Stat 342 Example 7

Suppose \( U_1 \) and \( U_2 \) are iid discrete r.v.'s with pmf

<table>
<thead>
<tr>
<th>( u )</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(u) )</td>
<td>.3</td>
<td>.4</td>
<td>.3</td>
</tr>
</tbody>
</table>

What is the distribution of \( \bar{U} = \frac{1}{2} (U_1 + U_2) \)? (This question is “What are possible values of \( \bar{U} \) and the corresponding probabilities?”)

\((U_1, U_2)\) are jointly discrete distributed with pmf as

\[
\begin{array}{c|ccc}
\hline
u_2 & 0  & 1  & 2  \\
\hline
2  & .09 & .12 & .09 \\
1  & .12 & .16 & .12 \\
0  & .09 & .12 & .09 \\
\hline
\end{array}
\]

and \( h(u_1, u_2) = \frac{1}{2} (u_1 + u_2) \) maps \( u_2 \) to \( u_1 \)

and we're looking for the pmf of \( h(U_1, U_2) \). We get this by for each possible \( u_i \) adding probabilities of \((u_1, u_2)\) pairs with average of \( u_i \) and \( u_2 \) equal to that value. That is

\[
P(\bar{U} = 0) = P[ U_1 = 0 \text{ and } U_2 = 0 ] = .09
\]

\[
P(\bar{U} = 1) = P[ U_1 = 0 \text{ and } U_2 = 1 ] + P[ U_1 = 1 \text{ and } U_2 = 0 ] = .24
\]

and so on, producing the pmf.
\[
\begin{array}{c|c}
\bar{u} & f(u) \\
0 & .09 \\
\frac{1}{2} & .12 + .12 = .24 \\
1 & .09 + .16 + .09 = .34 \\
\frac{3}{2} & .12 + .12 = .24 \\
2 & .09 \\
\end{array}
\]