I have neither given nor received unauthorized assistance on this exam.

KEY

Name

Date
1. Measurement and the subject of statistics are fundamentally related because
   a) measurement error has a negative effect on what of real world importance can be learned
      from (statistical) data collection and analysis.
   b) statistics can help quantify the impact of measurement error.
   c) how data are collected (statistical planning) affects what sources of measurement error impact
      (degrade) the real information content of (statistical) data.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) All of responses a) through c) are correct completions of the sentence.

2. Measurement and quality assurance are fundamentally related because
   a) without the ability to measure adequately, one cannot know what is going on in a process.
   b) modern automatic data acquisition systems allow huge amounts of information to be archived
      and used in guaranteeing good quality.
   c) quality of conformance typically cannot be reliably assessed without the ability to measure
      adequately.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) All of responses a) through c) are correct completions of the sentence.

3. A bimodal histogram of widget diameters is potentially indicative of
   a) two different machines or operators at work cutting those diameters.
   b) two different measuring methods being used to measure diameters.
   c) an opportunity for process improvement if the bimodality is traced to two different versions
      of some physical element of the cutting process.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) All of responses a) through c) are correct completions of the sentence.

4. Below are 4 (bullet) statements. How many of them are true?
   • Improvement of a measurement system used to observe a process is only a preliminary to
     improvement of process fundamentals.  ☑️
   • The "C" in the Six-Sigma DMAIC acronym refers to the use of automatic/digital
     "control" systems.  ☑️
   • Reduction of variation of essentially any kind typically makes products and processes
     work better.  ☑️
   • Benchmarking typically provides insight into what changes will enable an organization to
     leapfrog its competition in terms of effective business practice.  ☑️

   a) None of the statements are true.
   b) Exactly 1 statement is true.
   c) Exactly 2 statements are true.
   d) Exactly 3 of the statements are true.
   e) All of the statements are true.

5. If a measurement device is not "linear" (in the meaning of the word employed in metrology)
   a) average measurement is a quadratic function of measurand.
   b) there is no way to develop a calibration equation for the device.
   c) Neither a) nor b) above is a correct completion of the sentence.
   d) Both a) and b) are correct completions of the sentence.
6. Two blow-mold machines (#1 and #2) make nominally "identical" plastic bottles. Two nominally "identical" burst testers (#1 and #2) measure burst strength of some bottles produced. Of course, burst testing is destructive, and once tested, a bottle cannot be retested or used. Below are 4 (bullet) statements. How many of them are true?
   - It is impossible (by simply burst testing bottles directly from production) to ascertain the repeatability standard deviation of tester #1. Yes
   - It is impossible (by using either or both testers) to reliably assess the actual burst strength variation (measurand/part-to-part variation) produced by blow-mold machine #1. Yes
   - It is impossible to use the testers (#1 and/or #2) in any way to make a meaningful comparison of mean burst strengths produced by the blow-mold machines (#1 and #2). No
   - It is impossible to use bottles from (either or both) the blow-mold machines to compare tester biases.

   a) None of the statements are true.
   b) Exactly 1 statement is true.
   c) Exactly 2 statements are true.
   d) Exactly 3 of the statements are true.
   e) All of the statements are true.

7. Below are 4 (bullet) statements about range-based gauge R&R calculations. How many of them are true?
   - They are rarely used in industry. False
   - The usual "AIAG" (Automotive Industry Action Group) version of them is technically wrong in that the reproducibility standard deviation under the two-way random effects model is not correctly estimated. True
   - They lead easily to confidence limits for repeatability and reproducibility standard deviations. False
   - They are more reliable than corresponding ANOVA-based gauge R&R calculations. False

   a) None of the statements are true.
   b) Exactly 1 statement is true.
   c) Exactly 2 statements are true.
   d) Exactly 3 of the statements are true.
   e) All of the statements are true.

8. A reproducibility variance
   a) in a measurement context is the variance of operator-specific biases or mean measurements associated with a particular part.
   b) in a 0/1 or go/no-go context is the variance of operator-specific "call-1" rates $p_j$ associated with a specific part.
   c) measures operator-to-operator variation in measurement or "call-1" rates in a 0/1 or go/no-go context.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) All of responses a) through c) are correct completions of the sentence.
9. A gauge capability ratio
   a) compares process (or part-to-part) variation to a spread in engineering requirements $U - L$.
   b) should be small (no more than .1).
   c) compares process (or part-to-part) variation to R&R variation.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) None of responses a) through c) are correct completions of the sentence.

