Normal Plotting for Process Characterization

(Section 5.1 of Vardeman and Jobe)

When is a Data Set More Than Just a Data Set?

- Only when it comes from some kind of (physically) "stable" system
- Perfect consistency is too much to expect in this world
- If (by virtue of process monitoring and wise intervention) I am willing to claim that a data set represents a stable process, I may want to use it to characterize its output

Graphics for Process Shape Use the Notion of "Quantiles"

- For an ordered data set $x_1 \leq x_2 \leq \cdots \leq x_n$
  - and an integer *i* and *p=(i-.5)/n*, the ***p* quantile** of the data set is

$$Q\left(\frac{i-.5}{n}\right) = x_i$$

  - and a value *p* not of the form *=(i-.5)/n*, the ***p* quantile** of the data set is gotten by linear interpolation

Simple Example

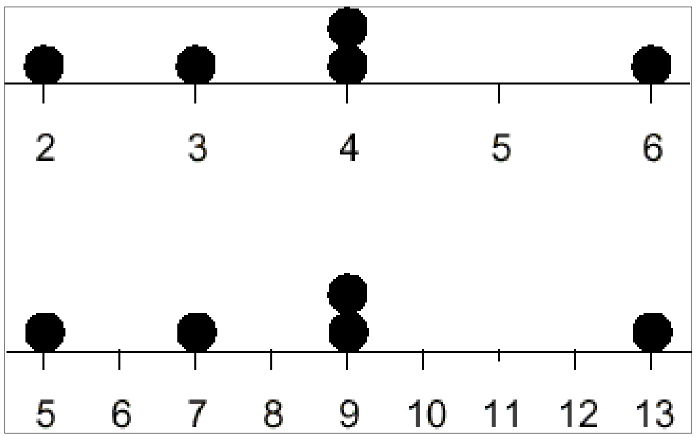
- Data set $\{2,3,4,4,6\}$

i	$p = \frac{i-.5}{5}$	$Q\left(\frac{i-.5}{5}\right) = x_i$
1	.1	$2 = Q(.1)$
2	.3	$3 = Q(.3)$
3	.5	$4 = Q(.5)$
4	.7	$4 = Q(.7)$
5	.9	$6 = Q(.9)$

and, e.g., $Q(.32) = \left(1 - \frac{.02}{.2}\right)3 + \left(\frac{.02}{.2}\right)4 = 3.1$

Key Insight and Application

- "Same shape" for two distributions is "linearly-related quantile functions"



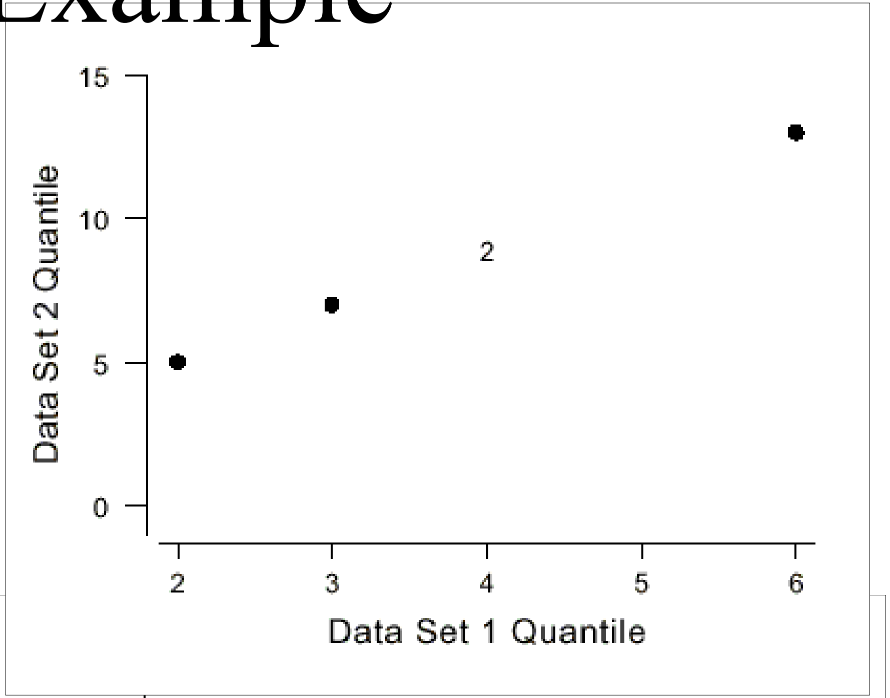
p	$Q_1(p)$	$Q_2(p)$
.1	2	5
.3	3	7
.5	4	9
.7	4	9
.9	6	13

