

**IE 361 Exam 1  
Fall 2007**

**I have neither given nor received unauthorized assistance on this exam.**

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Name

Date

This exam consists of 20 multiple choice questions. There is a single best answer for each question. **Circle EXACTLY ONE response** for each question on this answer sheet.

1.    A    B    C    D    E
2.    A    B    C    D    E
3.    A    B    C    D    E
4.    A    B    C    D    E
5.    A    B    C    D    E
6.    A    B    C    D    E
7.    A    B    C    D    E
8.    A    B    C    D    E
9.    A    B    C    D    E
10.   A    B    C    D    E
11.   A    B    C    D    E
12.   A    B    C    D    E
13.   A    B    C    D    E
14.   A    B    C    D    E
15.   A    B    C    D    E
16.   A    B    C    D    E
17.   A    B    C    D    E
18.   A    B    C    D    E
19.   A    B    C    D    E
20.   A    B    C    D    E

1. "Quality of conformance"
  - a) can in all cases only be achieved using state-of-the art processes
  - b) is concerned with the appropriateness of features and configuration of a product
  - c) has to do with small variation
  - d) Exactly 2 of the responses a) through c) are correct completions of the sentence.
  - e) None of the responses a) through c) are correct completions of the sentence.
  
2. The "continuous improvement" emphasis in modern quality assurance
  - a) essentially requires a "process orientation"
  - b) is a practical necessity in a competitive global business environment
  - c) can be thought of in ethical terms as "the way things 'should' be"
  - d) Exactly 2 of the responses a) through c) are correct completions of the sentence.
  - e) All of the responses a) through c) are correct completions of the sentence.
  
3. Several modes on a histogram showing measured diameters of axels produced by a turning process suggest
  - a) multiple versions of some element in the production process
  - b) multiple versions of the measurement process
  - c) multiple versions of *both* some element of the production process and of the measurement process
  - d) multiple versions of some element of the production process, or of the measurement process, or both
  - e) None of the responses a) through d) is a correct completion of the sentence.
  
4. Process analysis tools like flowcharts and Ishikawa diagrams
  - a) can help identify important data needs
  - b) are really only useful when working on very simple and technologically mature processes
  - c) should be employed early in a process improvement effort
  - d) Exactly 2 of the responses a) through c) are correct completions of the sentence.
  - e) All of the responses a) through c) are correct completions of the sentence.
  
5. Regarding basic concepts of metrology,
  - a) a problem with measurement validity can be addressed through a calibration study
  - b) a bathroom scale that is broken and always reads 150lbs is nevertheless a "precise" device
  - c) "accuracy" and "precision" are different words for the same concept
  - d) Exactly 2 of the responses a) through c) are correct completions of the sentence.
  - e) None of the responses a) through c) are correct completions of the sentence.
  
6. "Linearity" (in the sense that metrologists use the word) of a measurement device
  - a) holds if  $average\ measurement = 2 \times (measurand)$
  - b) holds if  $average\ measurement = 3 + measurand$
  - c) holds if  $average\ measurement = 3 + 2 \times (measurand)$
  - d) Exactly 2 of the responses a) through c) are correct completions of the sentence.
  - e) All of the responses a) through c) are correct completions of the sentence.

**Fact: The upper 2.5% point of the  $F_{2,2}$  distribution is about 39.0.**

7. The same specimen of 20lb bond paper is weighed  $n_1 = 3$  times on scale #1 and  $n_2 = 3$  times on scale #2, with resulting means and standard deviations  $\bar{y}_1 = 3.4760\text{g}$ ,  $\bar{y}_2 = 3.4633\text{g}$ ,  $s_1 = .0056\text{g}$ , and  $s_2 = .0133\text{g}$ . Based on 95% confidence limits made from these values, one can conclude

- that there is no clear difference in either scale biases or standard deviations
- that there is no clear difference in scale biases, but there is a clear difference in scale standard deviations
- that there is a clear difference in scale biases, but no clear difference in scale standard deviations
- that there are clear differences in both scale biases and in scale standard deviations
- One of a) through d) is a correct completion of the sentence and sample sizes are large enough to see definitively which scale is most accurate.

8. Two different specimens of 20lb bond paper are weighed respectively  $n_1 = 3$  and  $n_2 = 3$  times on the same scale with resulting means and standard deviations  $\bar{y}_1 = 3.4760\text{g}$ ,  $\bar{y}_2 = 3.2567\text{g}$ ,  $s_1 = .0056\text{g}$ , and  $s_2 = .0133\text{g}$ . Based on 95% confidence limits made from these values, one can conclude

- that there is no clear difference between weights of the specimens
- that there is a clear difference between weights of the specimens
- nothing about the true weight of specimen #1
- Both a) and c) are correct completions of the sentence.
- Both b) and c) are correct completions of the sentence.

**(For questions 9 and 10) A student group studied the densities of some  $\text{Nd}_2\text{O}_3$  pellets produced by a stable production process. Densities of 6 different pellets were measured and the resulting values had sample mean 5.70g/cc and standard deviation .80g/cc. The density of a 7<sup>th</sup> pellet was measured 5 times and the resulting values had mean 4.60g/cc and standard deviation .25g/cc.**

9. 95% confidence limits for a true mean pellet density for this process

- are  $5.70 \pm .83\text{g/cc}$
- are  $5.70 \pm 2.06\text{g/cc}$
- are  $4.60 \pm .31\text{g/cc}$
- are  $4.60 \pm .69\text{g/cc}$
- can not be determined from the given information

10. Approximate 95% confidence limits for a specimen-to-specimen standard deviation of actual pellet density

- can not be determined from the given information
- are .15g/cc and .72g/cc
- are .46g/cc and 2.18g/cc
- are .50g/cc and 1.96g/cc
- None of responses a) through d) is a correct completion of the sentence.

