Stat 330X Exam II

March 10, 2000 Prof. Vardeman

1. Suppose that the times till failure of circuit packs of a particular design are exponential with rate $\alpha = .4$ (per year).

a) Find the probability that a particular single circuit pack lasts at least 2.0 years.

b) Suppose that a switching system uses 20 of these circuit packs (simultaneously and independently). Let X be the number of the original circuit packs that are still installed and working in the switching system, 2.0 years into the life of the system. Use your answer to a) and find the probability that $X \leq 18$. What is the distribution of X? (Give both a name and appropriate parameter value(s).)

$X \sim$ ________________

$P[X \leq 18] =$ ________________

c) In a particular application, only one of these circuit packs is in service at a time. In addition to the in-service circuit pack, there are 2 back-up packs. (The first back-up is placed in service when the original pack fails and the second when the first replacement fails.) Let Y be the total operating time of the original and 2 back-up packs. What are the mean and variance of Y, and the probability that Y exceeds 5.0 years?

E$Y =$ ________________ Var$Y =$ ________________

$P[Y > 5.0] =$ ________________

d) If 100 of these circuit packs are purchased by a telecommunications company and placed into service (in independent applications), approximate the probability that the mean life of these 100 packs exceeds 2.2 years.

2. Measured resistances of leads on some small electronic components are normal with mean .2 and standard deviation .01 (units are $10^{-3}\Omega$).

a) Evaluate the probability that a single lead of this type has measured resistance above .189 ($10^{-3}\Omega$).

b) Use your answer to part a) and evaluate the probability that if I begin testing leads of this type, I need to test at least 3 leads in order to find one with measured resistance above .189 ($10^{-3}\Omega$).

3. Suppose that an experienced C++ programmer makes errors at a rate about .002 per line of code and that a Poisson process model is a decent approximate description of the generation of these errors. Consider a 1500 line program and assess the probability of at least 2 programming errors.

4. Suppose that you have at your disposal a Uniform $(0,1)$ random number generator. If I desire to simulate 100 "dart throws" at the square in the x-y plane defined by $0 < x < 2$ and $0 < y < 2$ (each throw "uniformly distributed over the square") how do you propose to use the uniform random number generator to do this? (Hint: x and y coordinates of the points are independent.)

5. If January interest rates on 1 year CD's over the next 5 years are $R_1, R_2, R_3, R_4,$ and R_5, \$1 invested in January and left to draw interest through 5 years will grow to $I = (1+R_1)(1+R_2)(1+R_3)(1+R_4)(1+R_5)$ at the end of 5 years. I am willing model the January rates as independent random variables, each uniform on $(.04, .06)$ and wish to know about the distribution of I. Attached to this exam is a Minitab printout useful in studying I. Use it to help you approximate the standard deviation of I and the probability that I is less than 1.256. Does it seem reasonable to treat I as normal? Why?

$\sqrt{\mathrm{Var} I} \approx$ ______________

$P[I < 1.256] \approx$ ______________

I approximately normal? yes/no (circle one) Explain:

6. Type A messages arrive at a telecommunications center according to Poisson process with rate $\lambda_A = 5$ per hour, while Type B messages arrive (independently) according to a Poisson process with rate $\lambda_B = 3$ per hour. We start observing the incoming message stream at midnight tonight.

a) What is the probability that the first message to arrive is a Type A message? (If you want partial credit should you be wrong, provide some rationale for your answer.)

b) Outline (naming a particular probability distribution and saying exactly how you would use it) how you would evaluate the probability that at least 10 messages (total) arrive between midnight and 1 AM.

c) Say carefully how you would use your answer to a) to evaluate the probability that among the first 10 messages to arrive, at least 5 are of Type A.

Mintab Printout for Question 5

```
MTB > Random 1000 c1-c5;
SUBC>   Uniform .04 .06.
MTB > let c1=1+c1
MTB > let c2=1+c2
MTB > let c3=1+c3
MTB > let c4=1+c4
MTB > let c5=1+c5
MTB > let c6=c1*c2*c3*c4*c5
MTB > Describe c6.
```

Descriptive Statistics

```
Variable       N     Mean   Median   TrMean    StDev   SEMean
C6          1000   1.2775   1.2765   1.2774   0.0161   0.0005

Variable     Min      Max       Q1       Q3
C6        1.2280   1.3271   1.2669   1.2890
```

```
MTB > GStd.
MTB > Histogram c6;
SUBC>   Start 1.23;
SUBC>   Increment .004.
```

Character Histogram

```
Histogram of C6   N = 1000
Each * represents 5 obs.

Midpoint   Count
 1.23000       2  *
 1.23400       1  *
 1.23800       5  *
 1.24200       7  **
 1.24600      12  ***
 1.25000      26  ******
 1.25400      40  ********
 1.25800      44  *********
 1.26200      62  *************
 1.26600      72  ***************
 1.27000     101  *********************
 1.27400     113  ***********************
 1.27800      94  *******************
 1.28200      86  ******************
 1.28600      71  ***************
 1.29000      67  **************
 1.29400      67  **************
 1.29800      40  ********
 1.30200      40  ********
 1.30600      18  ****
 1.31000      14  ***
 1.31400       8  **
 1.31800       5  *
 1.32200       2  *
 1.32600       3  *
```