

1. In the development of a new CPU, prototype chips are subjected to a combination of high temperature and high humidity intended to produce early failure.

a) Under these accelerated stress conditions, it is desirable that 80% of chips last at least 100 hours before failure. 400 chips of a candidate design are put on test and 105 of them fail before 100 hours. Assess the strength of the evidence in these figures to the effect that life for chips of this design is substandard. (Show all 5 steps of an appropriate significance test.)

b) Near the end of the development process, 45 prototype chips of a candidate design are fabricated and tested. These have a mean life (under accelerated stress conditions) of 131 hr with a standard deviation of life of 20 hr.

i) Give a 90% lower confidence bound for the mean life of these chips under these accelerated stress conditions.

ii) Based on your answer to i), would you expect to find a small  $p$ -value for testing  $H_0 : \mu = 100$  versus  $H_a : \mu > 100$ ? Say "yes" or say "no." Then explain.

2. Does a small  $p$ -value essentially assure one that the true parameter value is much different from the hypothesized one? Explain.

3. An ISU student project studied the relationship between Water/Cement Ratio (by volume),  $x$ , and 14-Day Compressive Strength,  $y$  (in psi), for Portland cement specimens. 9 specimens were made and tested. A Minitab output summarizing a regression analysis of the students' data is attached to this exam. Use it in answering the following questions.

a) Find and interpret the sample correlation between Water/Cement Ratio and 14-Day Compressive Strength.

Interpretation:

$r =$  \_\_\_\_\_

b) What fraction of the observed raw variability in Compressive Strength is accounted for by a linear equation in Water/Cement Ratio?

c) What is the least squares line through this  $(x, y)$  data set? Based on this line, what Water/Cement ratio would you use in an attempt to produce a specimen with compressive strength 2800 psi?

least squares line:

Water/Cement ratio for 2800 psi: \_\_\_\_\_

**Henceforth, adopt the normal simple linear regression model.**

d) Give a 90% two-sided confidence interval for increase in mean Compressive Strength that accompanies an increase of .1 in Water/Cement ratio. (Hint: How is this quantity related to the parameters in  $\mu_{y|x} = \beta_0 + \beta_1 x$  ?)

e) Give a 90% two-sided confidence interval for the mean Compressive Strength of specimens of this type with a .47 Water/Cement ratio.

f) Give a 90% two-sided prediction interval for the next Compressive Strength of a specimen of this type with a Water/Cement ratio of .45.

4. A secure digital communications system has a receiving station with a CPU that can decode messages at a rate of 1 per unit time, and a buffer that will hold up to 4 of these messages. Suppose that messages arrive according to a Poisson process with rate .7 per unit time. Making the assumption that decode times are exponential random variables, answer the following questions about the equilibrium behavior of the receiving station.

a) What fraction of the time is the CPU idle? What fraction of incoming messages are lost (i.e. arrive when the buffer is full)?

fraction of the time idle = \_\_\_\_\_ fraction of messages lost = \_\_\_\_\_

b) What is the mean number of messages in the buffer?

5. A company has 4 software systems that periodically need the attention of a systems analyst. The company employs 2 analysts. Suppose that when operating, each system crashes (putting it out of operation) at rate  $\lambda$  and that when working on a crashed system, an analyst returns it to working order (and back into operation) at rate  $\mu$ . (An analyst works on a single system at a time.) We wish to model this situation using a Birth and Death process with "State  $i$ " being the situation " $i$  software systems are down." Sketch an appropriate state transition diagram (putting appropriate rates on it).

6. A testing lab compares seek times for 2 models of hard disk drive. Tests on 30 Model A units produce a mean seek time of 9.35 ms and a standard deviation of .62 ms. Tests on 40 Model B units produce a mean seek time of 9.07 ms and a standard deviation of .44 ms. Give 90% two-sided confidence limits for the difference in (population) mean seek times. (No need to simplify.)

7. Disk drives will be tested for "seek time" as they come off a production line. Suppose that while these vary drive to drive, for a given production run, test results are normally distributed with  $\sigma = .2$  ms. What mean seek time is required so that only 1% of drives will have test results above 9.5 ms?

8. The following are 5 consecutive realizations of Uniform (0,1) random variables generated by Vardeman's HP-32S calculator:

.5819, .9509, .2222, .0290, .4162

a) Use the values above to generate a single realization,  $w$ , from a Binomial (5,.25) distribution. Indicate clearly what you've done.

$w =$  \_\_\_\_\_ Explanation:

b) Use the values above to generate 5 realizations from the Exponential ( $\alpha$ ) distribution with  $\alpha = 2$ . Indicate clearly what you've done to get these.

exponential realizations = \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

c) Use the values above to generate a single realization from the Erlang distribution with  $k = 5$  and  $\alpha = 2$ . Indicate clearly what you've done to get this.