(For questions 11 and 12) Below is part of a JMP report from a "one-way random effects" analysis of a study where an operator used the same gauge to measure the heights of 10 steel punches 3 times apiece. (The data were actually those of student 1 in Table 2.1 of SQAME.) Units used entering the data were .001 inches.

REML Variance Component Estimates						
Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Punch	1.7362514	1.9098765	1.0794185	-0.205784	4.0255367	63.454
Residual		1.1	0.3478505	0.643847	2.2938704	36.546
Total		3.0098765				100.000

11. In units of .001 inch, 95% confidence limits for the repeatability standard deviation in this context

- a) are not available based on the given information
- b) are 0 and 4.02
- c) are 0 and 2.01
- d) are .64 and 2.29
- e) are .80 and 1.51

12. In units of .001 inch, 95% confidence limits for the standard deviation of actual punch heights

- a) are not available based on the given information
- b) are 0 and 4.02
- c) are 0 and 2.01
- d) are .64 and 2.29
- e) are .80 and 1.51

13. Below is an ANOVA table from a small gauge R&R study where the same scale was used to measure the weights of  $I = 2$  pieces of 20lb bond paper,  $m = 3$  times each by  $J = 5$  different operators. The original data were in grams.

Source	SS	df	MS
Piece	.37386	1	.37386
Operator	.00061	4	.000152
Piece × Operator	.00013	4	.000032
Error	.00095	20	.000047
Total	.37555	29	

Based on the values in this table, a single number estimate of the reproducibility standard deviation,  $\sigma_{\text{reproducibility}}$ ,

- a) can not be determined
- b) is .0012g
- c) is .004g
- d) is .007g
- e) is .008g

14. In the context of problem 13, specifications on the weight of a piece of paper this size are *some value*  $\pm .080g$

so that the difference between the upper and lower specifications is .160g. It is possible to determine from the values in the ANOVA table here that  $\hat{\sigma}_{R\&R} \approx .008g$  and that  $\hat{v}_{R\&R} = 17$ . Approximate 95% confidence limits for a precision to tolerance ratio (a GCR) for checking conformance to these specifications

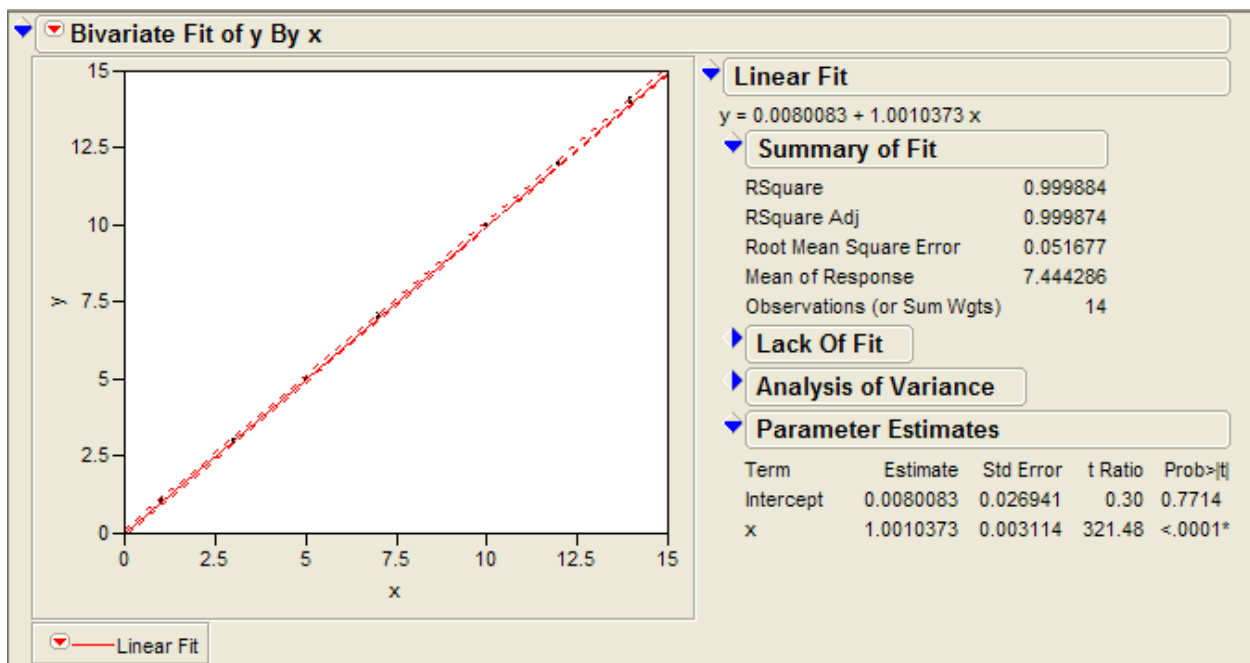
- are .00375 and .075 and indicate that the scale is adequate for this purpose
- are .00375 and .075 and call into question the adequacy of the scale for this purpose
- are .225 and .45 and indicate that the scale is adequate for this purpose
- are .225 and .45 and call into question the adequacy of the scale for this purpose.
- None of the responses a) through d) is a correct completion of the sentence.