here $Q_2(p) = 2Q_1(p) + 1$

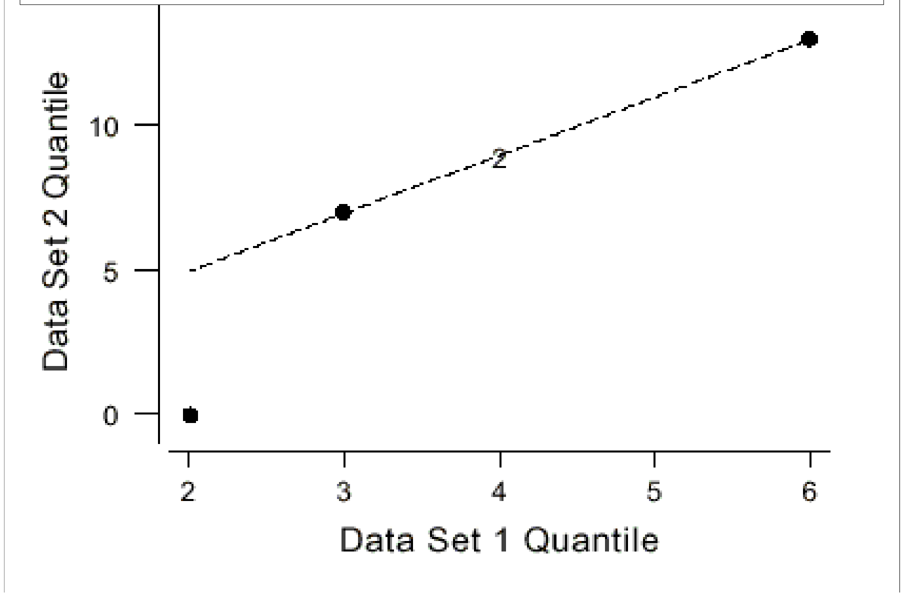
- So plots of $\left(Q_1(p_i), Q_2(p_i)\right)$ might be used to compare distribution shapes (*Q*-*Q* plots)

