \( \rho \) is negative and close to \(-1\), there is a strong negative linear relationship between Water/Cement Ratio and 14-Day Compressive Strength.

(c) Since this is a straight-line model, \( R^2 = r^2 = 0.968 \).
(d) The residuals are 30.11, −10.89, −.089, −22.89, 13.11, −26.89, 44.11, −.89, and −24.90.

The normal plot of residuals is fairly linear; this implies that the residuals are roughly bell-shaped. There are no outliers.

(e) For z = .48,

\[ \hat{y} = 4345.889 - 3160(.48) = 2829.09 \text{ psi}. \]

(f) R code:

```r
ratio <- c(rep(0.45, 3), rep(0.50, 3), rep(0.55, 3))
strength <- c(2954, 2913, 2923, 2743, 2779, 2739, 2652, 2607, 2583)
data <- data.frame(ratio, strength)
data
```

```r
## plot of the data with fitted straight line
plot(strength ~ ratio, data = data)
abline(lm(strength ~ ratio), xlim = c(0, 10), ylim = c(0, 10))
```
summary(data)
##    ratio     strength
##   Min. :0.45   Min. :2583
##   1st Qu.:0.45 1st Qu.:2652
##   Median :0.50  Median :2743
##   Mean :0.50   Mean :2766
##   3rd Qu.:0.55 3rd Qu.:2913
##   Max. :0.55   Max. :2954

#Correlation between x and y
cor(data)
##         ratio     strength
## ratio 1.0000000 -0.9836813
## strength -0.9836813  1.0000000

#regression coefficients
lm.out<- lm(strength~ratio)
lm.out

##
## Call:
## lm(formula = strength ~ ratio)
##
## Coefficients:
## (Intercept) ratio
##     4346    -3160
summary(lm.out)

## Call:
## lm(formula = strength ~ ratio)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -26.889 -22.889 -0.889  13.111  44.111
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4345.9      109.6   39.66  1.69e-09 ***
## ratio        -3160.0      218.5  -14.46  1.80e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26.76 on 7 degrees of freedom
## Multiple R-squared:  0.9676, Adjusted R-squared:  0.963
## F-statistic: 209.2 on 1 and 7 DF,  p-value: 1.799e-06

Section 1 Exercises 1a), 1b), 1c), 1d), 1f), 1h), and 2a) through 2f), Section 9.1

1. (a) See Exercise 3, Section 1, Chap. 4 for computations. Using equation (9-10),

$$ s_{LF}^2 = \frac{1}{8 - 2} (26940.69) = 4490.116, $$

so $s_{LF} = \sqrt{4490.116} = 67.01$, with 6 degrees of freedom associated with it. This measures
the baseline variation in molecular weight that would be observed for any fixed pot
temperature, assuming that model (9-4) is appropriate.

(b) The residuals were computed in Ex. 3, Sec. 1, Chap. 4. Use equation (9-12) to compute the
standardised residuals. $\bar{x} = 212.375$, and $\sum(x - \bar{x})^2 = 8469.875$. The rest of the
calculations are summarised below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\sqrt{1 - \frac{1}{8} - \frac{(x-212.375)^2}{8469.875}}$</th>
<th>$e$</th>
<th>$e^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>165</td>
<td>.78103</td>
<td>105.35535</td>
<td>2.01306</td>
</tr>
<tr>
<td>176</td>
<td>.84781</td>
<td>-21.12558</td>
<td>-.37186</td>
</tr>
<tr>
<td>188</td>
<td>.89714</td>
<td>-60.10477</td>
<td>-.99982</td>
</tr>
<tr>
<td>205</td>
<td>.93198</td>
<td>-97.57529</td>
<td>-1.56245</td>
</tr>
<tr>
<td>220</td>
<td>.93174</td>
<td>16.95072</td>
<td>.27150</td>
</tr>
<tr>
<td>235</td>
<td>.90253</td>
<td>14.47673</td>
<td>.23938</td>
</tr>
<tr>
<td>250</td>
<td>.84135</td>
<td>42.00275</td>
<td>.74503</td>
</tr>
<tr>
<td>260</td>
<td>.77924</td>
<td>.02009</td>
<td>.00038</td>
</tr>
</tbody>
</table>
The plots look almost exactly the same.