15. In a particular gauge R&R problem, 95% confidence limits on  $\sigma_{\text{repeatability}}$  are .1 to .3, while 95% confidence limits on  $\sigma_{\text{reproducibility}}$  are 1.8 to 2.4, all in the original units of measurement.

These results suggest

- differences between operator biases are a relatively important part of measurement variation
- single measurements of the same part by many operators would have a substantially larger variability than many measurements of a single part by a fixed operator
- creation of a standard measurement protocol and operator training might substantially reduce measurement variation
- Exactly 2 of responses a) through c) are correct completions of the sentence.
- All of responses a) through c) are correct completions of the sentence.

Below is a part of a JMP report from the analysis of a calibration data set. The data are from a weighing exercise, where  $n = 14$  specimens of "known" weights  $x$  were weighed on a scale, producing measurements  $y$ . All units were kg. Use the information on this report to answer questions 16 and 17.



**16.** In approximate terms, a specimen of known weight  $x = 9.00\text{kg}$  weighed tomorrow on this scale could be expected to weigh

- a)  $9.00 \pm 2(.052)\text{kg}$
- b)  $9.00 \pm 2(.027)\text{kg}$
- c)  $9.00 \pm 2(.003)\text{kg}$
- d) None of a) through c) is close to being an appropriate completion of this sentence.

**17.** "Linearity" of this scale (in the meaning of the word as employed in metrology) is evident from the report because

- a) the curves plotted on the graph are approximately straight lines
- b) 95% confidence limits for the slope,  $\beta_1$ , include 1.0
- c) 95% confidence limits for the slope,  $\beta_1$ , do not include 0
- d) 95% confidence limits for the intercept,  $\beta_0$ , include 0
- e) 95% confidence limits for the repeatability standard deviation will be based on  $\nu = 12$  df

**(For questions 18-20)** Here is a small hypothetical data set from a study of go/no-go inspection of 4 parts by 3 operators. Each operator (without being aware that a given part was being re-inspected) made 100 good/defective calls on each part. The values in the table are  $\hat{p}$  values (fractions of "defective" calls made by the operators).

	Operator 1	Operator 2	Operator 3
Part 1	.25	.20	.27
Part 2	.10	.15	.13
Part 3	.82	.90	.84
Part 4	.05	.21	.07

**18.** Considering only Part 1, single number estimates of  $\sigma_{\text{R\&R}}^2$  and  $\sigma_{\text{reproducibility}}^2$  are closest to

- a) .1824 and 0
- b) .1824 and .0013
- c) .1824 and .1824
- d) .4271 and 0
- e) .4271 and .0361

**19.** Still considering only Part 1, 95% confidence limits for the difference in long run fractions of "defective" calls that would be made by operators 1 and 2 (i.e.  $p_{11} - p_{12}$ )

- a) are  $.05 \pm .117$
- b) indicate that there is a detectable difference between how the operators perceive this part
- c) could be made narrower by increasing the number of "calls" the operators make on that part
- d) Exactly 2 of responses a) through c) are correct completions of the sentence.
- e) All of a) through c) are correct completions of the sentence.

**20.** Approximate 95% confidence limits for comparing the average (across many parts) fraction (across many calls) of "defective calls for operators 1 and 2 (operator 1 minus operator 2) are

- a)  $-.06 \pm .067$
- b)  $-.06 \pm .138$
- c)  $-.06 \pm .276$
- d)  $-.06 \pm .800$
- e) None of the responses a) through d) above is close to a correct answer.

**IE 361 Exam 2**  
**(With "Clarified Version" of Question #3)**  
**Fall 2007**

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- 1. A B C D E
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- 5. A B C D E
- 6. A B C D E
- 7. A B C D E
- 8. A B C D E
- 9. A B C D E
- 10. A B C D E
- 11. A B C D E
- 12. A B C D E
- 13. A B C D E
- 14. A B C D E
- 15. A B C D E
- 16. A B C D E
- 17. A B C D E
- 18. A B C D E
- 19. A B C D E
- 20. A B C D E

1. Control charting
  - a) is a form of Engineering Feedback Control.
  - b) compares individual process measurements to specifications in order to check for lack of process stability.
  - c) is most effective as a quality assurance tool when applied at the end of a production chain, a substantial distance in time and space from processes being monitored.
  - d) None of responses a) through c) are correct completions of the sentence.
  - e) Exactly 2 of responses a) through c) are correct completions of the sentence.
  
2. Control limits for a generic plotted statistic  $Q$ 
  - a) are typically derived from a probability model for  $Q$  appropriate when individual process outcomes are modeled as "independent draws from a fixed universe."
  - b) typically depend upon the size of the sample used to compute  $Q$ .
  - c) typically depend upon process parameters.
  - d) All of responses a) through c) are correct completions of the sentence.
  - e) Exactly 2 of responses a) through c) are correct completions of the sentence.
  
