“Statistical” (Probabilistic) Tolerancing

(Section 5.4 of Vardeman and Jobe)

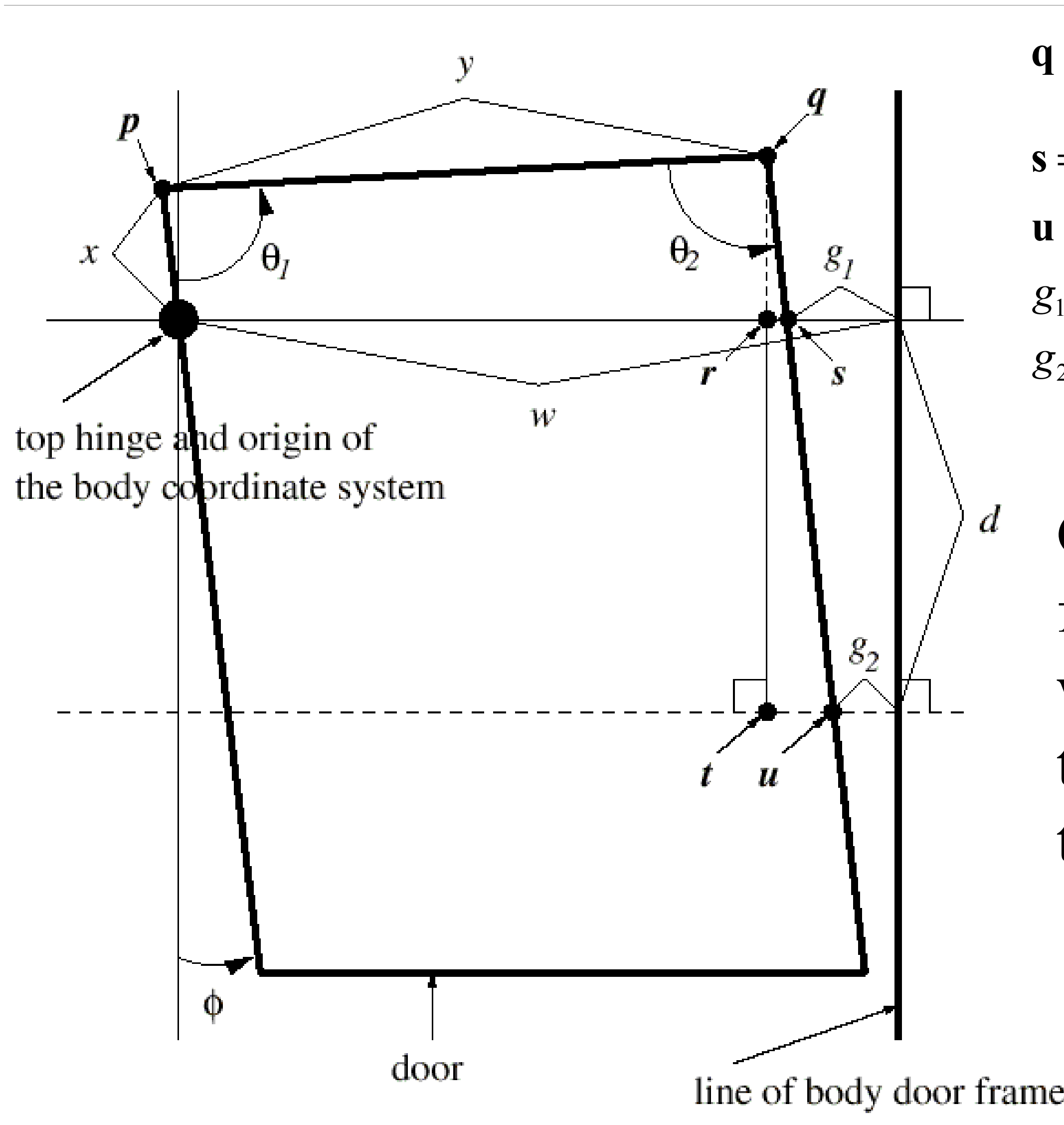
Piecing Together Information on Variation (to Predict Overall)

