Let's consider arguing directly that the pdf of \( V = -\ln U \)
for \( U \sim U(0,1) \) is the \( \text{Exp}(1) \) pdf.

First, the pdf of \( U \) is \( f_U(u) = I[0 < u < 1] \).

For \( h(u) = -\ln(u) \) for \( u \in (0,1) \)

\( h^{-1}(v) = \exp(-v) \) for \( v > 0 \)

and \( h'(u) = -\frac{1}{u} \) for \( u \in (0,1) \).

Then for \( v > 0 \)

\[
\frac{1}{|h'(h^{-1}(v))|} f(h^{-1}(v)) = \frac{1}{1 - \frac{1}{\exp(-v)}} I\left[0 < \exp(-v) < 1\right] = \exp(-v)
\]

the \( \text{Exp}(1) \) pdf.