3. In standards-given variables control charting with sample size  $n$  and process standards  $\mu$  and  $\sigma$ ,
  - a) the difference between  $UCL_{\bar{x}}$  and  $LCL_{\bar{x}}$  **always** shrinks as  $n$  increases (the spread in control limits gets smaller as sample size increases).
  - b) the difference between  $UCL_s$  and  $LCL_s$  **always** shrinks as  $n \geq 6$  increases (the spread in control limits gets smaller as sample size increases).
  - c) the difference between  $UCL_R$  and  $LCL_R$  **always** shrinks as  $n \geq 7$  increases (the spread in control limits gets smaller as sample size increases).
  - d) All of responses a) through c) are correct completions of the sentence.
  - e) Exactly 2 of responses a) and b) are correct completions of the sentence.

Below are sample means and ranges from 10 samples of size  $n = 5$ . Use the information from these samples to answer questions 4 through 6.

Sample	1	2	3	4	5	6	7	8	9	10	Sum
$\bar{x}$	11.0	11.9	9.7	10.0	11.1	11.3	11.0	11.5	11.1	12.6	111.2
$R$	2.1	6.0	3.9	4.5	3.9	5.8	6.0	5.2	2.9	6.8	47.1

4. Consider standards given control charting for both  $\bar{x}$  and  $R$ , with standards  $\mu = 11.0$  and  $\sigma = 2.0$ .
  - a) The  $\bar{x}$  chart and  $R$  chart both produce out-of-control signals.
  - b) The  $\bar{x}$  chart produces out-of-control signals, but the  $R$  chart does not.
  - c) The  $\bar{x}$  chart produces no out-of-control signals, but the  $R$  chart produces signals.
  - d) Neither chart produces out-of-control signals.
  
5. Consider retrospective control charting for both  $\bar{x}$  and  $R$ .
  - a) The  $\bar{x}$  chart and  $R$  chart both produce out-of-control signals.
  - b) The  $\bar{x}$  chart produces out-of-control signals, but the  $R$  chart does not.
  - c) The  $\bar{x}$  chart produces no out-of-control signals, but the  $R$  chart produces signals.
  - d) Neither chart produces out-of-control signals.

6. Ignore any lack of stability that you may have found in the sample ranges on the previous page and use the values there to estimate  $\sigma$ . Based on this estimate, in turn set an upper control limit for the sample standard deviation ( $s$ ) for an additional sample of size  $n = 4$ . (Remember that the samples represented in the table were of size  $n = 5$ .) This limit is closest to

- a) 2.02.
- b) 3.98.
- c) 4.23.
- d) 9.51.
- e) 9.96.

7. Below are 3 statements about control charting. How many of them are true?

- $R$  charting is less sensitive to process change than  $s$  charting.
  - Small sample sizes deprive one of the possibility of directly detecting unexpected improvements in process "spread" using ordinary three-sigma Shewhart control charts.
  - If one uses only an  $\bar{x}$  chart (and no  $R$  or  $s$  chart) it is impossible to detect a huge increase in process "spread."
- a) All 3 are correct.
  - b) Exactly 2 are correct.
  - c) Exactly 1 is correct.
  - d) None is correct.

8. In a particular "mean nonconformities per unit" context, a process standard rate is 1.5 nonconformities per single inspection unit. A redesign of some elements of the production process is intended to reduce that rate/improve the process. After implementation of the redesign, a standards given Shewhart control charting scheme will be set up to monitor nonconformities per unit based on  $k$  inspection units per sampling period.

- a) *Neither* a  $k = 2$  scheme nor a  $k = 8$  scheme *will* provide the ability to directly confirm a process improvement.
- b) A  $k = 2$  scheme *will not* provide the ability to directly confirm a process improvement, but a  $k = 8$  scheme *will* provide this ability.
- c) A  $k = 2$  scheme *will* provide the ability to directly confirm a process improvement, but a  $k = 8$  scheme *will not* provide this ability.
- d) *Both* a  $k = 2$  scheme and a  $k = 8$  scheme *will* provide the ability to directly confirm a process improvement.
- e) Not enough information is given here to tell whether any particular choice of  $k$  will provide the possibility of directly confirming process improvement.

9. Below are some records from inspection for counts of loose bolts (after torquing) on a model of large machine. (Each machine has 120 such bolts that are all checked.)

Machine	1	2	3	4	5	6	7	8	9	10	Sum
Loose Bolts	11	6	6	7	8	6	6	10	5	9	74

Appropriate retrospective control limits for the counts in the table above

- a) are  $7.4 \pm 7.9$ .
- b) are  $7.4 \pm 8.2$ .
- c) indicate process stability.
- d) a) and c) are both correct completions of the sentence.
- e) b) and c) are both correct completions of the sentence.

