Suppose the entries of \( \mathbf{x} = (x_1, x_2, \ldots, x_n) \) are iid Poisson \( \lambda \) and the inference problem concerns \( \lambda \). The joint pmf of \( \mathbf{x} \) is

\[
f(\mathbf{x} | \lambda) = \frac{e^{-\lambda} \lambda^{\sum x_i}}{\prod_{i=1}^{n} x_i!}
\]

Then treated as a function of \( \lambda \) for data \( \mathbf{x} \) plugged in, a log-likelihood function is

\[
\ln f(\mathbf{x} | \lambda) = -n \lambda + \left( \sum_{i=1}^{n} x_i \right) \ln \lambda - \sum_{i=1}^{n} \ln(x_i!)
\]

Then

\[
\frac{d}{d\lambda} \ln f(\mathbf{x} | \lambda) = -n + \frac{1}{\lambda}(\sum x_i)
\]

For \( \sum x_i = 0 \) this is negative and so the smallest possible value of \( \lambda \), namely \( \lambda = 0 \) maximizes the likelihood. For other \( \sum x_i \) one may set

\[
\frac{d}{d\lambda} \ln f(\mathbf{x} | \lambda) = 0
\]

and solve for \( \lambda \), obtaining \( \lambda = \bar{x} \). A cartoon for the \( \sum x_i > 0 \) case is below.

Thus in general the likelihood is maximized at \( \bar{x} \) and

\[
\hat{\lambda}_{MLE}(\mathbf{x}) = \bar{x}
\]