Suppose \( x \sim N(\mu, 1) \) and consider testing \( H_0: \mu = 0 \) vs \( H_a: \mu \neq 0 \). Here the generalized log likelihood ratio is

\[
\lambda(x) = \ln \frac{f(x | \hat{\mu}_{MLE})}{f(x | 0)} = \ln \frac{\exp -\frac{1}{2} (x-x)^2}{\exp -\frac{1}{2} (x-0)^2} = \frac{1}{2} x^2
\]

So likelihood ratio tests here will be of form

\[
a(x) = I \left[ \frac{1}{2} x^2 > # \right] \quad \text{for some # > 0}
\]

Notice here that under \( \mu = 0 \)

\[
P \left[ 2 \lambda(x) > c \right] = P \left[ 2 \left( \frac{1}{2} \right) x^2 > c \right] = P \left[ x^2 > c \right] = \chi^2 \text{ probability to the right of } c
\]

So that we get the "large n" \( \chi^2 \) behavior of the test statistic already when \( n = 1 \) ?

Rejecting \( H_0: \mu = 0 \) when \( x^2 > 3.841 \) gives exactly \( \alpha = 0.05 \) here.