10. Standards given control limits for  $\bar{x}$  based on samples of size  $n = 5$  are  $17 \pm 2$ .
- Corresponding standards given control limits for sample medians are  $17 \pm 2.4$ .
  - Corresponding engineering specifications on  $x$  must be  $17 \pm 2.7$ .
  - Corresponding standards given control limits for  $s$  are impossible to determined based on this information.
  - Exactly 2 of a) through c) are correct completions of the sentence.
  - All of a) through c) are correct completions of the sentence.
11. Plotted points "hugging the center line" on a Shewhart  $\bar{x}$  chart
- could be produced by an unexpected decrease in the process standard deviation,  $\sigma$ .
  - are the ideal, indicating a very stable process.
  - will fail to be directly detected by any of the "Western Electric Alarm Rules."
  - Exactly 2 of a) through c) are correct completions of the sentence.
  - All of a) through c) are correct completions of the sentence.
12. Five nominally identical production streams
- should be treated as a single stream for purposes of process monitoring.
  - can produce a "rational subgroup" if one outcome is sampled from each stream at a given period.
  - are best monitored using 5 separate control charts.
  - Exactly 2 of a) through c) are correct completions of the sentence.
  - All of a) through c) are correct completions of the sentence.
13. Process standards are  $\mu = 100$  and  $\sigma = 7$  and observations from the process are normally distributed. Below are 3 statements about a standards-given Shewhart  $\bar{x}$  chart for this process. How many of them are true?
- The chart has the same ARL when  $\mu = 100$  and  $\sigma = 7$  as does a standards given  $\bar{x}$  chart for a process with standards  $\mu = 15$  and  $\sigma = .7$  when in fact  $\mu = 15$  and  $\sigma = .7$ .
  - The ARL for this chart when  $\mu = 100$  and  $\sigma = 5$  is more than 370.
  - The ARL for this chart when  $\mu = 100$  and  $\sigma = 9$  is less than 370.
- All 3 are correct.
  - Exactly 2 are correct.
  - Exactly 1 is correct.
  - None is correct.
14. Flaws are generated in the production of carpet at a standard rate of 1 per 20 square yards of carpet. Certain 5 square yard rugs are inspected for flaws and a rug is considered to be non-conforming if it has any flaws on it. 10 of these rugs are inspected per hour. Let
- $X$  = the total number of flaws seen across the 10 rugs, and
- $Y$  = the number of non-conforming rugs among the 10
- Standards given control limits for  $X$  and  $Y$  are
- respectively  $2.5 \pm 4.7$  and  $2.5 \pm 4.1$ .
  - respectively  $2.5 \pm 4.7$  and  $2.2 \pm 4.5$ .
  - respectively  $2.5 \pm 4.7$  and  $2.2 \pm 3.9$ .
  - impossible to determine from the given information.
  - None of a) through d) is a correct completion of the sentence.

15. "Online"/ongoing process tweaking/adjustment
- is never an appropriate variation reduction methodology.
  - requires having a process "knob" that one can "turn" with predictable results in response to a process mis-adjustment.
  - is an alternative to control charting (having the same realm of application as Shewhart charting).
  - None of a) through c) is a correct completion of the sentence.
  - Exactly 2 of a) through c) are correct completions of the sentence.

16. 10 consecutive samples of size  $n = 1$  from a process are as below

Sample	1	2	3	4	5	6	7	8	9	10	Mean
$x$	1.01	2.03	2.98	3.97	4.95	5.97	6.97	8.07	8.96	9.97	5.488
$MR$		1.02	.95	.99	.98	1.02	1.00	1.10	.89	1.01	.9956

Further,

$$.9956 / 1.128 = .883$$

- A retrospective individuals chart based on the best available estimate of  $\sigma$  indicates there is process instability, and this estimate and analysis appear to be appropriate.
  - A retrospective individuals chart based on the best available estimate of  $\sigma$  indicates no process instability, and this estimate and analysis appear to be appropriate.
  - A retrospective individuals chart based on the best available estimate of  $\sigma$  indicates there is process instability, but the clear linear trend in the data makes it clear that this estimate is inappropriate.
  - A retrospective individuals chart based on the best available estimate of  $\sigma$  indicates no process instability, but the clear linear trend in the data makes it clear that this estimate is inappropriate.
17. Normal plots (normal quantile plots in JMP language)
- will always be approximately linear if a sample of size at least 30 is represented.
  - are tools for assessing the extent to which one should trust the confidence levels associated with the intervals for  $6\sigma$ ,  $C_p$ , and  $C_{pk}$  and the normal-based prediction and tolerance intervals of Chapter 5 of *SQAME*.
  - are tools for assessing whether  $C_p$  and  $C_{pk}$  are relevant descriptors of process capability.
  - Exactly 2 of responses a) through c) are correct.
  - All of responses a) through c) are correct.

Data from problem 5.15 of *SQAME* include measured diameters from  $n = 20$  3-inch rolled sheet metal saddles produced on a single day. These have  $\bar{x} = 3.0377$  in,  $s = .0606$  in,  $\min x_i = 2.969$  in, and  $\max x_i = 3.188$  in. Specifications on this diameter are  $3.00 \pm .20$  in. Use these facts to answer questions 18 through 20.

18. Assuming diameter to be normally distributed, what are limits that you are "at least 90% sure" would bracket one more (a 21<sup>st</sup>) saddle diameter?
- 2.969 in and 3.118 in.
  - $3.0377 \pm .1074$  in.
  - $3.0377 \pm .0606$  in.
  - either response a) or response b) could be correctly used.
  - either response a) or response c) could be correctly used.

19. If one assumes that the saddle-rolling process produces normally distributed diameters, 95% confidence limits for a capability ratio measuring current process performance
- a) are .57 and 1.21.
  - b) are .75 and 1.45 .
  - c) could be used to help evaluate whether a company "Six Sigma Program" goal is being met by seeing where the limits are relative to the value 2.0.
  - d) Both a) and c) are correct completions of the sentence.
  - e) Both b) and c) are correct completions of the sentence
20. Assuming diameter to be normally distributed, 95% confidence limits for the "process capability,"  $6\sigma$
- a) are .28 in and .53 in .
  - b) are .21 in and .77 in .
  - c) are .05 in and .09 in .
  - d) can not be determined from the information provided.
  - e) can be determined from the information provided, but are not close to any of the limits provided in a) through c).

