Suppose that \( x = (y_1, y_2, \ldots, y_n) \) has iid components where a pdf for \( y \) is:

\[
f(y|x) = \alpha y^{\alpha-1} I[0 < y < 1]
\]

Then for \( y \in (0,1) \):

\[
\frac{\partial}{\partial \alpha} \ln f(y|x) = \frac{1}{\alpha} + \ln y \quad \text{and}
\]

\[
\frac{\partial^2}{\partial \alpha^2} \ln f(y|x) = -\frac{1}{\alpha^2}
\]

So the log-likelihood is:

\[
\ln f(x|x) = \ln \left( \frac{1}{\alpha} \cdot \alpha y^{\alpha-1} \right) = n \ln x + (\alpha-1) \sum_{i=1}^{n} \ln y_i
\]

and the likelihood equation is:

\[
\frac{\partial}{\partial \alpha} \ln f(x|x) = \frac{n}{\alpha} + \sum_{i=1}^{n} \ln y_i = 0
\]

This has a unique solution \( \alpha = \frac{n}{\sum \ln y_i} \).

That is also the unique maximizer of the log-likelihood, i.e.

\[
\hat{\alpha}_{\text{MLE}} = \frac{n}{\sum_{i=1}^{n} y_i}
\]

Theory for the MLE says that \( \hat{\alpha}_{\text{MLE}} \) is approximately normal with mean \( \alpha \) and variance \( \frac{1}{n I_y(\alpha)} \).

From above \( \frac{\partial^2}{\partial \alpha^2} \ln f(y|x) = -\frac{1}{\alpha^2} \) so that

\[
I_y(\alpha) = -E_x \left( -\frac{1}{\alpha^2} \right) = \frac{1}{\alpha^2}
\]

and the variance of the approximating normal is
is $\frac{\alpha^2}{n}$. This gives valid but useless large
approximate confidence limits for $\alpha$ of

$$\hat{\alpha}_n^{\text{MLE}} \pm \frac{\alpha}{\sqrt{n}}$$

The first fix for this ("expected information" fix) is to replace $I_\nu(\alpha)$ with $I_\nu(\hat{\alpha}_n^{\text{MLE}})$ to give usable limits

$$\hat{\alpha}_n^{\text{MLE}} \pm \frac{\hat{\alpha}_n^{\text{MLE}}}{\sqrt{n}}$$

The second fix is to replace $n I(\alpha)$ with the negative curvature of the log-likelihood at the MLE. This is

$$\left(-\frac{n}{\alpha^2}\right) = \frac{n}{(\hat{\alpha}_n^{\text{MLE}})^2}$$

In this particular case the 2 fixes work out to produce the same result.