Module 4

A First Look at WinBUGS and Practical Bayes Computation

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"Real" professional Bayesians program their own MCMC algorithms, tailoring them to the models and data sets they face. The most widely used Bayes software available for non-specialists like you and me derives from the Biostatistics Unit at Cambridge University. The TM `Windows` version is `WinBUGS` and there is an open source version (that can be run in batch mode) called `OPENBUGS`. We'll illustrate `WinBUGS` in the balance of this workshop. `WinBUGS` has its own user manual and its own discussion list. These sources are far more authoritative than I will be. I am NOT a real expert with the system, and my intention is not to give you an exhaustive look at the software. Rather, it is my intention to give you a series of examples that will illustrate the power of the Bayes paradigm and the software in addressing important problems of inference in industry.

Using WinBUGS

In order to make a `WinBUGS` analysis, one must

- write and have the software **check** the syntax of a **model** statement for the problem,
- **load** any **data** needed for the analysis not specified in the model statement,
- **compile** the program that will run the Gibbs sampler, and
- one way or another (either by supplying them or by generating them from the model itself) provide **initial values** for the sampler(s)/ chain(s) that will be run.

One then

- **updates** the sampler(s) as appropriate,
- **monitors** the progress of the sampler(s), and
- ultimately **summarizes** what the sampler(s) indicate about the posterior distribution(s).

Example 1

As a first example, we will do the `WinBUGS` version of the small normal-normal model used in Module 2. The code for this is in the file

`BayesASQEx1.odc`

Remember that the model is

$$\begin{aligned} X &\sim \mathrm{N}(\theta, 1) \\ \theta &\sim \mathrm{N}(5, 2) \end{aligned}$$

(the prior variance is 2, so that the prior precision is .5) and we are assuming that $X = 4$ is observed.

Example 1 (cont.)

Here is the code:

```
model {
X~dnorm(theta,1)
Xnew~dnorm(theta,1)
theta~dnorm(5,.5)
#WinBUGS uses the precision instead of the variance or
#standard deviation to name its normal distributions
#so the prior variance of 2 is expressed as a prior
#precision of .5
}
#here is a list of data for this example
list(X=4.0)
#here are 4 possible initializations for Gibbs samplers
list(theta=7,Xnew=3)
list(theta=2,Xnew=6)
list(theta=3,Xnew=10)
list(theta=8,Xnew=10)
```

Example 1 (cont.)

The model is specified using the `Specification Tool` under the `Model` menu. One first uses the `check model` function, then the `load data` function to enter the `list(X=4.0)`, then (for example choosing to run 4 parallel Gibbs samplers) employs the `compile` function. To initialize the simulation, one may either ask `WinBUGS` to generate initial values from the model, or one at a time enter 4 initializations for the chains like those provided above.

The `Update Tool` on the `Model` menu is used to get `WinBUGS` to do Gibbs updates of a current sampler state (in this case, a current θ and value for X_{new}). The progress of the iterations can be watched and summarizations of the resulting simulated parameters (and new observations) can be produced using the `Sample Monitor Tool` under the `Inference` menu.

Example 1 (cont.)

Here are screen shots of what one gets for summaries of a fairly large number of iterations using the `history`, `density`, and `stats` functions of the `Sample Monitor Tool`. (One must first use the `set` function before updating, in order to alert `WinBUGS` to the fact that values of θ and X_{new} should be collected for summarization.)

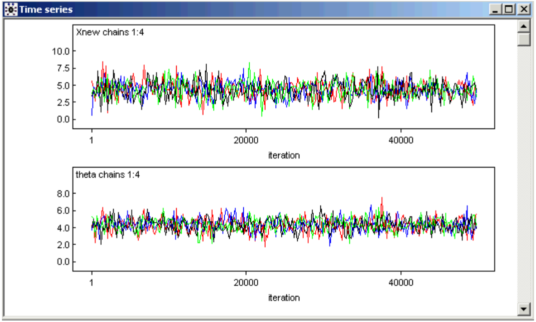


Figure: History plots for 50,000 iterations for 4 parallel chains (thinned to every 200th iteration for plotting purposes) for the toy normal-normal problem