**IE 361 Exam 3  
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- 19. A B C D E
- 20. A B C D E

In the experimental determination of the density of a particular metal specimen, a graduated cylinder is partially filled with water, the volume read, and the weight of the cylinder plus water determined. The specimen is then dropped into the cylinder and the volume (of water and specimen) and weight (of cylinder, water and specimen) are again determined. The specimen density is calculated as

$$D = \frac{W_2 - W_1}{V_2 - V_1}$$

(where the  $W$ 's are the two weights and the  $V$ 's are the two volumes). Originally,  $W_1 \approx 200$  g, and  $V_1 \approx 100$ cc. The metal specimen has a mass of roughly 60g and a volume of roughly 10cc. Suppose that the scale being used has an associated repeatability standard deviation of  $\sigma_w = .01$ g and the repeatability standard deviation associated with the use of the graduated cylinder in this way is  $\sigma_v = .3$ cc. Use this information when answering questions **1.** through **3.** below.

**1.** Using the "propagation of error" formula, a plausible standard deviation to associate with an experimentally determined density of this specimen

- a) is about .06g/cc.
- b) is about .25g/cc.
- c) is about .31g/cc.
- d) is about .33g/cc.
- e) is close to none of the responses a) through d) above.

**2.** If a specimen twice the size of the one referred to above is used in a second study to determine density, the propagation of error formula suggests

- a) the precision with which the density is determined will be improved.
- b) the precision with which the density is determined will stay the same.
- c) the precision with which the density is determined will be degraded.
- d) nothing about whether precision will improve or be degraded.

**3.** If one is interested in improving the precision of experimentally determined densities of specimens of this type, the propagation of error formula suggests

- a) improvement in precision of the weight measurement is most important.
- b) improvement in precision of the volume measurement is most important.
- c) improvement in precisions of the two types of measurements are about equally important.
- d) nothing about the relative importance of improving the two types of measurement.

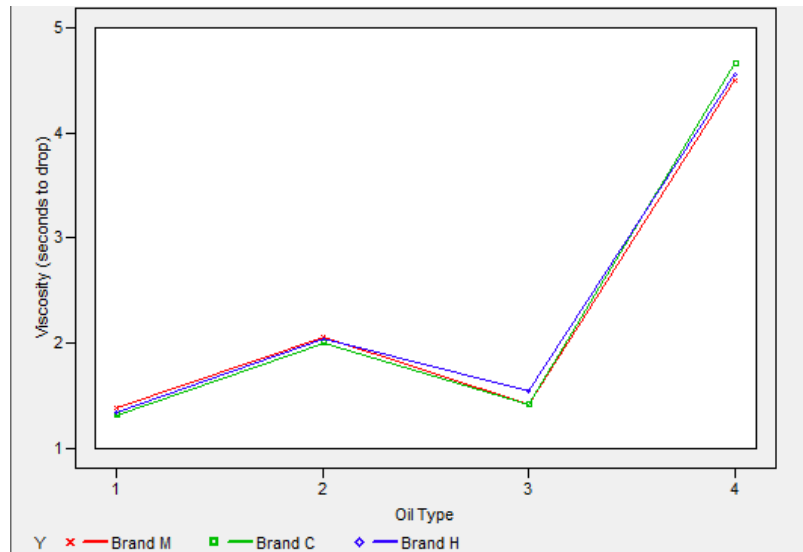
A metal drilling study was done to investigate the torque,  $y$  (ft-lb), required to rotate a drill into an aluminum alloy at various feed rates,  $x$  (in/rev). A summary of part of the data from the study is below. Use this information when answering questions **4.** through **6.** on the next page.

$x = .005$ in/rev	$x = .009$ in/rev	$x = .017$ in/rev
$n_1 = 1$	$n_2 = 4$	$n_3 = 1$
$\bar{y}_1 = 1.1$ ft-lb	$\bar{y}_2 = 2.08$ ft-lb	$\bar{y}_3 = 3.4$ ft-lb
	$s_2 = .13$ ft-lb	

4. The fact that only the  $x = .009$  in/rev condition was run more than once in this part of the study
- is a basic weakness in the data collection plan.
  - means that there is no way to check on the reasonableness of the "constant variance assumption" of the usual one-way normal model.
  - means that  $s_2$  given in the table must function as the "pooled estimate of standard deviation" for analyses based on the one-way normal model.
  - Exactly 2 of **a)** through **c)** are correct completions of the sentence.
  - All of **a)** through **c)** are correct completions of the sentence.
5. Two-sided confidence limits for  $\frac{(\mu_3 - \mu_2)}{.008} - \frac{(\mu_2 - \mu_1)}{.004}$  provide a way to ask whether  $y$  changes linearly with  $x$ . (Physically, there is probably no reason to expect this to be the case.) These limits are of the form
- $-80 \pm t \cdot s_{\text{pooled}} (113,281)$
  - $-80 \pm t \cdot s_{\text{pooled}} (15,625)$
  - $-80 \pm t \cdot s_{\text{pooled}} (337)$
  - $-80 \pm t \cdot s_{\text{pooled}} (125)$
  - None of **a)** through **d)** is close to correct.
6. By the standard of two-sided 95% confidence limits, the sample means in the table are "good to within" about
- .17 ft-lb
  - .21 ft-lb
  - .41 ft-lb
  - .41 ft-lb for the first and third, .21 ft-lb for the second
  - .41 ft-lb for the first and third, .10 ft-lb for the second

