1. ISU engineering statistics students conducted a study of the properties of liquid laundry detergents. In part of their study, they measured the amount of detergent that one needs to add to 45 ml of distilled water in order to reduce surface tension to the point that a particular small shirt button floating on the surface of the water will sink. Their data for two brands and some summary statistics are below.

<table>
<thead>
<tr>
<th>All Brand</th>
<th>ERA Brand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.20, 1.05, 1.11</td>
<td>.85, .81, .69</td>
</tr>
<tr>
<td>$\bar{x}_1 = 1.120$</td>
<td>$\bar{x}_2 = .783$</td>
</tr>
<tr>
<td>$s_1 = .075$</td>
<td>$s_2 = .083$</td>
</tr>
</tbody>
</table>

(a) What probability model assumptions must be made in order to do inference for the mean amount of "All" detergent that must be added in order to reduce the surface tension?

(b) Based on your assumptions from (a), give a 90% lower confidence bound for the mean amount of All detergent required to reduce the surface tension.

(c) Is there evidence of a difference in mean amounts of "All" and "ERA" required to reduce the surface tension of distilled water, if one is willing to run a 5% chance of committing a type I error? (Carefully SHOW all 7 steps!)
2. A testing laboratory is concerned with the mean lifetime of certain light bulbs, advertised to have a mean life of 800 hours. An exponential distribution with parameter $\lambda$ (and thus mean $1/\lambda$) will be used to describe bulb life.

Suppose an engineer suggests testing a single bulb till failure and rejecting $H_0$: the manufacturer's advertised lifetime is correct if the observed life is less than 600 hours.

(a) What is the type I error probability for this initial product testing method? (Give a number.)

(b) If in fact the mean lifetime is 700, what type II error probability is associated with this initial life testing method? (Again, give a number.)

An alternative method of life testing is proposed by a second engineer. She suggests testing $n = 36$ bulbs and rejecting the manufacturer's claim if $\bar{x} < 600$.

(c) Use the central limit theorem and approximate the type I error probability associated with this second method of life testing. (Hint: Both the exponential mean and standard deviation are $1/\lambda$.)
3. ISU engineering students purchased and measured the outside diameters of some spanner bushings. Below are the diameters recorded on 4 of these. (Student 1 and Student 2 both measured the same 4 bushings.) The units are inches.

<table>
<thead>
<tr>
<th>Bushing</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>.3690</td>
<td>.3700</td>
<td>.3695</td>
<td>.3700</td>
</tr>
<tr>
<td>Student 2</td>
<td>.3695</td>
<td>.3700</td>
<td>.3690</td>
<td>.3695</td>
</tr>
</tbody>
</table>

(a) The sample standard deviation of the Student 1 measurements is $s = .000479$ inch. Assuming that measurements are normally distributed, find 90% two-sided confidence limits for the population standard deviation of Student 1 measurements on bushings of this type.

(b) It is not possible to use the material of Devore Chapters 7 through 9 (or anything said in class) to make a confidence interval comparing the Student 1 and Student 2 standard deviations for measurements on bushings of this type. Explain why this is so.

(c) Give 90% two-sided confidence limits for the mean difference in Student 1 and Student 2 measurements of the diameter of this kind of bushing.

(d) The $p$-value for a test of the null hypothesis that there is no systematic difference in the diameter measurements produced by the two students is about .64. Is this consistent with the limits you produced in part (c)? Explain why or why not.
4. $n = 100$ newly minted US pennies were weighed. Attached to this exam is a part of a Minitab analysis of their weights (recorded to a multiple of .02 g). Included is a histogram and a normal plot of the weights.

(a) You may verify (by looking at the histogram) that 20 of the $n = 100$ pennies have weights below 3.10 g. Give 95% two-sided confidence limits for the fraction of all newly minted pennies with weights below 3.10 g.

(b) Suppose that it was desirable to estimate the fraction of all newly minted pennies with weights below 3.10 g using a 99% two-sided confidence interval of length no more than .02. What sample size do you recommend for a future sample to produce such an interval?

(c) Say what the 2nd plot on the printout indicates about the shape of the penny weight distribution.

(d) The $n = 100$ penny weights have $\bar{x} = 3.108$ g and $s = .043$ g. Give two-sided 90% prediction limits for the weight of a single additional newly minted penny.
MTB > Histogram C1.

Character Histogram

Histogram of C1   N = 100

<table>
<thead>
<tr>
<th>Midpoint</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>1 *</td>
</tr>
<tr>
<td>3.02</td>
<td>4 ****</td>
</tr>
<tr>
<td>3.04</td>
<td>4 ****</td>
</tr>
<tr>
<td>3.06</td>
<td>4 ****</td>
</tr>
<tr>
<td>3.08</td>
<td>7 ******</td>
</tr>
<tr>
<td>3.12</td>
<td>17 ********************</td>
</tr>
<tr>
<td>3.14</td>
<td>17 ********************</td>
</tr>
<tr>
<td>3.16</td>
<td>13 ************</td>
</tr>
<tr>
<td>3.18</td>
<td>6 ******</td>
</tr>
<tr>
<td>3.20</td>
<td>2 **</td>
</tr>
<tr>
<td>3.22</td>
<td>1 *</td>
</tr>
</tbody>
</table>

MTB > GPro.
MTB > Sort C1 c1;
SUBC>   By c1.
MTB > Set c2
DATA>   1(.005 : .995 / .01 )1
DATA>   End.
MTB > InvCDF C2 c3;
SUBC>   Normal 0.0 1.0.
MTB > GStd.
MTB > Plot C1 C3;
SUBC>   Symbol '*'.