10. Modern process-oriented quality assurance
    a) is fundamentally an activity for upper management and rarely reaches to front line employees.
    b) employs a wide spectrum of tools, ranging from very simple ones like Ishikawa diagrams and histograms to fairly complicated tools of statistical analysis.
    c) is not really applicable to the service sector.
    d) Only responses a) and b) are correct completions of the sentence.
    e) Only responses b) and c) are correct completions of the sentence.
11. A gauge block (a standard for length) that is made to be exactly 2.000 inch is measured
    $n = 5$ times with a particular caliper with the results (in inches) below

    $2.011, 2.015, 2.010, 2.009, 2.010$

    Find 95% confidence limits for the bias of the caliper for measuring 2.000 inch.

    \[
    \bar{y} = 2.011 \text{ inch} \\
    s = .0023 \text{ inch}
    \]

    Confidence limits for the mean measurement (mean value of $y$) are

    $\bar{y} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

    i.e.

    $2.011 \pm 2.776 \frac{.0023}{\sqrt{5}}$

    i.e.

    $2.011 \pm .0029$

    So 95% limits for the bias are

    $(2.011 - 2.000) \pm .0029$

    i.e.

    $.011 \pm .0029$ inch
12. A particular machined dimension was measured twice for each of 5 machined parts (units were inches). A NQT printout for a one-way random effects analysis run on the 10 resulting measurements is below.

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Var Ratio</th>
<th>Component Var</th>
<th>Std Error</th>
<th>95% Lower</th>
<th>95% Upper</th>
<th>Pct of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>1.1875</td>
<td>0.0000019</td>
<td>1.9751e-6</td>
<td>-1.971e-6</td>
<td>5.7712e-6</td>
<td>54.286</td>
</tr>
<tr>
<td>Residual</td>
<td>0.0000016</td>
<td>1.0119e-6</td>
<td>6.2342e-7</td>
<td>9.6245e-6</td>
<td>45.714</td>
<td></td>
</tr>
</tbody>
</table>

What are 95% confidence limits for the part-to-part standard deviation of actual dimension?

95% limits for \( \sigma_p^2 \) are 0 and 5.7712 \( \times \) 10\(^{-6} \) so 
95% limits for \( \sigma_p \) are 0 and 2.4023 \( \times \) 10\(^{-3} \) inch.

What are 95% confidence limits for the measurement standard deviation?

95% limits for \( \sigma_{\text{measurement}}^2 \) are 6.2342 \( \times \) 10\(^{-7} \) and 9.6245 \( \times \) 10\(^{-6} \)
So 95% limits for \( \sigma_{\text{measurement}} \) are 7.8957 \( \times \) 10\(^{-4} \) and 3.1023 \( \times \) 10\(^{-3} \) inch.
13. In a standard gauge R&R study on a precision balance, AVOVA-based calculations produce

\[ \hat{\sigma}_{R&R} = 0.0024 \text{ g} \quad \text{and} \quad \nu_{R&R} = 17.2 \]

Give approximate 95% confidence limits for the standard deviation that would be associated with many operators each making a single measurement of the weight of a single specimen on this balance.

Use \[ \hat{\sigma} \sqrt{\frac{\hat{\sigma}^2}{\chi^2_v}} \] and \[ \hat{\sigma} \sqrt{\frac{\hat{\sigma}^2}{\chi^2_c}} \]. Here this is

\[ 0.0024 \sqrt{\frac{17}{30.131}} \quad \text{and} \quad 0.0024 \sqrt{\frac{17}{7.564}} \]

\[ 0.0018 \text{ g} \quad \text{and} \quad 0.0036 \text{ g} \]

Tomorrow, an operator weighs a specimen and gets the value 19.823 g. Propose and defend what you consider to be an appropriate "± uncertainty" to associate with this value.

\[ \pm \text{ some multiple of } \hat{\sigma}_{R&R} \]

seems appropriate since \( \hat{\sigma}_{R&R} \) measures variability coming from both repeatability and random selection of operators. Perhaps

\[ \pm 3 \hat{\sigma}_{R&R} \]

Motivated by a normal distribution assumption for measurement errors.

\[ \pm 3(0.0024) \quad \text{i.e.} \quad \pm 0.0072 \text{ g} \]
Below is a small table representing part of the results from Go/NO-Go "gauge R&R" study, based on $m = 10$ conforming/non-conforming calls per part by each operator.