An ISU student group did a viscosity study for 3 different Brands of motor oil of 4 different Types/weights. They measured the time (in seconds) required for a particular ball to drop a fixed distance through samples of oil at room temperature. Each measurement was repeated  $m = 10$  times. (The students almost certainly did *not* use 10 different cans of oil of each type, so the variability that they observed was really purely repeatability variation and *likely does not reflect can-to-can variation in oil viscosity for cans of these types*. We will simply take this fact for what it is worth, and remember that strictly speaking conclusions drawn here apply to the 12 cans of oil tested, and not more generally to all cans of these types from these companies.) The pooled standard deviation from the 120 measurements made by the group was  $s_{\text{pooled}} = .12$  sec. The following is a table of sample means of the measurements and there is a plot of these means on the next page. Use them in answering questions **7.** through **9.**

	Type 1 (10W30)	Type 2 (SAE30)	Type 3 (10W40)	Type 4 (20W50)
Brand M	1.385	2.066	1.414	4.498
Brand C	1.319	2.002	1.415	4.662
Brand H	1.344	2.049	1.544	4.549



7. The "2 standard deviation margin of error" associated with any single mean in the study (as representing the can from which the oil was drawn) is about .076 sec. This fact and the table and plot above suggest

- a) that there are clearly detectable viscosity differences between oil Types.
- b) that differences in viscosity between Brands for oil Types 1 and 2 are not statistically detectable.
- c) that (even if they are barely statistically detectable) Brand by Type interactions are small in comparison to Type main effects.
- d) All of a), b) and c) are correct completions of the sentence.
- e) Exactly two of a), b) and c) are correct completions of the sentence.

8. The difference between the Oil Type 2 and Oil Type 1 average sample means is about .690 sec. This difference

- a) has a "2 standard deviation margin of error" around .076 sec, clearly indicating a detectable difference in Type 1 and Type 2 main effects.
- b) has a "2 standard deviation margin of error" around .076 sec, indicating there is no detectable difference in Type 1 and Type 2 main effects.
- c) has a "2 standard deviation margin of error" around .062 sec, clearly indicating a detectable difference in Type 1 and Type 2 main effects.
- d) has a "2 standard deviation margin of error" around .062 sec, indicating there is no detectable difference in Type 1 and Type 2 main effects.
- e) None of a) through d) is a correct completion of the sentence.

9. The "2 standard deviation margin of error" associated with differences in Brand main effects

- a) is about .054 sec
- b) is about .062 sec
- c) is about .076 sec
- d) is about .240 sec
- e) is none of responses a) through d).

**10. Two-factor interactions in a two-way factorial study**

- a) are measures of what that can be seen in factor combination means beyond what is explainable in terms of an overall mean and the factors acting "individually."
- b) are measures of "lack of parallelism" that would be seen on an "interaction plot" of means.
- c) are "small" for simple systems where one can say what changing levels of Factor A does to mean response without having to specify what level of Factor B is under discussion.
- d) Exactly 2 of a) through c) are correct completions of the sentence.
- e) All of a) through c) are correct completions of the sentence.

Consider a hypothetical balanced  $2^3$  complete factorial data set with  $m = 4$  observations per combination producing sample means and standard deviations as in the table below.

Combination	$\bar{y}$	$s$	$\ln s$
(1)	7	.08	-2.5
a	9	.08	-2.5
b	7	.22	-1.5
ab	9	.22	-1.5
c	11	.08	-2.5
ac	13	.08	-2.5
bc	11	.22	-1.5
abc	13	.22	-1.5

Use the values in this table in answering questions **11.** through **15.**

**11.** If one assumes that the experimental response variable,  $y$ , has the same standard deviation for every fixed combination of levels of the three factors A, B and C, an estimate of this standard deviation

- a) is .08
- b) is .150
- c) is .166
- d) is .22
- e) is not close to any of a) through d) above.

**12.** Continuing to assume that the response has the same standard deviation for every fixed combination of levels of the factors, a "margin of error" based on 95% two-sided confidence limits for any of the fitted effects produced by applying the Yates algorithm to the sample means (the  $\bar{y}$ 's)

- a) is .055
- b) is .060
- c) is .171
- d) is .310
- e) is .342

**13.** Still continuing under the constant standard deviation model, the Yates algorithm shows that (in terms of the factorial effects)

- a) only the A main effects on mean response are statistically detectable.
- b) only the B main effects on mean response are statistically detectable.
- c) only the C main effects on mean response are statistically detectable.
- d) only the main effects on mean response of exactly 2 of the 3 factors are statistically detectable.
- e) only the main effects on mean response of exactly 2 of the 3 factors and the corresponding 2-factor interactions are statistically detectable.

**14.** If one wants to entertain the possibility that variability in  $y$  for fixed levels of the factors changes combination-to-combination, a possible way to proceed is to apply the Yates algorithm to the 8 values of  $s$  (or better, the 8 values of  $\ln s$ ) in the table. There is no "replication" of  $s$  values and thus no way to find a "margin of error" to attach to the values output in this second use of the Yates algorithm and judge statistical significance. But the more-or-less-obvious conclusion of using the algorithm to analyze response variance is that

- a) only the A main effects on response variability are large.
- b) only the B main effects on response variability are large.
- c) only the C main effects on response variability are large.
- d) only the main effects on response variability of exactly 2 of the 3 factors are large.
- e) only the main effects on response variability of exactly 2 of the 3 factors and the corresponding 2-factor interactions are large.