Character Plot

```
-                                                       *
3.200+                                               **
-                                                  2****
C1 -                                                    33433*
-                                                            232222
-                                                                344444*
-                                                              33343*
-                                                              223
-                                                              22
3.120+                                                **2
-                                                  **2
-                                                * **2
-                                               *  **
-                                              **
2.960+                                            **
---+---------+---------+---------+---------+--------
-2.0    -1.0    0.0     1.0     2.0
```

C3
1. ISU engineering statistics students conducted a study of the properties of liquid laundry detergents. In part of their study, they measured the amount of detergent that one needs to add to 45 mL of distilled water in order to reduce surface tension to the point that a particular small algal biomass floating on the surface of the water will sink. Their data for two brands and some summary statistics are below.

<table>
<thead>
<tr>
<th>Detergent</th>
<th>All Brands</th>
<th>ERA Brand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.20</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>85, 81, 89</td>
<td>82, 81, 89</td>
</tr>
<tr>
<td></td>
<td>1.120</td>
<td>0.783</td>
</tr>
<tr>
<td></td>
<td>0.775</td>
<td>0.483</td>
</tr>
</tbody>
</table>

(a) What probability model assumptions must be made in order to do inference for the mean amount of "All" detergent that must be added in order to reduce the surface tension? Use the sample data and the model assumptions to make the inference. All detergents act the same.

(b) Based on your assumptions from (a), give a 95% lower confidence bound for the mean amount of All detergent required to reduce the surface tension. Use $\bar{x} - t_{\alpha/2} \frac{s}{\sqrt{n}}$. i.e., $1.120 - 0.882$. 

(c) Is there evidence of a difference in mean amounts of "All" and "ERA" required to reduce the surface tension of distilled water, if one is willing to take a 5% chance of committing a type I error? (Carefully show all 7 steps)

1. State the null and alternative hypotheses.
2. Identify the test statistic, the test statistic, and the rejection region.
3. Compute the value of the test statistic.
4. Calculate the p-value of the test statistic.
5. Make a decision to either reject or fail to reject $H_0$.
6. Conclude.
7. State the conclusion in context.

(d) The samples come from normal populations with unknown means and the same unknown variance $\sigma^2$.

2. A testing laboratory is concerned with the mean lifetime of certain light bulbs, advertised to have a mean life of 800 hours. An exponential distribution with parameter $\lambda$ and mean $1/\lambda$ will be used to describe bulb life. Suppose an engineer suggests testing a single bulb till failure and rejecting $H_0$ if the manufacturer's advertised lifetime is correct if the observed life is less than 400 hours.

(a) What is the type I error probability for the initial product testing method? (Give a number.)

(b) If in fact the mean lifetime is 700, what is the type II error probability associated with this initial life testing method? (Again, give a number.)

An alternative method of life testing is proposed by a second engineer. She suggests testing $n = 30$ bulbs and rejecting the manufacturer's claim if $\bar{x} < 600$.

(c) Use the central limit theorem and approximate the type I error probability associated with this second method of life testing. (Hint: Both the exponential mean and standard deviation are $1/\lambda$.)

3. ISU engineering students purchased and measured the inside diameters of some spanner bushings. Below are the diameters recorded on 4 of these. (Student 1 and Student 2 both measured the same 4 bushings.) The units are inches.

<table>
<thead>
<tr>
<th>Bushing</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>1.36</td>
<td>1.35</td>
<td>1.37</td>
<td>1.36</td>
</tr>
<tr>
<td>Student 2</td>
<td>1.35</td>
<td>1.36</td>
<td>1.36</td>
<td>1.36</td>
</tr>
</tbody>
</table>

(a) The sample standard deviation of the Student 1 measurements is $s = 0.00670$ inch. Assuming that measurements are normally distributed, find 90% two-sided confidence limits for the population standard deviation of Student 1 measurements on bushings of this type.

(b) It is not possible to use the material of Exercise 7 through 9 (for anything said in class) to make a confidence interval comparing the Student 1 and Student 2 standard deviations for measurements on bushings of this type. Explain why this is so.

(c) If we use the sample standard deviations to compute a confidence interval, what will be the sample size $n$ required to reach a 90% confidence interval of length no more than 0.2. What sample size do you recommend for a future sample to produce such an interval?

(d) The $n = 100$ newly minted US pennies were weighed. Attached to this exam is a part of a Minitab analysis of their weights (recorded to a multiple of 0.02 g). Included is a histogram and a normal plot of the weights.

(e) You may verify (by looking at the histogram) that 20 of the $n = 100$ pennies have weights below 3.10 g. Give 95% two-sided confidence limits for the fraction of all newly minted pennies with weights below 3.10 g.

(f) Suppose that it was desirable to estimate the fraction of all newly minted pennies with weights below 3.10 g using a 99% two-sided confidence interval of length no more than 0.2. What sample size do you recommend for a future sample to produce such an interval?

(g) Say what the 2nd plot on the printout indicates about the shape of the penny weight distribution.

(h) The plot is reasonably linear and therefore indicates that the sample of penny weights looks like it could well have come from a normal distribution.

(i) The value of $\bar{x}$ is given on the printout and the corresponding upper 95% confidence limit is $1.6694$ (from Minitab).

(j) The $n = 100$ penny weights have $\bar{x} = 3.108$ and $s = 0.043$ g. Give two-sided 95% prediction limits for the weight of a single additional newly minted penny.