Simple Example

- *Q-Q* plot for the two artificial data sets



- *Q-Q* plot if the smallest value in data set #2 is changed to 0



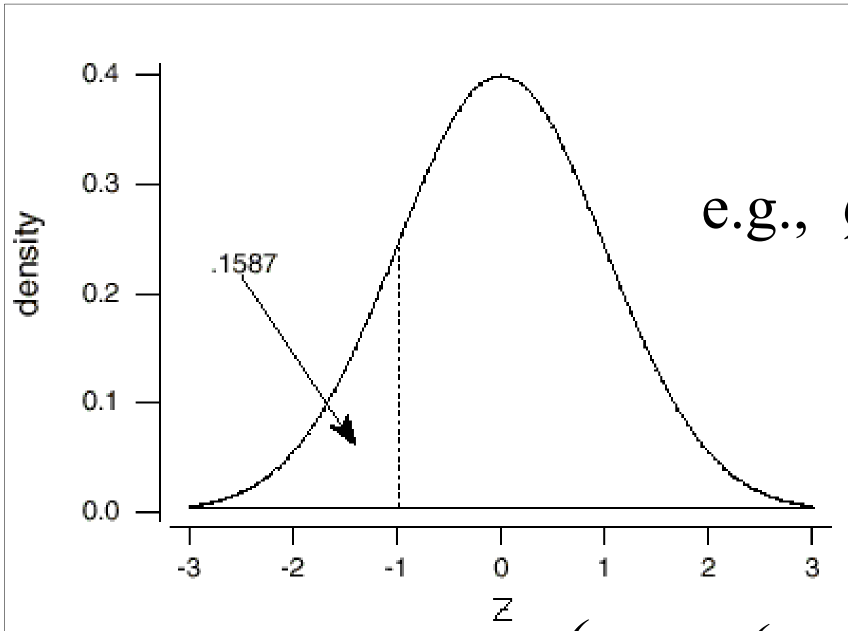
Probability Plotting

- The most important version of *Q-Q* plotting is that where the 2nd distribution is a theoretical one (rather than that of a second data set) … a *Q-Q* plot is then a "probability plot" (and one is checking to see if the shape of the data set matches the theoretical shape)
- For an ordered data set $x_1 \leq x_2 \leq \cdots \leq x_n$ plot

$$\left(x_i, Q_{\text{theoretical}}\left(\frac{i-.5}{n}\right)\right)$$

Normal Plotting

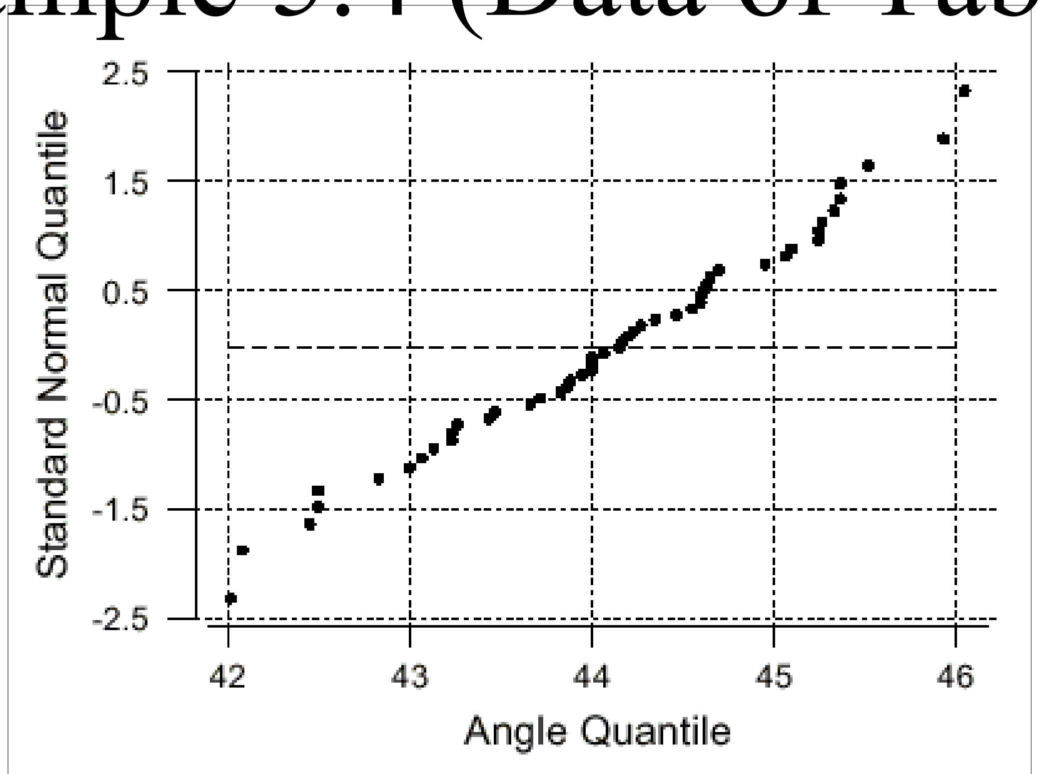
- The case where the theoretical distribution is standard normal is "normal plotting"



