Stat 231 Exam 3
Spring 2010

I have neither given nor received unauthorized assistance on this exam.

Name Signed    Date

Name Printed
This exam concerns analyses of some data collected by ISU students Flood and Shankwitz in a study of tempering (heat treating) of a particular grade of stainless steel. 48 slugs of this material were pre-processed to a fairly uniform hardness, and then tempered at various combinations of time $x_1$ (in minutes) and temperature $x_2$ (in °F). (3 slugs were tested at each of the 16 combinations of $x_1 = 5, 50, 150, 500$ and $x_2 = 800, 900, 1000, 1100$.) The response $y$ recorded by the students was the increase in Brinell hardness as a result of tempering.

One might consider expressing time on a log scale. Below are plots of the 16 time-temperature combinations the students employed, with $x_1$ on the both the original and log scales.

The first few lines of the JMP data table are as below.
And here is some information on a variety of fits made to these data using the Fit Model routine in its "Stepwise" mode.

Some summaries of the data are:

\[
\sum_{i=1}^{48} y_i = -119 \quad \text{and} \quad \sum_{i=1}^{48} (y_i - \bar{y})^2 = 575.9792
\]

\[
\sum_{i=1}^{48} x_{2i} = 45,600 \quad \text{and} \quad \sum_{i=1}^{48} (x_{2i} - \bar{x}_2)^2 = 600,000
\]

Various JMP reports are scattered at appropriate points in the body of this exam and others are collected at end of the exam. Use these as appropriate to help you answer the questions here.
Begin by considering the possibility of explaining hardness increase, $y$, in terms of only temperature, $x_2$. In particular, models

$$y = \beta_0 + \beta_1 x_2 + \varepsilon \quad \text{(Model 1)}$$

$$y = \beta_0 + \beta_1 x_2 + \beta_2 x_2^2 + \varepsilon \quad \text{(Model 2)}$$

might be of interest. Two relevant JMP reports follow.

\[ \text{7 pts} \]

a) What is the value of the sample correlation $r$ between $y$ and $x_2$?
b) What are 95% two-sided confidence limits for the standard deviation of hardness increase at a fixed temperature based on Model 1? (Upper and lower 2.5% points of the \( \chi^2_{46} \) distribution are 66.62 and 29.16. Plug in completely, but you need not simplify.)

c) Under Model 1, give 95% two-sided confidence limits for the change in mean \( y \) that accompanies a 100 °F increase in temperature (\( x_2 \)).

d) Under Model 1, what are 95% two-sided confidence limits for the mean hardness increase of slugs tempered at 1000 °F? (Plug in completely, but you need not simplify.)

e) What is the value of a test statistic, a reference distribution, and a \( p \)-value for comparing Models 1 and 2?

Value of test statistic:

Reference distribution:

\( p \)-value:
Based on the information on the JMP reports on page 4, which of Model 1 and Model 2 seems to you to be most attractive and why? (Circle the "best" model below and then explain.)

Best Model: Model 1 or Model 2

Reasoning:

Now consider modeling $y$ in terms of both $x_1$ and $x_2$. Attached to the back of this exam are several JMP reports that may use as you answer the rest of the questions on this exam. Below are several plots for the model

$$y = \beta_0 + \beta_1 \ln(x_1) + \beta_2 x_2 + \beta_3 (\ln(x_1))^2 + \beta_4 x_2^2 + \beta_5 \ln(x_1) \cdot x_2 + \epsilon$$

(Model 3)
g) Do the plots on page 6 indicate any serious problems with Model 3? Explain.

h) What about the "Stepwise" Fit Model report on page 3 indicates that expressing time on the log scale is more effective for modeling purposes that expressing it simply in minutes?

i) What is the value of an F statistic and degrees of freedom for testing whether overall Model 3 has any effectiveness in the prediction of the hardness increase provided by tempering.

\[ F = \ldots \]  
\[ df = \ldots, \ldots \]

j) What is the value of the sample correlation between \( y \) and \( \hat{y} \) (based on Model 3)?
Consider the conditions $x_1 = 300$ and $x_2 = 950$. As it turns out, JMP indicates that $SE_y = .5187$ for Model 3 and this set of conditions. What then are 95% two-sided prediction limits for the next hardness increase, $y_{new}$, for a slug tempered at this set of conditions? (Plug in completely, but you need not simplify.)

Give the value of an F statistic and the degrees of freedom for testing $H_0: \beta_3 = \beta_4 = \beta_5 = 0$ in Model 3.

If the hypothesis in part l) (immediately above) is true, give 95% two-sided confidence limits for the increase in mean $y$ that is associated with a 100 °F increase in temperature for time held fixed.