I have neither given nor received unauthorized assistance on this exam.

KEY

Name Signed

Date

Name Printed
1. US 5 cent pieces have weights that are roughly normal with mean $\mu = 5.00$ g and standard deviation $\sigma = .06$ g. What is the probability that $n = 4$ of these coins taken from my pocket have a total weight of more than 20.06 g? (Hint: If it helps, express this probability in terms of the sample mean weight of the $n = 4$ coins.)

$$P(\text{total} > 20.06) = P(\bar{X} > 5.015)$$

$\bar{X} \sim \text{Normal with mean } m_{\bar{X}} = \mu = 5.00$

$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{.06}{\sqrt{4}} = .030$

$z = \frac{5.015 - 5.00}{.03} = .5$

$p(z > .5) \approx .3085$

2. An object of mass $M$ spinning in a plane around a point a distance $R$ away from its center of mass has rotational inertia

$$I = MR^2$$

If $M$ is modeled as random with mean 1 kg and standard deviation .01 kg and $R$ is modeled as random (and independent of $M$) with mean 1 m and standard deviation .01 m, what are an approximate mean and standard deviation for $I$? (It's actually possible to find exact values for these, but if you don't see how to get exact values, provide approximate values.)

$$E(I) \approx MM \left(\frac{MR}{2}\right)^2 = 1 \text{ kgm}^2$$

$$\text{Var}(I) \approx \left(\frac{\partial I}{\partial M}\right)^2 \text{Var} M + \left(\frac{\partial I}{\partial R}\right)^2 \text{Var} R$$

$$= (MR_0)^2 \text{Var} M + (2\mu_M \mu_R)^2 \text{Var} R$$

$$= 1 (.01)^2 + 4 (.01)^2 = 5 (.01)^2$$

$$\mu_I \approx 1 \text{ kgm}^2 \quad \sigma_I \approx \sqrt{5} (.01) \text{ kgm}^2$$
3. Drill depths in a sample of $n = 15$ pump housings had $\bar{x} = 1.2909$ inch and $s = .0007$ inch and an approximately normal relative frequency histogram.

7 pts a) Give end-points of a two-sided interval that you are "95% sure" contains the mean drill depth for these housings. (PLUG IN COMPLETELY, but there is no need to simplify.)

Use $\bar{x} \pm t \frac{s}{\sqrt{n}}$

$$1.2909 \pm 2.145 \frac{.0007}{\sqrt{15}}$$

7 pts b) Give end-points of a two-sided interval that you are "95% sure" will bracket a single additional drill depth. (PLUG IN COMPLETELY, but there is no need to simplify.)

Use $\bar{x} \pm t \sqrt{1 + \frac{1}{n}}$

$$1.2909 \pm 2.145 \sqrt{1 + \frac{1}{15}} \left( .0007 \right)$$

6 pts c) Give end-points of a two-sided interval that you are "95% sure" contains 99% of all drill depths for these housings. (PLUG IN COMPLETELY, but there is no need to simplify.)

Use $\bar{x} \pm t_{0.005} s$

$$1.2909 \pm 3.878 \left( .0007 \right)$$

(from Table A.6 of Devore)
4. In a visual inspection process, two different inspectors check (among a very large number of newly-produced vials that they inspect during an 8-hour work shift) 100 glass vials that are known by a supervisor to be defective and that have been marked by her with ink visible only under ultraviolet light. If the inspectors fail to catch and remove these marked vials from the production stream, the supervisor can identify and remove them after ordinary inspection is complete. The more experienced inspector catches 75 of the defective vials, while the less experienced inspector catches 65 defective vials.

13 pts a) Is there conclusive evidence (with \( \alpha = .05 \)) that the more experienced inspector is better at identifying defective vials than the less experienced employee? (Carefully employ the whole 7-step format.)

1. The quantity of interest is the difference in rates of detection of defective vials.

2. \( H_0: \hat{P}_{exp} - \hat{P}_{not} = 0 \)

3. \( H_a: \hat{P}_{exp} - \hat{P}_{not} > 0 \)

4. The test statistic will be:

\[
z = \frac{\hat{P}_{exp} - \hat{P}_{not}}{\sqrt{\frac{\hat{P}(1-\hat{P})}{n_{exp}} + \frac{1}{n_{not}}}}
\]

5. We'll reject for large \( z \) i.e. \( z > 1.645 \) for \( \alpha = .05 \)

6. The samples give:

\[
z = \frac{.75 - .65}{\sqrt{(.7)(.3) \left(\frac{1}{100} + \frac{1}{100}\right)}} = 1.54
\]

7. Do not reject \( H_0 \). This is not (with \( \alpha = .05 \)) conclusive evidence that the experienced inspector has a higher detection rate for defective vials.