- Sometimes geometry or physical theory gives me an equation for a variable of interest in terms of other, more basic variables

$$U = g(X, Y, \ldots, Z)$$

- If I have information on how the inputs vary, I can sometimes infer how U varies

Car Door Example



$$\mathbf{p} = (-x\sin f, x\cos f)$$

$$\mathbf{q} = \mathbf{p} + \left(y\cos\left(f + \left(q_1 - \frac{p}{2} \right) \right), y\sin\left(f + \left(q_1 - \frac{p}{2} \right) \right) \right)$$

$$\mathbf{s} = \left(q_1 + q_2 \tan(f + q_1 + q_2 - p), 0 \right)$$

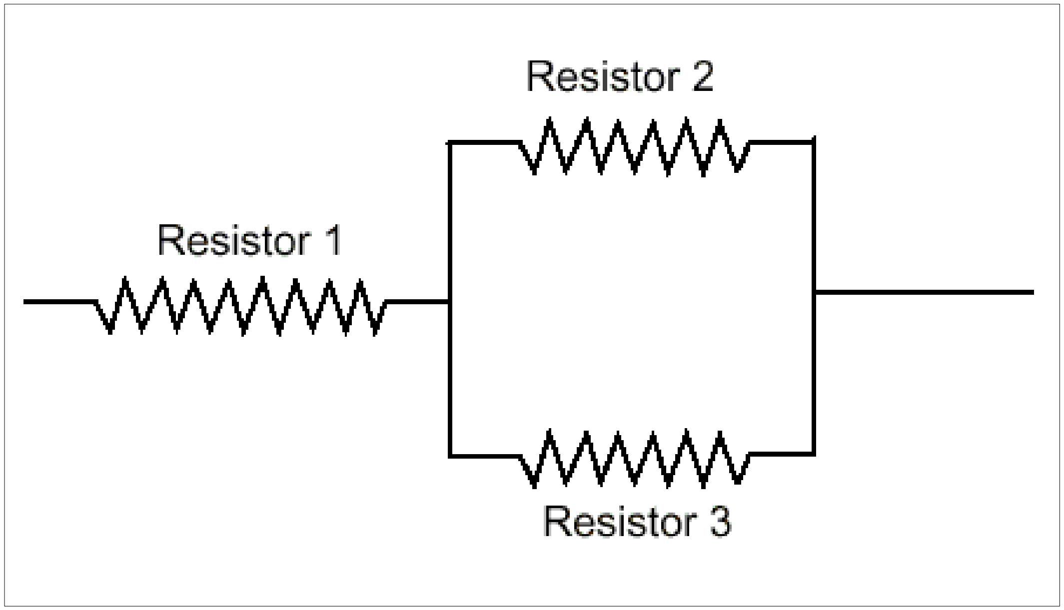
$$\mathbf{u} = (q_1 + (q_2 + d)\tan(f + q_1 + q_2 - p), -d)$$

$$g_1 = w - s_1$$

$$g_2 = w - u_1$$

Given nominals and “sigmas” for x, y, w, f, q_1 and q_2 what can I say about the the nominals and “sigmas” for the gaps, g_1 and g_2 ?

Example 5.8



$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$\mu_{R_1} = 100\Omega \text{ and } \mu_{R_2} = \mu_{R_3} = 200\Omega$$