**15.** The kind of analysis outlined in questions **11.** through **14.** suggests that if one wants large mean  $y$  with small variability in  $y$ , levels of the factors should be set as

- a) A high, B low, C high
- b) A either low or high, B low, C high
- c) A high, B either low or high, C high
- d) A high, B low, C either low or high
- e) None of responses **a)** through **d)** above is appropriate.

An ISU student group did a project with a company involving the operation of a large injection molding machine. What follows is a simplification and slight modification of some aspects of that project. The company was interested in improving the output of the machine (in terms of pounds of plastic extruded per hour). In some (undisclosed) units,  $y$  is a measure of this process yield. The students considered the impact of 6 experimental factors on  $y$ . These were A (bulk density of raw material), B (amount of water added to the raw material), C (amperage supplied to the crammer/auger), D (extruder screw speed), E (extruder front end temperature), and F (extruder back end temperature). Each factor had a low and a high level. An unreplicated  $2^{6-2}$  study produced 16 values of  $y$ . The two generators used to choose levels of Factors E and F to go with combinations of levels of Factors A through D were

$$E \leftrightarrow ABC \text{ and } F \leftrightarrow BCD$$

**16.** The generators above indicate that in the experimentation, when Factors A and B were at their high levels and Factors C and D were at their low levels, the levels of the other 2 factors used were

- a) E low and F low
- b) E low and F high
- c) E high and F low
- d) E high and F high
- e) unknown, but randomly chosen.

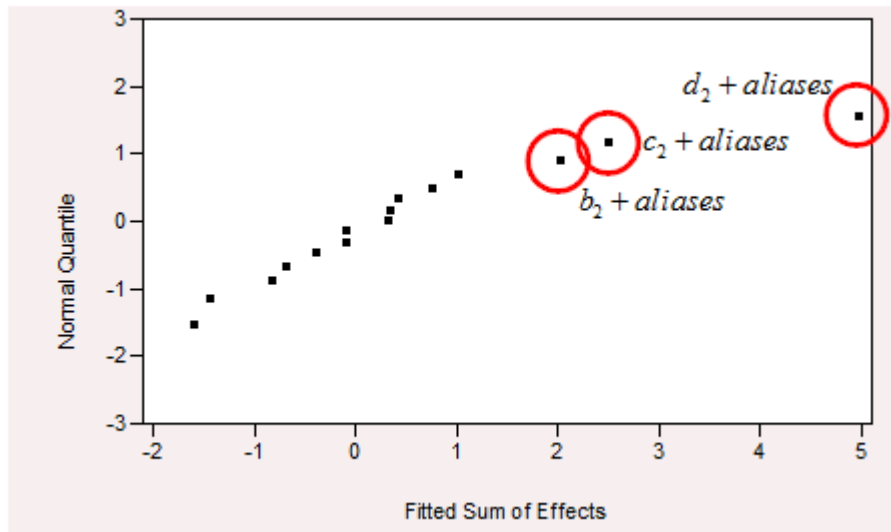
**17.** The "defining relation" for the study

- a) is  $I \leftrightarrow ABCE$
- b) is  $EF \leftrightarrow AD$
- c) is  $I \leftrightarrow ABCE \leftrightarrow BCDF$
- d) is  $I \leftrightarrow ABCE \leftrightarrow BCDF \leftrightarrow ADEF$
- e) is none of the answers **a)** through **d)**.

**18.** When the Yates algorithm is applied to the 16 values of yield,  $y$ , (placed into Yates standard order as regards levels of Factors A,B,C, and D ignoring levels of Factors E and F), a fitted sum of effects that involves the E main effect can be found in the last column of the table of calculations

- a) on the 7<sup>th</sup> row.
- b) on the 8<sup>th</sup> row.
- c) on the 15<sup>th</sup> row.
- d) on the 16<sup>th</sup> row.
- e) None of the responses a) through d) is a correct completion of the sentence.

A Normal plot of the last 15 estimates produced by the Yates algorithm when used as described in question 18. is below.



**19.** The message conveyed by the normal plot is that

- a) if the extruder actually behaves in a relatively simple fashion, the high level of D, and probably also the high levels of B and C should be used in order to produce large yield.
- b) there are clearly no main effects of any of the Factors A, E, and F.
- c) yield for a fixed combination of levels of Factors A,B,C,D, and E can NOT be treated as normally distributed.
- d) Exactly 2 of responses a) through c) are correct completions of the sentence.
- e) All of responses a) through c) are correct completions of the sentence.

**20.** Suppose that long experience with the extruding machine suggests that running it with fixed levels of the experimental factors produces a long run standard deviation of yield around  $\sigma = 4.0$ . This can be used to develop a sensible "2 standard deviation margin of error" for any fitted sum of effects. By the standard of such a margin and the values indicated on the plot

- a) there are no statistically detectable sums of effects.
- b) only the sum of the D main effect and its aliases is statistically detectable.
- c) only the two sums of the D main effect and its aliases and the C main effect and its aliases are statistically detectable.
- d) there are three statistically detectable sums of effects corresponding to D, C, and B main effects and their respective aliases.
- e) None of the responses a) through d) are correct completions of the sentence.