Suppose that \((x, y)\) has a joint pdf \(f(x, y)\) on \([0, 1]^2\) with joint pdf

\[
f(x, y) = \begin{cases} 
  (x + y) & \text{if } (x, y) \in [0, 1]^2 \\
  0 & \text{otherwise}
\end{cases}
\]

Then, for any \(x \in [0, 1]\)

\[
f(y | x) = \begin{cases} 
  \frac{(x + y)}{\int_{y} (x + y) \, dy} & \text{for } y \in [0, 1] \\
  0 & \text{otherwise}
\end{cases}
\]

So for \(0 \leq x \leq 1\),

\[
E[y | x] = \int_{0}^{1} y \cdot f(y | x) \, dy
\]

\[
= \int_{0}^{1} y \left( \frac{x + y}{2x + 1} \right) \, dy
\]

\[
= \frac{1}{3} \left( \frac{3x + 2}{2x + 1} \right)
\]

So, the optimal predictor of \(y\) based on \(x\) under SEL is

\[
\hat{y}^{opt} = E[y | x] = \frac{1}{3} \left( \frac{3x + 2}{2x + 1} \right)
\]

It's also possible to find an AEL optimal predictor of \(y\) based on \(x\). For this one needs the median of a conditional dsn of \(y | x\). To this end, consider the conditional cdf of \(y | x\). This is (for \((x, y) \in [0, 1]^2\))
\[ F(y|x) = \int_0^y 2 \left( \frac{x+t}{2x+1} \right) dt = \cdots = \left( \frac{2}{2x+1} \right) \left( \frac{y^2}{2} + xy \right) \]
a quadratic function of \( y \). The conditional median of \( y|x \), \( \hat{y}(x) \), solves

\[
.5 = F(y|x) = \left( \frac{2}{2x+1} \right) \left( \frac{y^2}{2} + xy \right)
\]
i.e.

\[ 0 = y^2 + 2xy - (x + \frac{1}{2}) \]

Use of the quadratic formula and the fact that \( \hat{y} \) must be positive to conclude that

\[
\hat{y}_{\text{opt}}(x) = \hat{y}(x) = -x + \sqrt{x^2 + (x + \frac{1}{2})^2}
\]