e.g., $Q_z(.1587) = -1.00$

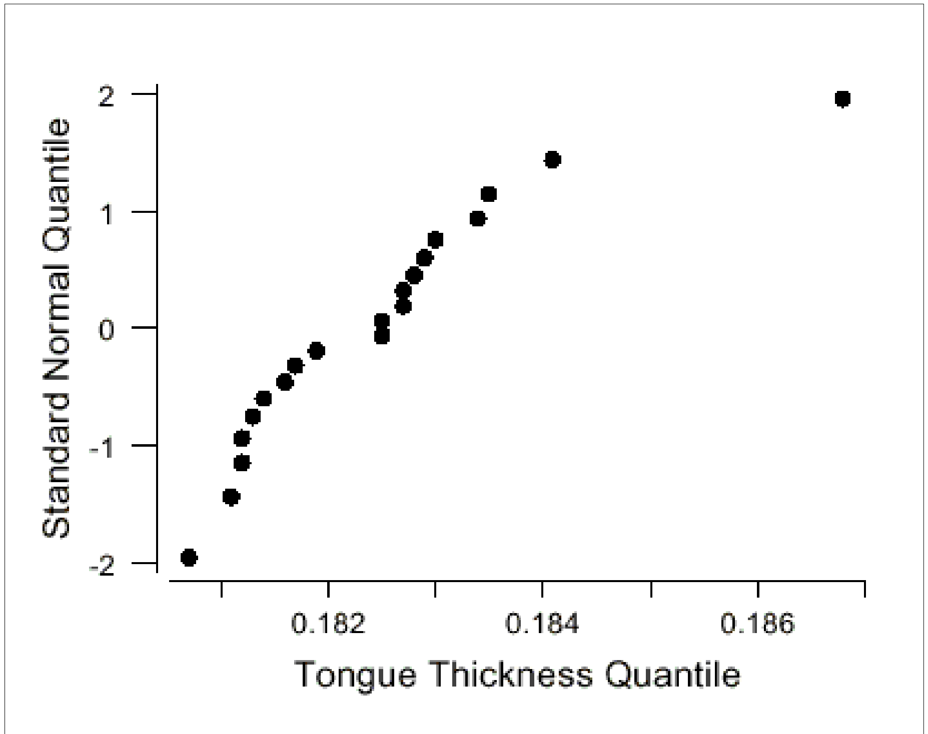
- Plotting points $\left(x_i, Q_Z\left(\frac{i-.5}{n}\right)\right)$ gives a way of checking on "normal shape"

Example 5.4 (Data of Table 5.7)



- This plot is fairly linear, so the distribution shape is "normal" … if the drilling process is stable, one may treat it as generating normally distributed angles

Example 5.1 (Data of Table 5.1)



- Here the data distribution is long-tail right (skewed right) in comparison to the normal shape … a normal model would not be a good one for describing tongue thickness

Normal Plotting in Practice

- It's rarely done by finding standard normal quantiles and plotting by hand
  - For decades special (normal) "probability paper" has been used … on it, plot points

$$\left(x_i, 100\left(\frac{i-.5}{n}\right)\right) \text{ or } \left(x_i, \frac{i-.5}{n}\right)$$