(c) This is $\beta_1$. Use equation (9-17). For 90% confidence, the appropriate $t$ is $t = Q_\alpha(.95) = 1.943$ from Table B-4. The resulting interval is

$$23.49827 \pm 1.943 \frac{67.01}{\sqrt{8469.875}}$$

$$= 23.49827 \pm 1.414696$$

$$= [22.08, 24.91].$$

(d) Use equation (9-24). The appropriate $t$ is the same as the one in part (c). The resulting interval for the mean at $x = 212$ is

$$1807.063 \pm 1.943(67.01)\sqrt{\frac{1}{8} + \frac{.140625}{8469.875}}$$

$$= 1807.063 \pm 46.03471$$

$$= [1761.03, 1853.10].$$

The resulting interval for the mean at $x = 250$ is

$$2699.997 \pm 1.943(67.01)\sqrt{\frac{1}{8} + \frac{1415.641}{8469.875}}$$

$$= 2699.997 \pm 70.37134$$

$$= [2629.63, 2770.37].$$
(f) Use equation (9-26). For a 90% one-sided interval, appropriate $t$ is $t = Q_t(90) = 1.440$ from Table B.4. The resulting lower prediction bound at $x = 212$ is

$$1807.063 - 1.440(67.01)\sqrt{1 + \frac{1}{8} + \frac{.140625}{8469.875}}$$

$$= 1807.063 - 102.346$$

$$= 1704.72.$$  

The resulting bound for the mean at $x = 250$ is

$$2699.997 - 1.440(67.01)\sqrt{1 + \frac{1}{8} + \frac{1415.641}{8469.875}}$$

$$= 2699.997 - 109.6846$$

$$= 2590.31.$$  

(h) Using the general form given in Table 9-6.

<table>
<thead>
<tr>
<th>Source</th>
<th>$SS$</th>
<th>$df$</th>
<th>$MS$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>4676798</td>
<td>1</td>
<td>4676798</td>
<td>1041.58</td>
</tr>
<tr>
<td>Error</td>
<td>26941</td>
<td>6</td>
<td>4490</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4703739</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The $p$-value is

$$P(\text{an } F_{1,6} \text{ random variable } > 1041.58),$$

which is less than .001, according to Tables B.6 ($Q_{1,6}(.999) = 35.51$). This is overwhelming evidence that the mean average molecular weight is related to the pot temperature. (The model used is an improvement over the model $y = \beta_0 + \epsilon$.)

(a) In Ex. 3, Sec. 1, Ch. 4, $b_1 = -3160$ and $b_0 = 4345.889$. The necessary computations for $s_{LP}$ (the residuals) are also given there. Using equation (9-10),

$$s_{LP}^2 = \frac{1}{9-2}(5010.889) = 715.84,$$

so $s_{LP} = \sqrt{715.84} = 26.755$ psi, with 7 degrees of freedom associated with it. This measures the baseline variation in 14-day compressive strength that would be observed for any fixed water/cement ratio, assuming that model (9-4) is appropriate. Using equation (7-7), $s_p = 26.890$ psi. These two estimates are very close, giving no indication
that the model is inappropriate.

(b) The residuals were computed in Ex. 3, Sec. 1, Ch. 4. Use equation (9-12) to compute the standardized residuals. $\bar{x} = .50$, and $\sum (x - \bar{x})^2 = .015$. The rest of the calculations are summarized below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\sqrt{1 - \frac{1}{9} \cdot \frac{(x - .50)^2}{.015}}$</th>
<th>$e$</th>
<th>$e^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.45</td>
<td>.8498</td>
<td>30.1111</td>
<td>1.3243</td>
</tr>
<tr>
<td>.45</td>
<td>.8498</td>
<td>-10.8889</td>
<td>-.4789</td>
</tr>
<tr>
<td>.45</td>
<td>.8498</td>
<td>-.8889</td>
<td>-.0391</td>
</tr>
<tr>
<td>.50</td>
<td>.9428</td>
<td>-22.8889</td>
<td>-.9074</td>
</tr>
<tr>
<td>.50</td>
<td>.9428</td>
<td>13.1111</td>
<td>.5198</td>
</tr>
<tr>
<td>.50</td>
<td>.9428</td>
<td>-26.8889</td>
<td>-1.0660</td>
</tr>
<tr>
<td>.55</td>
<td>.8498</td>
<td>44.1111</td>
<td>1.9400</td>
</tr>
<tr>
<td>.55</td>
<td>.8498</td>
<td>-.8889</td>
<td>-.0391</td>
</tr>
<tr>
<td>.55</td>
<td>.8498</td>
<td>-24.8889</td>
<td>-1.0946</td>
</tr>
</tbody>
</table>

