

2. Two possible configurations of a receiver in a secure digital communications system are as follows.

Configuration A: a single double-speed processor

Configuration B: two single-speed processors

If a message arrives and a processor is available, it goes immediately to the processor for decoding, otherwise it is lost. Messages arrive according to a Poisson process with rate 1 message per time unit. A single speed processor decodes messages at a mean rate of 1 message per time unit.

Which configuration decodes the largest fraction of incoming messages? (Find both loss rates.)

3. In the context of the previous problem, consider adding a one-message buffer to Configuration A. If a new message arrives while the processor is busy and the buffer is empty, the message is stored in the buffer and immediately transferred to the processor when the processor becomes available.

a) Sketch an appropriate birth and death process state transition diagram for this situation.

b) What fraction of incoming messages are decoded using this configuration?

4. Attached to this exam is a Minitab output summarizing a simulation of the distribution of the sample mean \bar{U} of 5 iid Uniform (0,1) random variables. (Note that $n = 100$ realizations of \bar{U} have been generated.)
- a) Probability theory tells us that $\mathbf{m} = E\bar{U} = .5$. If we did not know this, we might have used the $n=100$ realizations to make a 90% two-sided confidence interval for \mathbf{m} . Find that interval. Does it cover $\mathbf{m} = .5$?
- b) It is tedious to evaluate $p = P[\bar{U} \leq .55]$ exactly, but it is easy to approximate this value via simulation. Give a 90% two-sided confidence interval for this unknown probability.
- c) What number of realizations, n , do you recommend for another simulation if one wishes to estimate the probability in b) to within .01 with confidence 95%?
- d) The smallest of the 100 realizations of \bar{U} was approximately .173 and the largest was about .776. What confidence should one have that the interval (.173,.776) brackets 90% of the actual (theoretical) distribution of \bar{U} ?

5. Miscellaneous Short Answer.

- a) For a particular queueing system composed of two subsystems, individuals arrive at the first subsystem a rate of 1 per unit time and spend an average of 2 time units in the system (in queue and being serviced). 50% of these individuals then leave the system while the other 50% enter the second subsystem and spend an average of 3 time units in the second subsystem. On average, how many individuals are in the system at equilibrium?
- b) For a sample of size $n = 100$, what i do you recommend in order to make the interval (i th smallest data point, i th largest data point) have at least 90% confidence as a prediction interval for one additional data point?
- c) Interpret the sentence “(3.4,5.7) is a 90% confidence interval for $\mu_1 - \mu_2$.” for someone who knows nothing about statistical inference.

Worksheet size: 100000 cells
 MTB > Random 100 c1-c5;
 SUBC> Uniform 0.0 1.0.
 MTB > RMean c1-c5 c6.
 MTB > Describe C6.

Descriptive Statistics

Variable	N	Mean	Median	TrMean	StDev	SEMean
C6	100	0.5131	0.5145	0.5155	0.1314	0.0131

Variable	Min	Max	Q1	Q3
C6	0.1728	0.7759	0.4324	0.5981

MTB > GStd.
 MTB > Histogram C6;
 SUBC> Start .16;
 SUBC> Increment .02.

Character Histogram

Histogram of C6 N = 100

Midpoint	Count
0.1600	0
0.1800	2 **
0.2000	0
0.2200	1 *
0.2400	0
0.2600	2 **
0.2800	0
0.3000	1 *
0.3200	2 **
0.3400	1 *
0.3600	4 ****
0.3800	4 ****
0.4000	1 *
0.4200	5 *****
0.4400	7 *****
0.4600	7 *****
0.4800	8 *****
0.5000	2 **
0.5200	8 *****
0.5400	7 *****
0.5600	5 *****
0.5800	5 *****
0.6000	5 *****
0.6200	4 ****
0.6400	4 ****
0.6600	3 ***
0.6800	3 ***
0.7000	0
0.7200	3 ***
0.7400	2 **
0.7600	2 **
0.7800	2 **

MTB > GprO.