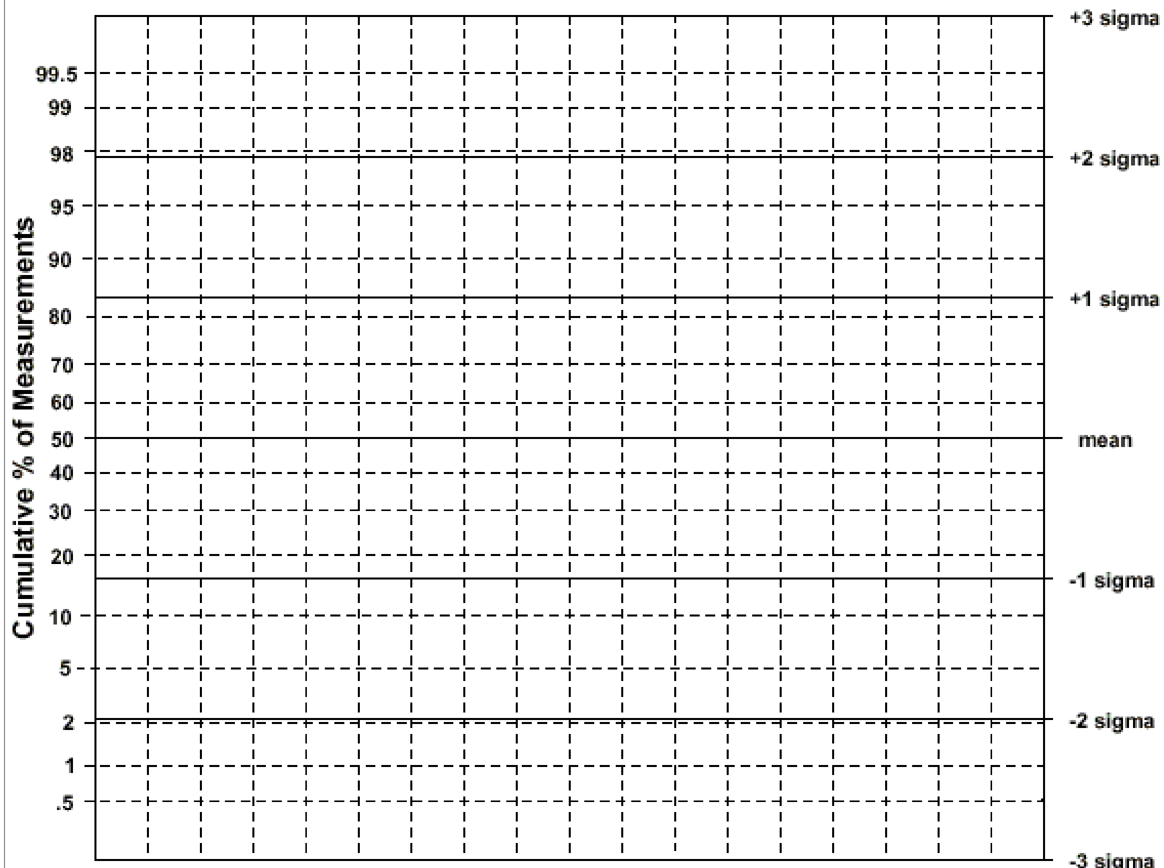
  - These days any decent statistical package will do it automatically
- For linear plots, means and standard deviations can be read from from the graph