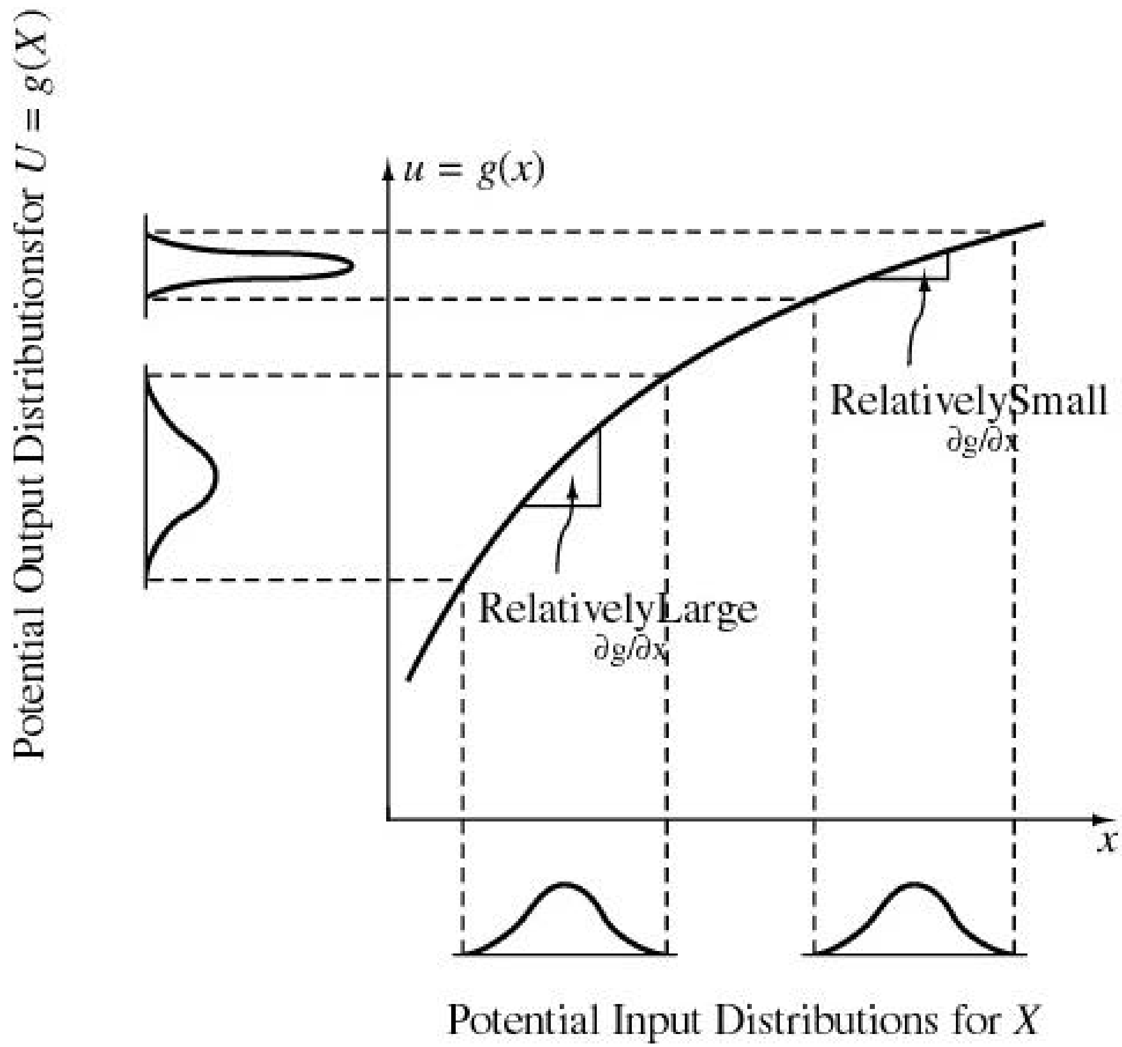
$$\sigma_{R_1} = 2\Omega \text{ and } \sigma_{R_2} = \sigma_{R_3} = 4\Omega$$

What about R?