<table>
<thead>
<tr>
<th>Operator 1</th>
<th>Operator 2</th>
<th>Operator 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1</td>
<td>.2</td>
<td>.4</td>
</tr>
<tr>
<td>Part 2</td>
<td>.4</td>
<td>.4</td>
</tr>
<tr>
<td>Part 3</td>
<td>.1</td>
<td>.2</td>
</tr>
<tr>
<td>Part 4</td>
<td>.2</td>
<td>.4</td>
</tr>
<tr>
<td>Part 5</td>
<td>.3</td>
<td>.3</td>
</tr>
</tbody>
</table>

Give 95% approximate limits for comparing Operators 1 and 2 on Part 1. Is there a clear difference in how they judge Part 1? Explain.

$$\hat{p}_1 - \hat{p}_2 \pm z \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{m} + \frac{\hat{p}_2(1 - \hat{p}_2)}{m}}$$

$$(.2 - .4) \pm 1.96 \sqrt{\frac{(.2)(.8)}{10} + \frac{(.4)(.6)}{10}}$$

$$-.2 \pm .41$$

There is no clear difference in "call-1" rates for these 2 operators on this part since this interval covers 0.

Give 95% confidence limits for comparing Operators 2 and 3 across all parts. Is there a clear difference in how they judge these parts? Explain.

$$\bar{y} \pm z \sqrt{\frac{S^2}{n}}$$

$$.04 \pm 2.776 \frac{.0548}{\sqrt{15}}$$

$$i.e., .04 \pm .068$$

Since this interval covers 0, there is no clear evidence that these operators differ in their average "call-1" rates across parts of this kind.
In a small calibration study, a metric caliper is used to measure some steel gauge blocks twice each. Gauge block sizes $x$ are in inches, while measurements $y$ are in mm. A NQT report for a simple linear regression of $y$ on $x$ is below. (The least squares line and 95% two-sided prediction limits for $y_{\text{new}}$ are shown on the plot.)

If tomorrow a value of 4.5 mm is read on the caliper, give 95% confidence limits for the true size of the dimension being measured (in inches).

From the figure above $(.17, .19)$ (inches) is appropriate.

We have identified the SLR parameter $\sigma$ as a kind of repeatability standard deviation. Describe a (fairly unusual, but nevertheless possible) scenario for the collection of the calibration data, where a more natural interpretation is as an R&R standard deviation.

The tacit assumption made in calling $\sigma$ a "repeatability" standard deviation has been that a single operator does all the measuring. If each of the 20 different measurements were made by a different operator, then $\sigma$ would represent both repeatability and reproducibility variation.
I have neither given nor received unauthorized assistance on this exam.

KEY

_________________________________________________
Name                Date
The first 8 question are 5-point multiple choice questions. Circle the letter corresponding to the single best response for each of these questions.

1. Statistical Process Control and Engineering Feedback Control
   a) are competing or rival technologies, amounting to different solutions for the same problem.
   b) can both be applied to reduce process variation.
   c) both require knowledge of engineering specifications in order to implement.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) None of responses a) through c) are correct completions of the sentence.

2. A "sample" or "rational subgroup" in a variables control charting application is a group of measurements
   a) representing a short enough time period that it is plausible to treat them as coming from "the same" process conditions.
   b) modeled as random draws from a fixed distribution/population/universe.
   c) can sometimes be of size \( n = 1 \).
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) All of responses a) through c) are correct completions of the sentence.

3. Control limits for an "Individuals" chart
   a) are those for "an \( n = 1 \) x-bar chart."
   b) are based directly on product performance requirements.
   c) are equivalent to engineering specifications for individual measurements.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) All of responses a) through c) are correct completions of the sentence.