$$\hat{m} = \text{horizontal intercept}$$

$$\hat{s} = 1/\text{slope}$$

Capability Analysis Sheet

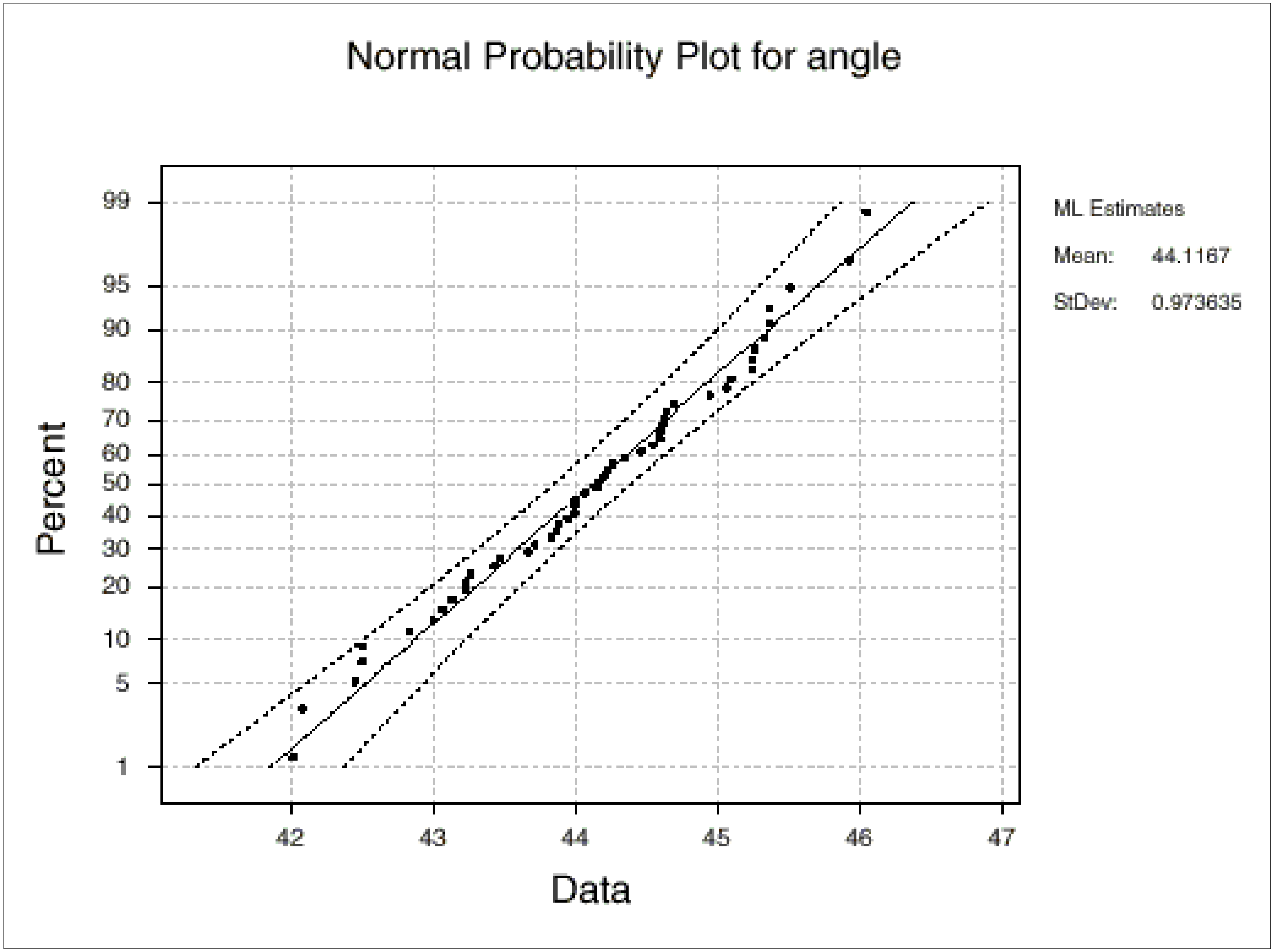
Supplier	Date
Part	Specifications
Operation	3 sigma =
Technician	4 sigma =
Characteristic	Units