Methods for Studying U (for Independent $X, Y, \ldots, Z$)

- Exact mean and variance for linear g
  - Equations (5.23) and (5.24)
  - Helpful for simple tolerance stack-up calculations
- Approximations for mean and variance (based on a linearization) for other g
  - Equations (5.26) and (5.27)
  - Easily done on something like Mathcad
  - Variance involves both the input variances AND the derivatives/rates of change

Var$U$ and Derivatives



Example 5.9

- Nice little tolerance stack-up problem

$$U = Y - X_1 - X_2 - X_3 - X_4$$

$$s_U^2 = 1^2 s_Y^2 + (-1)^2 s_{X_1}^2 + (-1)^2 s_{X_2}^2 + (-1)^2 s_{X_3}^2 + (-1)^2 s_{X_4}^2$$

- *U* was head space in a carton designed to hold 4 units of product

Mathcad for Example 5.8

$$R(R1, R2, R3) := R1 + \frac{R2 \cdot R3}{R2 + R3}$$

$R1 := 100 \qquad \sigma 1 := 2$

$R2 := 200 \qquad \sigma 2 := 4$

$R3 := 200 \qquad \sigma 3 := 4$

$$\frac{d}{dR1} R(R1, R2, R3) = 1.00$$

$$\frac{d}{dR2} R(R1, R2, R3) = .25$$

$$\frac{d}{dR3} R(R1, R2, R3) = .25$$

$$\sigma R := \sqrt{\left(\frac{d}{dR1} R(R1, R2, R3)\right)^2 \cdot \sigma 1^2 + \left(\frac{d}{dR2} R(R1, R2, R3)\right)^2 \cdot \sigma 2^2 + \left(\frac{d}{dR3} R(R1, R2, R3)\right)^2 \cdot \sigma 3^2}$$

$\sigma R = 2.449$

Simulation … the Easiest Method for Studying U

- Any decent statistical package will let you simulate sets of $X, Y, \ldots, Z$, compute U's, and then look at descriptive statistics for the simulated values

Minitab for Example 5.8

```
MTB > Random 1000 c1;
SUBC>   Normal 100 2.
MTB > Random 1000 c2 c3;
SUBC>   Normal 200 4.
MTB > let c4=c1+((c2*c3)/(c2+c3))
MTB > Describe c4.
```

Calc>>Random Data>>Normal

Calc>>Random Data>>Normal

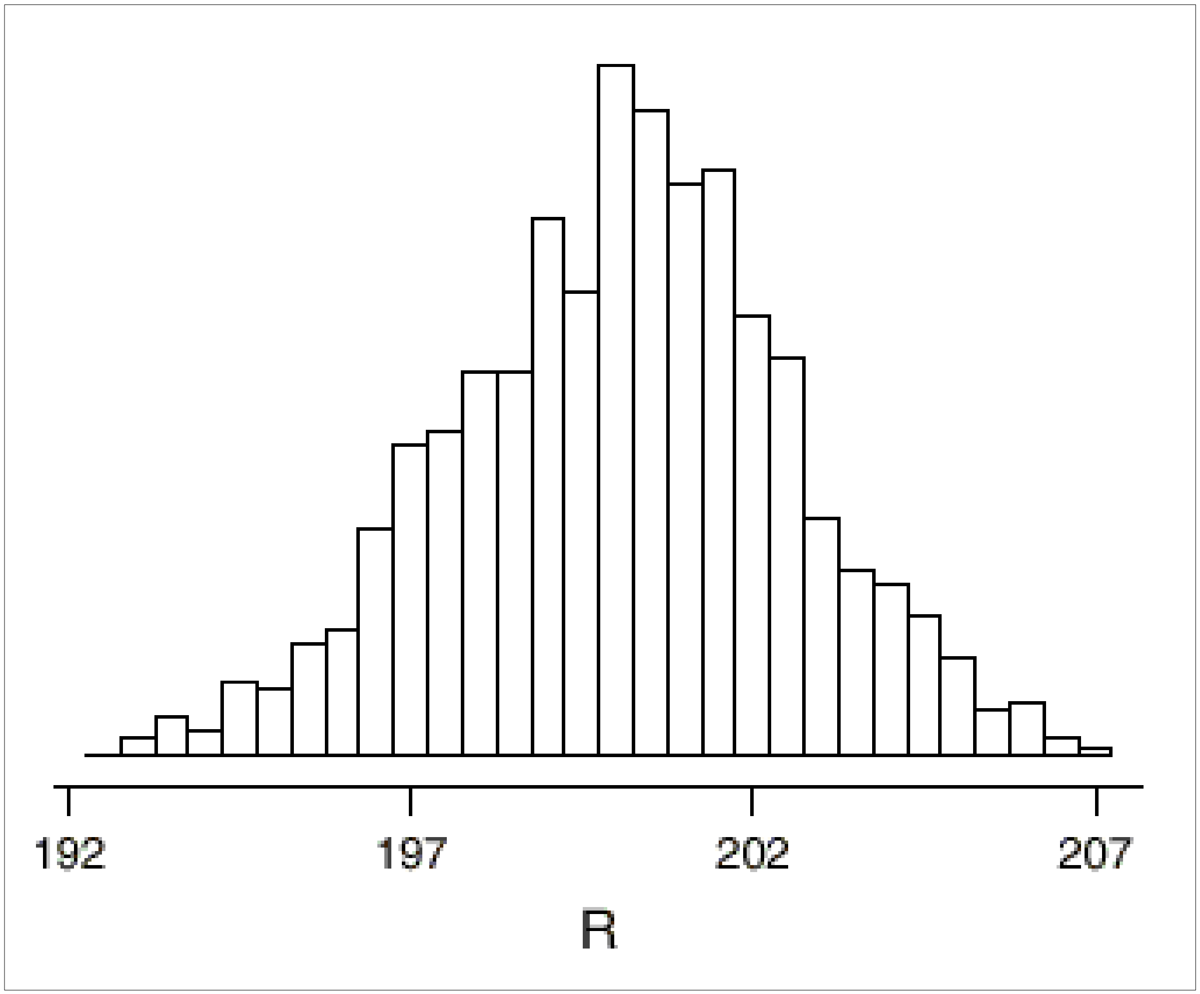
Calc>>Calculator
Stat>>Basic Statistics>>Display Basic