4. The ARL of a particular process monitoring scheme
   a) is a theoretical tool for quantifying what the scheme will do under some model for process behavior.
   b) is typically smaller for "not ALL-OK" circumstances than for "All-OK" circumstances.
   c) is typically smaller for "drastically not ALL-OK" circumstances than for "barely not ALL-OK" circumstances.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) All of responses a) through c) are correct completions of the sentence.

5. Normal plotting is useful in process capability analysis because
   a) it provides graphical ways of estimating both process mean and standard deviation.
   b) it can provide evidence that the process has been stable and thus that there is such a thing as 'capability.'
   c) it can provide evidence that the typical process capability measures are practically relevant descriptions of the process.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence.
   e) All of responses a) through c) are correct completions of the sentence.
6. Below are 4 (bullet) statements about Statistical Process Control. How many of them are true?
   a) SPC would be better named "Statistical Process Monitoring."
   b) SPC provides a basis for capability analysis (in that without process stability/consistency it makes no sense to ask what it can be expected to do).
   c) SPC provides a basis for experimentation for process improvement (in that without process stability/consistency there is no baseline against which to judge the effectiveness of physical process changes).
   d) SPC is part of Six Sigma methodology in terms of making sure that processes that have been improved stay improved.
   a) All of the statements are true.
   b) Exactly 3 of the statements are true.
   c) Exactly 2 of the statements are true.
   d) Exactly 1 of the statements is true.
   e) None of the statements are true.

7. Below are 4 (bullet) statements about statistical intervals. How many of them are true?
   a) Prediction intervals are intended to locate "most of" a process output.
   b) Tolerance intervals are intended to locate a single additional value from a process.
   c) Confidence intervals for a mean are intended to locate the center of a process distribution.
   d) Intervals made from minimum and maximum sample values can be used as prediction and tolerance intervals even when a process distribution is clearly not normal.
   a) All of the statements are true.
   b) Exactly 3 of the statements are true.
   c) Exactly 2 of the statements are true. **The last 2 are true**
   d) Exactly 1 of the statements is true.
   e) None of the statements are true.

8. Sets of "extra alarm rules" (like the set of "Western Electric Rules")
   a) are meant to allow detection of interpretable patterns on control charts.
   b) increase ARLs when used (in addition to the basic "one point outside control limits" rule).
   c) provide for standardization of how different individuals in an organization might look for patterns on a control chart.
   d) Exactly 2 of responses a) through c) are correct completions of the sentence. **a) and c) are true**
   e) None of responses a) through c) are correct completions of the sentence.
9. Below are means and ranges of samples of size $n = 4$ journal diameters on tractor axels from a grinding process (units are $10^{-4}$ mm above 4.9000 mm).

<table>
<thead>
<tr>
<th>Sample</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}$</td>
<td>815</td>
<td>815</td>
<td>810</td>
<td>778</td>
<td>748</td>
<td>725</td>
<td>778</td>
<td>790</td>
<td>785</td>
<td>795</td>
<td>7839</td>
</tr>
<tr>
<td>$R$</td>
<td>37</td>
<td>37</td>
<td>53</td>
<td>46</td>
<td>53</td>
<td>64</td>
<td>46</td>
<td>75</td>
<td>22</td>
<td>22</td>
<td>455</td>
</tr>
</tbody>
</table>

Is there evidence of process instability in these $\bar{x}$ and $R$ values? (Show appropriate calculations and explicitly say whether there is any evidence of instability in means and ranges.)

**$\bar{x}$ chart**

- Retrospective limits are $\bar{x} \pm A_2R$ i.e. $783.9 \pm 0.729(45.5)$ i.e. $783.9 \pm 33.2$ units are $10^{-4}$ mm above 4.9000mm

- $UCL_{\bar{x}} = 817.1$ $LCL_{\bar{x}} = 750.7$

**$R$ chart**

- Retrospective limits are $D_4R = UCLR$ i.e. $2.282(45.5) = 103.9 \times 10^{-4}$ mm

Evidence of instability? (Circle the correct response.)