7 pts b) Give 95% two-sided confidence limits for the difference in long-run fractions of defective vials detected (experienced minus inexperienced) by these two inspectors. (PLUG IN COMPLETELY, but there is no need to simplify.)

Use:

\[
\hat{P}_{exp} - \hat{P}_{not} = z \sqrt{\frac{\hat{P}_{exp}(1-\hat{P}_{exp})}{n_{exp}} + \frac{\hat{P}_{not}(1-\hat{P}_{not})}{n_{not}}}
\]

\[
.75 - .65 \pm 1.96 \sqrt{\frac{(104)(.274)}{104} + \frac{(104)(.374)}{104}}
\]

4
5. A student project concerned measurement of resistivity of a type of copper wire at two different temperatures. Seven pieces of this were used in their study, and measured resistances at 0.0°C and at 21.8°C are in the following table. (Units are \(10^{-8} \, \Omega m\).)

<table>
<thead>
<tr>
<th>Wire</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.8°C</td>
<td>1.72</td>
<td>1.56</td>
<td>1.68</td>
<td>1.64</td>
<td>1.69</td>
<td>1.71</td>
<td>1.72</td>
</tr>
<tr>
<td>Resistivity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0°C</td>
<td>1.52</td>
<td>1.44</td>
<td>1.52</td>
<td>1.52</td>
<td>1.56</td>
<td>1.49</td>
<td>1.56</td>
</tr>
<tr>
<td>Resistivity</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Give a 95% lower confidence bound for the mean increase in resistivity of this wire associated with an increase in temperature from 0.0°C to 21.8°C. (PLUG IN COMPLETELY, but there is no need to simplify.)

\[
\overline{d} = 0.1586, \quad s_d = 0.0393
\]

\[
\text{use} \quad \overline{d} - t \frac{s_d}{\sqrt{n}}
\]

\[
0.1586 - 1.943 \left( \frac{0.0393}{\sqrt{7}} \right)
\]

10 pts

6. Here is a NQT normal plot for angles in degrees (to a flat machined surface) of holes drilled in 50 small high-precision parts via Electrical Discharge Machining. There are then 5 statements about the plot. **Circle the letters** corresponding to each of the statements that is true.

a) The angle distribution is approximately uniform.

b) The mean (or median) hole angle is slightly above 44°.

c) The standard deviation of hole angle is about 4°.

d) It is probably "safe" to use the "t" prediction limits for a 51st hole angle.

e) It is definitely not "safe" to use the tolerance limits presented in class to locate the sizes of most hole angles.
7. (This problem is about the logic/concepts of hypothesis testing. It is NOT an application of any of the standard methods on the posted summary of one- and two-sample inference methods.)

A radiation detector is used to measure the danger to humans working in a particular room. For illustration, we'll model

\( X \) = the number of radioactive "clicks" registered in a standard time period as Poisson with mean \( \lambda \). Presume that \( \lambda = 2 \) is "safe" but larger means (indicating higher radiation levels) are not.

a) Suppose that one determines to declare that there is an unsafe radiation level if \( X \geq 3 \).

\[ \begin{align*}
\text{i) What } \alpha \text{ is being employed? (Explain if you hope for any credit if your number is wrong!)} \\
\mathbb{P}_{\lambda=2} \left[ X \geq 3 \right] &= 1 - \mathbb{P}_{\lambda=2} \left[ X \leq 2 \right] \\
&= 1 - .677 = .323
\end{align*} \]

\[ \begin{align*}
\text{ii) What is } \beta \text{ if in fact the radiation level ( } \lambda \text{ ) is twice the maximum safe level? (Explain if you hope for any credit if your number is wrong!)} \\
\mathbb{P}_{\lambda=4} \left[ X \leq 2 \right] &= .283
\end{align*} \]

b) What \( p \)-value for a test of \( H_0 : \lambda = 2 \) is associated with an observation \( X = 4 \)? (Explain if you hope for any credit if your number is wrong!)

\[ \begin{align*}
\mathbb{P}_{\lambda=2} \left[ X \geq 4 \right] &= 1 - \mathbb{P}_{\lambda=2} \left[ X \leq 3 \right] \\
&= 1 - .857 = .143
\end{align*} \]