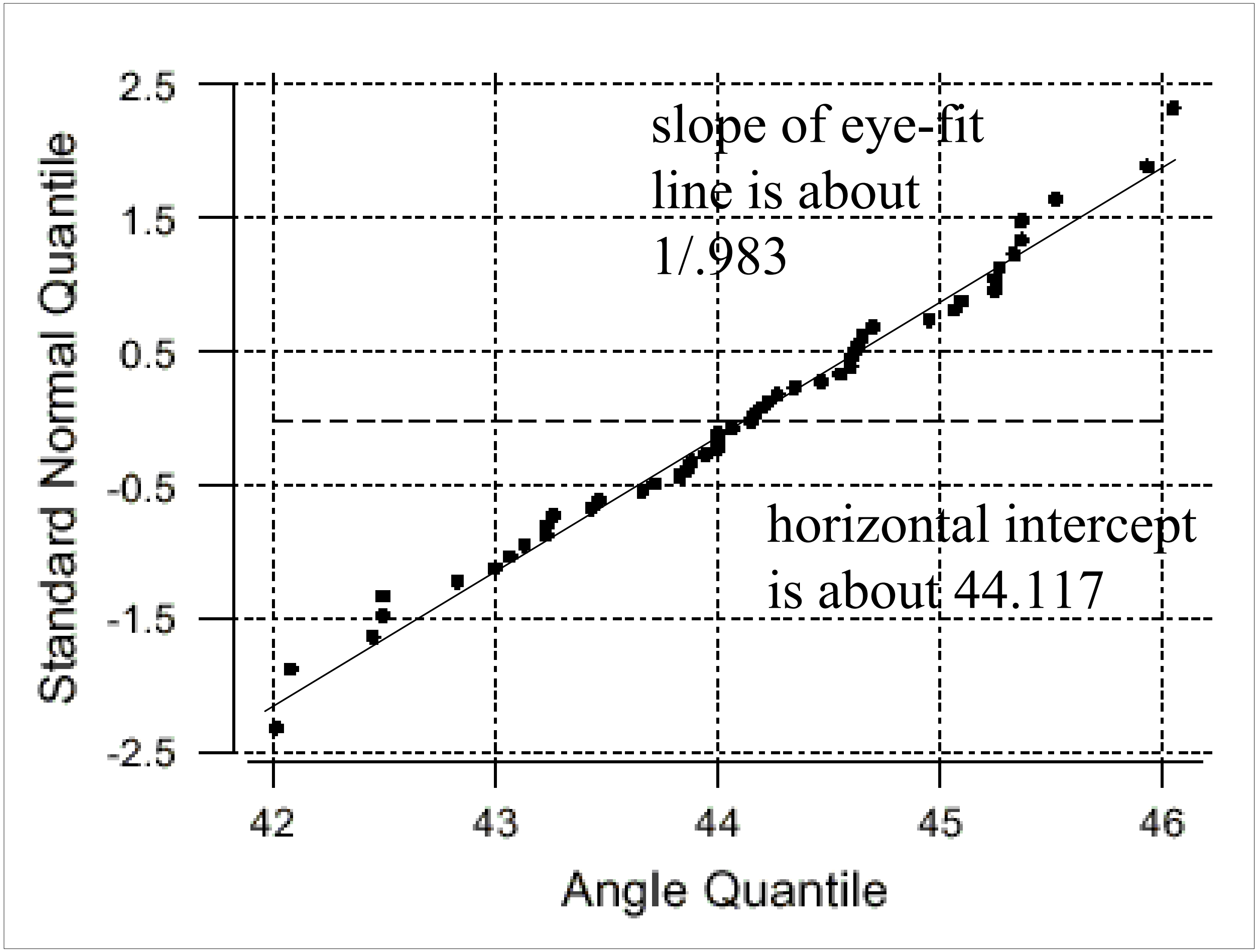
value																	
frequency																	
add as shown																	
divide by 2N																	

← plotting positions

Minitab Normal Plot of Angles



Graphical Estimates



Workshop Exercises

- Find *.1*,*.3*,*.5*,*.7* and *.9* standard normal quantiles and use them to make a normal plot of the small data set on slide 4 on regular graph paper, plotting points

$$\left(x_i, Q_Z\left(\frac{i-.5}{n}\right)\right)$$

  - Read an approximate mean and standard deviation off your plot
- Normal plot the data set from slide 4 on the capability analysis sheet of slide 12
  - Compare to your plot from above

Workshop Exercises

- Interpret the Minitab normal plot of weights of new pennies (recorded to the nearest .02 gram)