Erlang realization = \_\_\_\_\_ Explanation:

9. A data base is segmented as below in terms of "type of record" and "completeness of record." Suppose that somehow one will select a single record "at random" from this data base.

	Type A	Type B	Type C
Complete	200 records	350 records	450 records
Incomplete	50 records	50 records	100 records

a) Evaluate  $Pr[\text{Type A or Type B record}]$ .

b) Evaluate  $Pr[\text{Type A or Type B record} \mid \text{Complete record}]$ .

c) Are the events "Type A or Type B record" and "Complete Record" independent? Explain.

10. A discrete distribution has probability mass function given below.

$x$	2	3	4	5
$p(x)$	.1	.2	.2	.5

a) If  $X$  has this distribution, find  $EX$ .

**As it turns out,  $\text{Var}X = 1.09$ .**

b) If  $X_1, X_2, \dots, X_{100}$  are independent with the distribution above, what is the approximate distribution of  $\bar{X} = \frac{1}{100} \sum X_i$ ?

# Printout for Problem 3

MTB > print c1 c2

## Data Display

Row	ratio	strength
1	0.45	2954
2	0.45	2913
3	0.45	2923
4	0.50	2743
5	0.50	2779
6	0.50	2739
7	0.55	2652
8	0.55	2607
9	0.55	2583

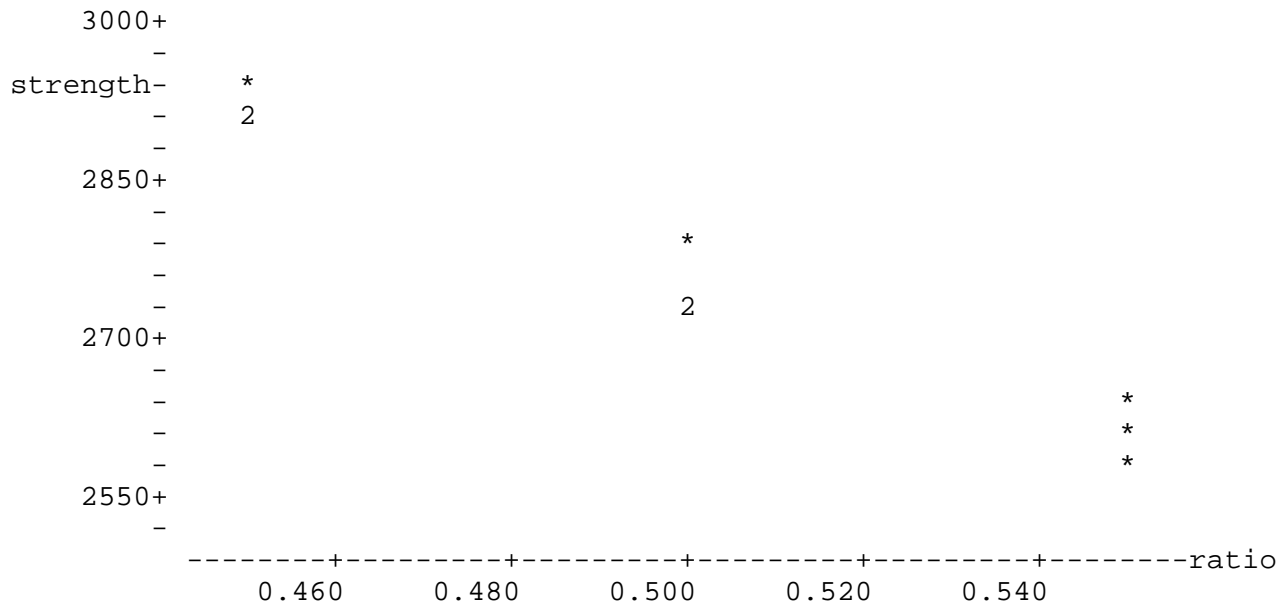
MTB > GStd.

\* NOTE \* Character graphs are obsolete.

MTB > Plot 'strength' 'ratio';

SUBC> Symbol '\*'.  
Symbol '2'.

## Plot



MTB > GPro.

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MTB > Regress 'strength' 1 'ratio';
SUBC>   Constant;
SUBC>   Predict .47;
SUBC>   Confidence 90;
SUBC>   Brief 2.

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## Regression Analysis

The regression equation is  
strength = 4346 - 3160 ratio

Predictor	Coef	StDev	T	P
Constant	4345.9	109.6	39.66	0.000
ratio	-3160.0	218.5	-14.47	0.000

S = 26.76      R-Sq = 96.8%      R-Sq(adj) = 96.3%

## Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	149784	149784	209.24	0.000
Residual Error	7	5011	716		
Total	8	154795			

## Predicted Values

Fit	StDev Fit	90.0% CI	90.0% PI
2860.69	11.07	( 2839.72, 2881.66)	( 2805.83, 2915.54)