

Stat 330X Exam II

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Prof. Vardeman

1. Some jobs submitted for processing on a particular CPU have fatal programming errors, while others do not. Suppose that the long run fraction of jobs with fatal programming errors is  $p = .05$ .

a) Find (under appropriate model assumptions) the probability that among the next 10 jobs submitted there are less than 3 with fatal errors.

b) One begins monitoring the processing of jobs and lets

$X =$  the number of jobs processed before the first with a fatal error .

Find (again under appropriate model assumptions)  $P[X \leq 25]$ .

c) One begins monitoring the processing of jobs and lets

$Y =$  the total number of jobs without errors processed before the 2nd job with a fatal error .

Find (under appropriate model assumptions)  $EY$  and  $\text{Var}Y$ .

2. Hits on a popular Web page occur according to a Poisson Process with rate  $\lambda = 10$  hits/min. One begins observation at exactly noon tomorrow (WOI standard time).

a) Evaluate the probability of 2 or less hits in the first minute.

b) Evaluate the probability that the time till the first hit exceeds 10 seconds.

c) Evaluate the mean and the variance of the time till the 4th hit.

d) Evaluate the probability that the time till the 4th hit exceeds 24 seconds.

e) The number of hits in the first hour is Poisson with mean 600. You would like to know the probability of more than 650 hits. Exact calculation isn't really feasible. So approximate this probability and justify your approximation.

3. A physical roulette wheel has 38 (supposedly equally likely) slots in it. It's your job to program a "video" roulette game. Someone else has provided you with a good Uniform  $(0, 1)$  random number generator that you may use. Carefully describe how you will simulate successive spins of the wheel.

4. The log-normal distribution is a continuous distribution sometimes used to describe non-negative random variables.  $X \sim \text{log-normal}(\mu, \sigma^2)$  means that  $Y = \ln(X) \sim \text{normal}(\mu, \sigma^2)$ .

a) Suppose that the lives (in years) of a model of disk drive are log-normal with  $\mu = 1$  and  $\sigma^2 = .25$ . For  $X$  the life of such a drive, evaluate  $P[1 < X < 2]$ . (Note that this is  $P[0 < \ln(X) < .693]$ .)

b) V Corporation's average debt in the period of fiscal 2000 will be  $D$  and the average interest rate the company will pay during that period will be  $R$ . Roughly, then,

$$I = \text{the interest on debt (in dollars) that V Corp will pay in 2000} = DR .$$

In assessing uncertainty in her projection of  $I$ , the company's Chief Financial Officer models  $D$  and  $R$  as independent,  $D \sim \text{log-normal}$  with  $\mu = 16$  and  $\sigma = 1$  and  $R \sim \text{log-normal}$  with  $\mu = -2.81$  and  $\sigma = .16$ . Evaluate the CFO's probability that less than \$500,000 will be paid in interest.

5. Suppose that  $U_1, U_2$  and  $U_3$  are independent Uniform  $(0, 1)$  random variables and distributional properties of  $Y = \min(U_1, U_2, U_3)$ ,  $W = \max(U_1, U_2, U_3)$  and  $R = W - Y$  are of interest. Attached to this exam is a printout from a Minitab session conducted to study these variables. (On that printout "RMinimum" and "RMaximum" are minimum and maximum values in a given row.)

a) Use the printout to deduce approximate values for the following:

$$EW \approx \underline{\hspace{2cm}} \qquad EY \approx \underline{\hspace{2cm}} \qquad ER \approx \underline{\hspace{2cm}}$$

$$\text{Var}W \approx \underline{\hspace{2cm}} \qquad \text{Var}Y \approx \underline{\hspace{2cm}} \qquad \text{Var}R \approx \underline{\hspace{2cm}}$$

$$P[R > .5] \approx \underline{\hspace{2cm}}$$

b) Do your approximate values of  $\text{Var}W$  and  $\text{Var}Y$  sum to something close to your approximate value of  $\text{Var}R$ ? Do you expect them to? Explain.

6. A continuous distribution of interest has distribution function

$$F(y) = \begin{cases} 0 & \text{if } y \leq 0 \\ 1 - (1 - y)^3 & \text{if } 0 < y < 1 \\ 1 & \text{if } y \geq 1 \end{cases}$$

Describe as completely as possible how you would simulate a realization of  $Y$  with this distribution.

Minitab Printout for Stat 330X Exam II, Problem 5

```
MTB > Random 1000 C1-C3;
SUBC> Uniform 0.0 1.0.
MTB > RMinimum C1 C2 C3 C4.
MTB > RMaximum C1 C2 C3 C5.
MTB > Let C6 = C5 - C4
MTB > Describe C4 C5 C6.
```

Descriptive Statistics

Variable	N	Mean	Median	TrMean	StDev	SE Mean
C4	1000	0.25134	0.21293	0.23860	0.19324	0.00611
C5	1000	0.75152	0.79377	0.76452	0.19303	0.00610
C6	1000	0.50017	0.50961	0.50039	0.21395	0.00677

Variable	Minimum	Maximum	Q1	Q3
C4	0.00002	0.90188	0.09468	0.36765
C5	0.12254	0.99850	0.63536	0.91337
C6	0.01594	0.98028	0.33519	0.66442

```
MTB > GStd.
* NOTE * Character graphs are obsolete.
```

```
MTB > Histogram C6;
SUBC> Start .025;
SUBC> Increment .05.
```

Histogram

Histogram of C6 N = 1000  
Each \* represents 2 observation(s)

Midpoint	Count
0.0250	4 **
0.0750	22 *****
0.1250	22 *****
0.1750	42 *****
0.2250	59 *****
0.2750	53 *****
0.3250	73 *****
0.3750	71 *****
0.4250	69 *****
0.4750	75 *****
0.5250	70 *****
0.5750	93 *****
0.6250	77 *****
0.6750	71 *****
0.7250	68 *****
0.7750	43 *****
0.8250	39 *****
0.8750	30 *****
0.9250	15 *****
0.9750	4 **