Descriptive Statistics

Variable	N	Mean	Median	TrMean	StDev	SE Mean
c4	1000	200.09	200.19	200.10	2.45	0.08

Variable	Minimum	Maximum	Q1	Q3
c4	192.83	207.13	198.50	201.69

More Mintab for Example 5.8



Workshop Door Exercise

c1=*x*, c2=*y*, c3=*w*, c4=phi
c5=theta1, c6=theta2, *d=40*
(units of cm and radians)

```
MTB >  Random 1000 c1;
SUBC>   Normal 20 .01.
MTB > Random 1000 c2;
SUBC>   Normal 90. .01.
MTB > Random 1000 c3;
SUBC>   Normal 90.4 .01.
MTB > Random 1000 c4;
SUBC>   Normal 0 .001.
MTB > Random 1000 c5 c6;
SUBC>   Normal 1.570796 .001.
MTB > let c7=-c1*sin(c4)
MTB > let c8=c1*cos(c4)
MTB > let c9=c7+c2*cos(c4+(c5-1.570796))
MTB > let c10=c8+c2*sin(c4+(c5-1.570796))
MTB > let c11=c9+c10*tan(c4+c5+c6-3.141593)
MTB > let c12=c9+(c10+40)*tan(c4+c5+c6-3.141593)
MTB > let c13=c3-c11
MTB > let c14=c3-c12
```

More Door

```
MTB > Describe 'g1' 'g2'.
```

Descriptive Statistics

Variable	N	Mean	Median	TrMean	StDev	SE Mean
g1	1000	0.40053	0.40035	0.40040	0.03078	0.00097
g2	1000	0.40138	0.39775	0.40144	0.09153	0.00289

Variable	Minimum	Maximum	Q1	Q3
g1	0.31109	0.51216	0.37877	0.42168
g2	0.12174	0.75907	0.34043	0.46280

- What does this analysis show about the gaps?
- How could you study the difference in gaps, *g1-g2,* and/or any relationship between gaps?