Stat 330X Exam I

September 25, 1999 — Prof. Vardeman

1. A continuous random variable, X, has distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^3 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 < x \end{cases}$$

a) What is $P[X = .1]$?

b) What is $P[X \leq .1]$?

c) Evaluate EX. (Hint: What is $f(x)$?)

2. Here is a small table specifying the joint probability mass function for discrete random variables X and Y. Use it to answer the following questions.

		$x=1$	$x=2$	$x=3$
y	3	.1	.2	.1
	2	.1	.1	.1
	1	.1	0	.2

a) Give the marginal probability mass functions for X and Y.

b) Using your answer to a), find EX.

c) Are X and Y independent? Explain.

d) Evaluate $P[X = Y]$.

e) Find the conditional probability mass function for Y given that $X = 2$. Use it to evaluate E$[Y|X = 2]$.

3. A systems analyst wants to spot-check some large files for corruption by comparing to copies on a back-up tape. The files are organized into 3 folders. Folder A has 2 files in it, 1 of which is corrupted. Folder B has 4 files in it, 1 of which is corrupted. Folder C has 4 files in it, none of which is corrupted.

Suppose first that the analyst ignores the file directory structure and selects 2 files at random (without replacement) from the 10 to check.

a) What is the probability that neither of the files the analyst checks is corrupted?

b) What is the probability that neither of the files is corrupted or both are from Folder C?

c) What is the conditional probability that both are from Folder C given that neither is corrupted?

Now suppose that instead of sampling as above, the analyst does the following: He selects a single folder at random and from that folder he selects a single file at random.

d) What is the probability that the file is corrupted?

e) What is the conditional probability that the file is from Folder A, given that it is corrupted?

4. A "parallel system" consists of 3 independent identical components, one of which must function if the system is to function. (All must fail if the system is to fail.)

a) Suppose that at a particular point in time, the probability that any particular component functions is .8. What is the probability that the system operates at that time?

b) Consider a dynamic version of this problem. For $i = 1, 2, 3$ let $X_i =$ the time till failure of component i. Notice that for $t > 0$

$$P[\text{component } i \text{ is working at time } t] = P[X_i > t] \quad .$$

Suppose that for $t > 0$,

$$P[X_i > t] = \exp(-t)$$

Let $Y =$ the time till system failure. Using the independence of X_1, X_2 and X_3, give a formula for the distribution function of Y, $F(t) = P[Y \leq t]$.

5. A digital communications system transmits information encoded as strings of 0's and 1's. As a means of reducing transmission errors, each digit in a message string is repeated twice. Hence the message string {0 1 1 0} would (ideally) be transmitted as {00 11 11 00} and if digits received in a given pair don't match, one can be sure that the pair has been corrupted in transmission.

Suppose that when each individual digit in a "doubled string" like {00 11 11 00} is transmitted, there is a probability p of transmission error and that whether or not a particular digit is correctly transferred is independent of whether any other one is correctly transferred.

Suppose first that the single pair {00} is transmitted.

a) Find the probability that the pair is correctly received.

b) Find the probability that what is received has obviously been corrupted.

c) Find the conditional probability that the pair is correctly received given that it is not obviously corrupted.

Suppose now that the "doubled string" {00 00 11 11} is transmitted and that the string received is not obviously corrupted.

d) What is then a reasonable assignment of the "chance" that the correct message string (namely {0 0 1 1}) is received? (Hint: Use your answer to part c).)