![Residuals vs. Water/Cement Ratio](image1)

![Standardized Residuals vs. Water/Cement Ratio](image2)

![Residuals vs. Fitted Values](image3)

![Standardized Residuals vs. Fitted Values](image4)
For each of the three types of plots, the residuals and standardized residuals look almost exactly the same.

(c) First make a confidence interval for $\beta_1$, and then multiply the endpoints by $.1$. Use equation (9-17). For 90\% confidence, the appropriate $t$ is $t = Q_t(.95) = 1.895$ from Table B-4. The resulting interval for $\beta_1$ is

$$-3160.0 \pm 1.895 \frac{26.755}{\sqrt{.015}}$$

$$= -3160.0 \pm 413.9729$$

$$= [-3573.973, -2746.027] \text{ psi.}$$

Multiplying each endpoint by $.1$, the resulting interval for $.1\beta_1$ is $[-357.4, -274.6]$ psi.

(d) This can be done using the test statistic (9-16) or with (9-34). Using (9-16),

1. $H_0$: $\beta_1 = 0$.
2. $H_a$: $\beta_1 \neq 0$.
3. The test statistic is given by equation (9-16), with $\# = 0$. The reference distribution is the $t_7$ distribution. Observed values of $t$ far above or below zero will be considered as evidence against $H_0$.
4. The samples give

$$t = \frac{-3160.0}{\frac{26.755}{\sqrt{.015}}} = -14.47.$$ 

5. The observed level of significance is

$$2P(\text{a } t_7 \text{ random variable } < -14.47)$$

$$= 2P(\text{a } t_7 \text{ random variable } > 14.47)$$

$$= 2(\text{less than } .0005)$$

which is less than $.001$, according to Table B-4 (14.47 is greater than $Q(.9995) = 5.408$). This is overwhelming evidence that the mean compressive strength is related to the water/cement ratio. (The given model is an improvement over the model $y = \beta_0 + \epsilon$.)

Using (9-34),

1. $H_0$: $\beta_1 = 0$. 

2. \( H_a: \beta_1 \neq 0 \).
3. The test statistic is given by equation (9.34). The reference distribution is the \( F_{1,7} \) distribution. Large observed values of \( F \) will be considered as evidence against \( H_0 \).
4. The samples give \( SSE = (n - 2)(s_L) = 5010.889 \) and \( SSTot = 154794.9 \), so \( SSR = SSTot - SSE = 149784 \).

\[
f = \frac{149784}{715.84} = 209.24.
\]

5. The observed level of significance is

\[
P(\text{an } F_{1,7} \text{ random variable } > 209.24)
\]

which is less than .001, according to Tables B-6 (209.24 is greater than \( Q(.999) = 29.24 \)). This is overwhelming evidence that the mean compressive strength is related to the water/cement ratio. (The given model is an improvement over the model \( y = \beta_0 + \epsilon \).)

Note that the \( F \) statistic is equal to the square of the \( t \) statistic.

(e) Use equation (9-24). For 95% confidence, the appropriate \( t \) is \( t = Q_7(.975) = 2.365 \) from Table B-4. The resulting interval for the mean at \( z = .5 \) is

\[
2765.889 \pm 2.365(26.755)\sqrt{\frac{1}{9} + \frac{0}{.015}}
\]

\[
= 2765.889 \pm 21.09202
\]

\[
= [2744.8, 2787.0] \text{ psi.}
\]

(f) Use equation (9-26). The appropriate \( t \) is the same one used in part (e). The resulting prediction interval at \( z = .5 \) is

\[
2765.889 \pm 2.365(26.755)\sqrt{1 + \frac{1}{9} + \frac{0}{.015}}
\]

\[
= 2765.889 \pm 66.69884
\]

\[
= [2699.2, 2832.6] \text{ psi.}
\]