Node statistics

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
Xnew	4.33	1.289	0.00282	1.804	4.332	6.859	1	200000
theta	4.331	0.8144	0.001688	2.731	4.33	5.924	1	200000

Figure: Summary statistics for 50,000 iterations for 4 parallel chains for the toy normal-normal problem

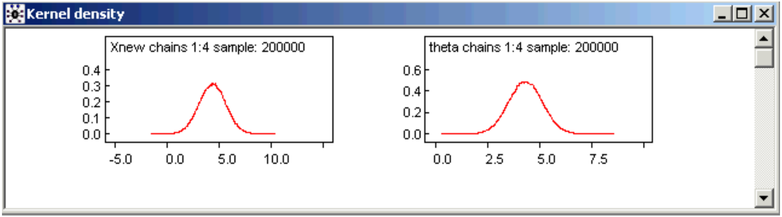


Figure: Approximate densities estimated from 50,000 iterations for 4 parallel chains for the toy normal-normal problem

Example 1 (cont.)

The first figure shows no obvious differences in the behaviors of the 4 chains (started from the fairly "dispersed" initializations indicated in the example code), which is of comfort if one is worried about the possibility of Gibbs sampling failing. The last 2 figures are in complete agreement with the pencil and paper analyses of this problem offered in Module 2. Both the posterior for θ and the posterior predictive distribution of X_{new} look roughly "normal" and the means and standard deviations listed in the "node statistics" are completely in line with posterior means and standard deviations. In fact, these can be listed in tabular form for comparison purposes as on the next panel.

Example 1 (cont.)

Table 1 Theoretical (Pencil and Paper Calculus) and MCMC (Gibbs Sampling) Means and Standard Deviations for the Toy Normal-Normal Example

	θ	X_{new}
Theoretical Posterior Mean	4.333	4.333
MCMC Posterior Mean	4.331	4.333
Theoretical Posterior Std Dev	$\sqrt{\frac{2}{3}} = .8165$	$\sqrt{\frac{2}{3}+1} = 1.291$
MCMC Posterior Std Dev	.8144	1.289

This first example is a very "tame" example, the effect of the starting value for the Gibbs sampling is not important, and the samplers very easily produce the right posteriors.

Functions of Parameters

One of the real powers of simulation as a way of approximating a posterior is that it is absolutely straightforward to approximate the posterior distribution of any function of the parameter vector θ, say $h(\theta)$. One simply plugs simulated values of θ into the function observes the resulting relative frequency distribution.

Example 1 (cont.)

Continuing the normal-normal example, a function of θ that could potentially be of interest is the fraction of the X distribution below some fixed value, say 3.0. (This kind of thing might be of interest if X were some part dimension and 3.0 were a lower specification for that dimension.) In this situation, the parametric function of interest is

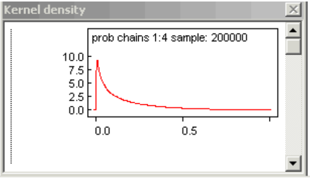
$$h(\theta) = \Phi\left(\frac{3.0-\theta}{1}\right) = P\left[Z \leq \frac{3.0-\theta}{1}\right]$$

and by simply adding the line of code

```
prob<-phi(3.0-theta)
```

to the previous model statement, it is easy to get a picture of the posterior distribution of the fraction of the X distribution below 3.0 similar to that in the figure on the next panel.

Example 1 (cont.)



Node statistics

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
prob	0.1511	0.1636	3.455E-4	0.001726	0.09175	0.6062	1	200000

Figure: The posterior distribution of $prob = \Phi\left(3.0 - \theta\right)$

The posterior mean for this fraction of the X distribution is about 15%, but very little is actually known about the quantity. If one wanted 95% posterior probability of bracketing $prob = \Phi\left(3.0 - \theta\right)$, a so-called 95% Bayes *credible interval* (running from the lower 2.5% point to the upper 2.5% point of the approximate posterior distribution) would be

$$(.001726, .6062)$$

Example 2

As a second simple well-behaved example, consider a fraction non-conforming context, where one is interested in

$$X = \text{the number non-conforming in a sample of } n = 50$$

and believes *a priori* that

$$p = \text{the process non-conforming rate}$$

producing X might be appropriately described as having mean .04 and standard deviation .04.

