In a two independent normal samples model
\[ x_{11}, x_{12}, \ldots, x_{1n_1} \text{ are iid } \mathcal{N}(\mu_1, \sigma_1^2) \text{ independent} \]
\[ x_{21}, x_{22}, \ldots, x_{2n_2} \text{ that are iid } \mathcal{N}(\mu_2, \sigma_2^2) \]
An important quantity for elementary inference is then
\[
F = \frac{S_1^2}{S_2^2} = \frac{(n_1-1)S_1^2}{\left(\frac{\sigma_1^2}{\sigma_1^2}\right)/(n_1-1)} \cdot \frac{\sigma_1^2}{\sigma_2^2} - \frac{(n_2-1)S_2^2}{\left(\frac{\sigma_2^2}{\sigma_2^2}\right)/(n_2-1)}
\]
This then has the form
\[
\frac{(W_1/n_1)}{(W_2/n_2)} - \frac{\sigma_1^2}{\sigma_2^2}
\]
for \( W_1 = \frac{(n_1-1)S_1^2}{\sigma_1^2} \sim \chi^2_{n_1-1} \) independent of
\[ W_2 = \frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1} \]
This motivates the definition of the \( F_{n_1, n_2} \) distribution as the distribution of
\[
\frac{W_1/n_1}{W_2/n_2}
\]
for \( W_1 \sim \chi^2_{n_1} \) independent of \( W_2 \sim \chi^2_{n_2} \). As it turns out, this definition leads to a pdf of the form
\[
f(x) = \frac{1}{B\left(\frac{n_1}{2}, \frac{n_2}{2}\right)} \left(\frac{n_1}{n_2}\right)^{n_1/2} x^{\frac{n_1}{2} - 1} \left(1 + \frac{n_1}{n_2} x\right)^{-\frac{n_1+n_2}{2}} \mathbb{I}[x>0]
\]
and probabilities are available by integrating with this form and are tabulated and produced by statistical software.
Sina \( \frac{S_1^2}{S_2^2} = \left(\frac{\sigma_1^2}{\sigma_2^2}\right) \sim F_{n_1-1, n_2-1} \text{ r.v.} \)
\[
\frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1}
\]

and one can find numbers \# and \#\# so that (e.q.)

\[
P\left[ \# < \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} < \#\# \right] = .95
\]

i.e.

\[
P\left[ \# < \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} < \#\# \right] = .95
\]

This event is exactly the event

\[
\frac{s_1}{s_2\sqrt{\#\#}} < \frac{\sigma_1}{\sigma_2} < \frac{s_1}{s_2\sqrt{\#}}
\]

That is, a 95% confidence interval for \( \frac{\sigma_1}{\sigma_2} \) is

\[
\left( \frac{s_1}{s_2\sqrt{\#\#}}, \frac{s_1}{s_2\sqrt{\#}} \right)
\]