Module 1

Basic concepts and introduction to probability modeling of measurement error

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March 4, 2008

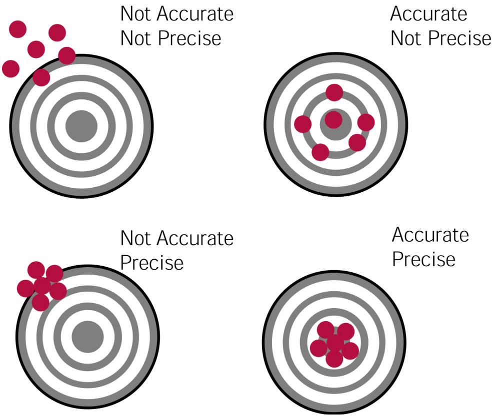
Motivation for this Workshop

- Characteristics of measurement limit/prescribe how one should interpret the output of statistical methods and what can be learned from data.

- Even very simple statistical methods have things to say about measurement quality and how basic properties of measurement error can be quantified.

Basic Concepts

- A measurement or measuring method is said to be **valid** if it usefully or appropriately represents the feature of the measured object or phenomenon that is of interest.
- A measurement system is said to be **precise** if it produces small variation in repeated measurement of the same object or phenomenon.
- A measurement system is said to be **accurate** (or sometimes **unbiased**) if on average it produces the true or correct values of quantities of interest.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Simple Measurement Error Model

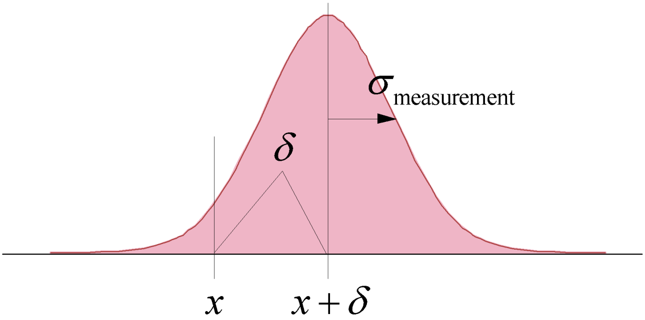

Figure: Distribution of a measurement y (measurand x, bias or systematic error δ, and random error $\epsilon = y - x$)

The Measurement Error Model in Symbols

A measurement is

$$y = x + \epsilon$$

where the measurement error ϵ has mean δ (the bias) and standard deviation $\sigma_{\text{measurement}}$. This produces

$$\mu_y = x + \delta \quad \text{and} \quad \sigma_y = \sigma_{\text{measurement}}$$

(Of course, ideally $\delta = 0$.) Calibration aims to eliminate bias.

Measurement Device "Linearity"

This is the situation where bias does *not* depend upon the measurand (δ is independent of x).

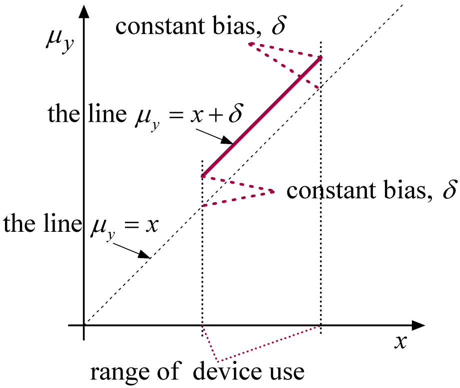

Figure: Measurement device "linearity" is "constant bias"

Modeling When the Measurand Varies (as in QC Applications)

Suppose that x varies and itself has mean

$$\mu_x$$

and standard deviation

$$\sigma_x \quad .$$

Then with constant bias (and assuming independence of x and ϵ) what is observed, y, has

$$\mu_y = \mu_x + \delta$$

and

$$\sigma_y = \sqrt{\sigma_x^2 + \sigma_{\text{measurement}}^2}$$

Measurement error shifts the perceived average of x by δ and inflates the perceived variability.

The Effect of Measurement Error When the Measurand Varies

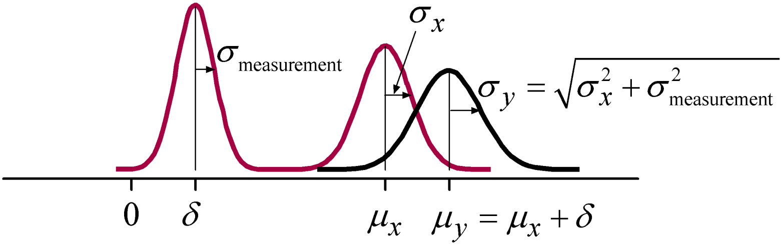

Figure: The effect of measurement error on measurement of varying x

A Way of Potentially "Removing" the Effect of Measurement Imprecision

The relationship

$$\sigma_x = \sqrt{\sigma_y^2 - \sigma_{\text{measurement}}^2}$$

suggests a means of estimating "part"/measurand variation σ_x.

Based on

- (single) measurements y for several parts that produce a sample standard deviation s_y, and
- several measurements on a single part that produce a sample standard deviation s, a plausible estimator of σ_x is

$$\hat{\sigma}_x = \sqrt{\max\left(0, s_y^2 - s^2\right)}$$

A Basic Insight

The previous slide is a first illustration of a basic insight that will run through all we do today:

> *How sources of physical variation interact with a data collection plan governs what of practical importance can be learned from a data set, and in particular, how measurement error is reflected in the data set.*