The model for the observable data here will be

$$X \sim \text{Binomial}\,(50, p)$$

Example 2 (cont.)

The figure below shows a convenient prior density for p that has the desired mean and standard deviation. This is the so-called Beta distribution with parameters $\alpha = .92$ and $\beta = 22.08$.

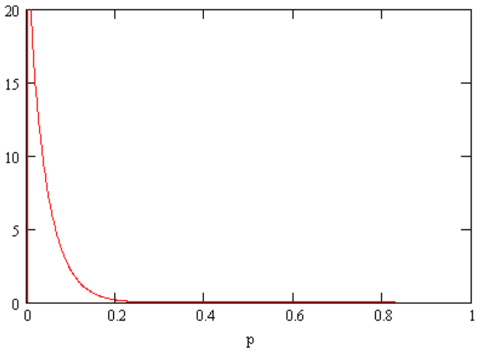


Figure: Beta density with parameters $\alpha = .92$ and $\beta = 22.08$

Example 2 (cont.)

The general Beta density is

$$g(p) \propto p^{\alpha-1}(1-p)^{\beta-1}$$

and it is common in Bayes contexts to think of such a prior as contributing to an analysis information roughly equivalent to α "successes" (non-conforming items in the present context) and β "failures" (conforming items in the present context). So employing a Beta$(.92, 22.08)$ prior here is roughly equivalent to assuming prior information that a single non-conforming item has been seen in 23 inspected items.

The code in the file

`BayesASQEx2.odc`

can be used to find a $X = 4$ posterior distribution for p and posterior predictive distribution for

$$X_{\text{new}} = \text{the number non-conforming in the next 1000 produced}$$

Example 2 (cont.)

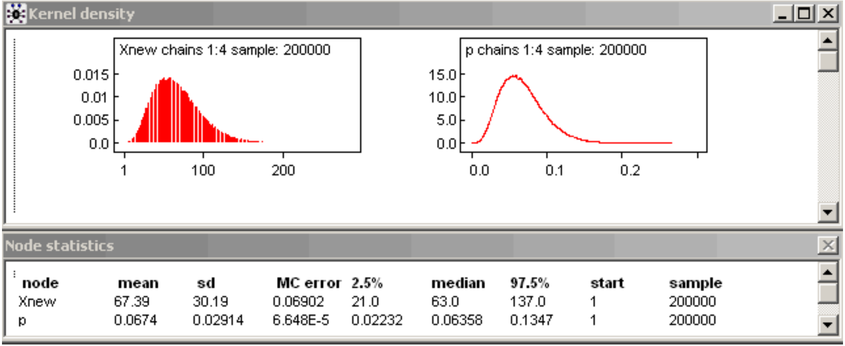
(One is, of course, assuming the stability of the process at the current p.) The code is

```
model {
 X~dbin(p,50)
 p~dbeta(.92,22.08)
 Xnew~dbin(p,1000)
}
#here are the data for the problem
list(X=4)
#here are 4 possible initializations for Gibbs samplers
list(p=.1,Xnew=50)
list(p=.5,Xnew=20)
list(p=.001,Xnew=30)
list(p=.7,Xnew=2)
```

Example 2 (cont.)

This is a "tame" problem (that could actually be solved completely by pencil and paper) and the 4 initializations all yield the same view of the posterior. The figure on the next panel provides an approximate view of the posterior distribution of p and the posterior predictive distribution of the number of non-conforming items among the next 1000 produced. (Note that in retrospect, it is no surprise that these distributions have essentially the same shape. $n = 1000$ is big enough that we should expect the sample fraction non-conforming among the 1000 to be about p, whatever that number might be. Thus X_{new} should have a posterior predictive distribution much like a posterior for $1000p$.)

Example 2 (cont.)



node	mean	sd	MC error	2.5%	median	97.5%	start	sample
Xnew	67.39	30.19	0.06902	21.0	63.0	137.0	1	200000
p	0.0674	0.02914	6.648E-5	0.02232	0.06358	0.1347	1	200000

Figure: Approximate posterior distributions for p and for X_{new} (based on $n = 1000$) upon observing $X = 4$ non-conforming among 50 using a Beta$(.92, 22.08)$ prior