

1. Consider an M/M/2/4 queuing system. In what follows, work from first principles and DO NOT try to simply plug and chug with the formulas on pages 679 and 680 of Allen. You may for definiteness think of a communications system with 2 processors for decoding messages and a buffer that will hold 2 messages. Assume that messages arrive at a rate  $\lambda = 1$  per second and that an individual processor decodes messages at a rate of  $\mu = .4$  per second.
- a) Carefully draw the birth and death process state transition diagram for this problem below and mark on the arcs the appropriate transition rates.
- b) Find the steady state probability that there are no messages in the system.
- c) What fraction of messages are lost?
- d) Find the steady state mean number of messages in the buffer.
- e) Find the steady state mean time that *an entering* message spends in the system. (Hint: What is the steady state conditional distribution of the number of messages "ahead of" an entering message, and what are the conditional mean times in the system for the new message for each possible number of messages ahead of it?)

2. A television marketing firm has a phone bank that is purposely much larger than is needed to handle the typical volume of incoming phone orders, as the payoff for each call received is huge in comparison to the cost of hiring an operator and paying for a phone line, and there is the fear that a call lost because of a busy signal will not be redialed. In fact there are 50 operators at work at the phone bank, calls arrive at rate of 1 per minute and require on average 2 minutes to complete.

a) Approximate the steady state mean number of operators busy at this phone center.

b) Approximate the steady state probability that 2 or more operators are busy at this phone center.

3. A company has 3 machines that each break down at exponentially distributed intervals of mean 1 hour. When one of these machines is down, only one repairman can work on it at a time, and while it is down the company loses \$50/hour in profit. Repairs require 1 hour on average to complete and the repair times are exponentially distributed. Repairmen can be hired for \$30/hour (and must be paid regardless of whether there is a broken machine to work on). Do you recommend that the company employ 1 repairman or 2 repairmen? Show your whole analysis.

3. Below are measured gains on  $n = 30$  amplifiers (in dB).

7.81 7.95 8.01 9.09 9.35 9.89 9.95 9.96 10.23  
10.52 10.60 10.69 10.71 11.71 11.93 12.21 12.22 12.82  
13.06 13.16 13.20 13.45 13.47 13.64 14.21 15.30 15.46  
15.52 16.17 25.43

These values have mean  $\bar{x} = 12.257$  and standard deviation  $s = 3.406$ .

- a) Give 90% 2-sided confidence limits for the mean measured gain of all such amplifiers. (No need to simplify.)
- b) If one wants to estimate the mean gain to within .2 dB with confidence 95%, what sample size do you recommend?
- c) Give an approximate 90% lower confidence bound for the fraction of such amplifiers with measured gains below 10 dB.
- d) What confidence do you have that a single additional amplifier of this type will have measured gain between 8.01 dB and 15.52 dB?
- e) What confidence do you have in 8.01 dB as a lower tolerance bound for 90% of measured gains for all such amplifiers?

4. Simulations on 2 complicated queueing systems were made to compare several steady state characteristics of the two systems. Below are some summary statistics for  $n_1 = 500$  simulation runs for System 1 and  $n_2 = 600$  simulation runs for System 2.

System 1	System 2
Large $t$ Queue Length	Large $t$ Queue Length
$\bar{x}_1 = 13.5$	$\bar{x}_2 = 10.1$
$s_1 = 4.5$	$s_2 = 3.2$
Large $t$ Server Availability	Large $t$ Server Availability
$\hat{p}_1 = 429/500$	$\hat{p}_2 = 503/600$

(The availability figures are the sample fractions of the runs where at least one server was available at the time point checked.)

a) Give 95% approximate 2-sided confidence limits for the difference in large  $t$  mean queue lengths. (No need to simplify.)

b) Give 95% approximate 2-sided confidence limits for the difference in large  $t$  server availabilities. (No need to simplify.)

c) If I am not satisfied with the precision provided in b) and want to conduct a second study to estimate the difference in large  $t$  availability to within .01 with 99% confidence using a conservative interval based on a sample size  $n$  for both systems, what  $n$  do you recommend?