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1. A business analyst forecasts supply and demand for a particular item using ideas from probability. In fact, to describe

S = the number of items that will be available in a future period

and D = the number of items needed in the same future period,

this person uses a joint distribution with joint probability mass function:

$d \backslash s$	0	1	2	3
0	.10	.01	.01	.01
1	.06	.07	.03	.02
2	.04	.10	.10	.04
3	.05	.05	.09	.09
4	.02	.02	.04	.05

a) Find the marginal distribution this person is using for S .

b) In this model, $ES = 1.42$, $\text{Var}S = 1.2036$, $ED = 2.1$ and $\text{Var}D = 1.49$ and $\text{Cov}(S, D) = .548$. Use these facts and find the mean and variance of the random variable $D - S$.

c) Evaluate $P[D > S]$ using this person's model.

d) An "unmet demand" variable, U , might be defined as

$$U = \begin{cases} D - S & \text{if } D > S \\ 0 & \text{otherwise.} \end{cases}$$

Write out (but don't bother to simplify) a sum giving EU under this person's model.

2. When ready to configure a PC order, a consumer must choose a Processor Chip, a Motherboard, a Drive Controller and a Hard Drive. The choices are:

<u>Processor Chip</u>	<u>MotherBoard</u>	<u>Drive Controller</u>	<u>Hard Drive</u>
700 MHz	Premium	Premium	Premium
600 MHz	Standard	Standard	Standard
500 MHz		Economy	Economy
400 MHz			

a) Suppose initially that all components are compatible with all components. How many different configurations are possible?

Suppose henceforth that:

- 1) a Premium MotherBoard is needed to run a 600 or 700 MHz Processor,
- 2) a Premium MotherBoard is needed to run a Premium Drive Controller,
- 3) a Premium Drive Controller is needed to run a Premium Hard Drive.

b) How many permissible configurations are there with a Standard MotherBoard?

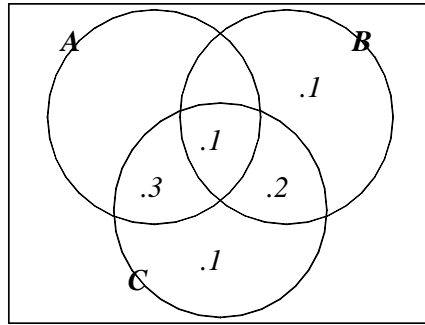
c) How many permissible configurations are there total? Explain carefully.

3. For a "series system" of k identical and independent components, all k must function at time t for the system to function. Suppose that the continuous random variable

$$T_i = \text{time of failure of component } i$$

has cumulative probability function $F(t)$ for each i . For a series system of k components, express, in terms of $F(t)$, the probability that **the system** "dies" a point beyond time t .

4. Here is a Venn diagram with some probabilities of events marked on it. In addition to the values marked on the diagram it is the case that, $P[B] = .4$ and $P[C|A] = .8$.



a) Finish filling in the probabilities on the diagram. That is, evaluate the 3 probabilities below:

$$P[A \cap B \cap \overline{C}] = \underline{\hspace{2cm}}$$

$$P[A \cap \overline{B} \cap \overline{C}] = \underline{\hspace{2cm}}$$

$$P[\overline{A \cup B \cup C}] = P[\overline{A} \cap \overline{B} \cap \overline{C}] = \underline{\hspace{2cm}}$$

b) Use the probabilities on the diagram (and your answers to a)) and evaluate $P[A \cap B]$.

c) Use the probabilities on the diagram and evaluate $P[B|C]$.

d) Based on the information provided here, are the events B, C independent events? Explain.

5. A continuous random variable X has probability density

$$f(x) = \begin{cases} \frac{x}{2} & \text{for } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

a) Evaluate $F(1)$.

b) Find EX .

6. Suppose that a discrete random variable X has probability mass function

x	0	1	2	3	4	5
$p(x)$	1/32	5/32	10/32	10/32	5/32	1/32

a) Find EX and $\text{Var}X$.

b) In some contexts, it may be useful to define a "conditional distribution of X given that $X \geq 2$." Fill in the table below specifying such a conditional distribution. (Hint: How would you evaluate, for example, $P[X = 3 | X \geq 2]$?)

x	2	3	4	5
$p(x X \geq 2)$				