In means? yes/no

In ranges? yes/no

Samples 5 and 6
10. In the context of problem 9, ignoring any evidence of process instability you found there, do the following.

Give an estimate of the grinding process standard deviation of tractor axel journal diameters.

\[ \hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{45.5}{2.059} = 22.1 \times 10^{-4} \text{ mm} \]

Based on your estimate of process standard deviation above, give \( n = 5 \) control limits for future (real-time) monitoring of sample medians, \( \bar{x} \).

Use \( \mu \pm 3\bar{x} \) substituting \( \bar{x} \) for \( \mu \) and \( \hat{\sigma} \) for \( \sigma \).

This is

\[ 783.2 \pm 3(1.157) \frac{22.1}{\sqrt{5}} \text{ c.c.} \]

\[ \text{UCL}_\bar{x} = 819.4 \text{ and LCL}_\bar{x} = 748.4 \text{ units are } 10^{-4} \text{ mm above 7.3000} \]
11. A paper collating machine jams at a standard rate of 1 jam per 10 minutes of running time. As means of process monitoring two different options are being considered. Find appropriate (3-sigma) control limits for these two options.

The first option is control charting

\( X = \) the number of jams encountered in 1 hour of running time

This is a "c-chart" setting where \( \lambda \) for 60 minutes is \( 6 \) jams in 10 minutes, i.e. \( \lambda = 6 \). So

\[
UCL_X = 6 + 3\sqrt{6} = 13.3 \text{ jams}
\]

\( \text{no LCL}_X \) as \( 6 - 3\sqrt{6} < 0 \).

The second option is control charting

\( Y = \) the number of minutes in an hour of running time that have at least 1 jam in them

This is an np charting scenario, where

\[
p = P[\text{at least 1 jam in 1 min}]
\]

\[
= 1 - P[\text{no jams in 1 min}]
\]

\[
\frac{1}{10(1)} = \frac{1}{10} \text{ jams in 1 min is Poisson with mean}
\]

\[
\lambda = 0.1
\]

So control limits are

\[
np \pm 3\sqrt{np(1-p)} \quad \text{i.e.}
\]

\[
60(0.0952) \pm 3\sqrt{60(0.0952)(0.9048)}
\]

\[
5.71 \pm 6.82
\]

i.e. \( UCL_Y = 12.53 \) min and \( \text{there is no LCL}_X \) (since \( 5.71 < 6.82 \)).
12. Vardeman's brown bag has $\mu = 5.0$ and $\sigma = 1.715$, while his blue bag has $\mu = 10.0$ and $\sigma = 3.47$. In $\bar{x}$ charting with $n = 4$ and the brown bag standards, what is the ARL if in fact one is sampling from the blue bag?

Brown bag standards given control limits are:

$$UCL_{\bar{x}} = 5.0 + 3 \frac{1.715}{\sqrt{4}} = 7.573$$

and

$$LCL_{\bar{x}} = 5.0 - 3 \frac{1.715}{\sqrt{4}} = 2.427$$

Then when sampling the blue bag $\mu_{\bar{x}} = \mu_{\bar{x}} = 10.0$ and $\sigma_{\bar{x}} = \sigma_{\bar{x}} = 1.735$ and

$$q = P[\bar{x} \text{ plots outside limits}] = P[\bar{x} < \frac{2.427 - 10}{1.735} + P[\bar{x} > \frac{7.573 - 10}{1.735}$$

$$\approx 0 + .9192$$

And $ARL = \frac{1}{q} = \frac{1}{.9192} = 1.09$
13. Paper helicopter drop times for a single helicopter dropped $m = 10$ times produce $s = .120$ sec, while (single) drop times for $n = 10$ other helicopters of the same design produce $s_p = .160$ sec. You may use (without showing the calculations needed to get them) the (Module 4) facts that an estimated production process standard deviation (not inflated by measurement noise) is $\hat{\sigma}_x = .106$ sec with approximate degrees of freedom $\nu = 1.31$. If specifications on helicopter drop times are some value $\pm .30$ sec

give approximate 95% confidence limits for the capability index $C_p$ that are free of the influence of measurement noise.

\[
\hat{\sigma}_x \sqrt{\frac{2}{\chi^2_{\nu, \text{upper}}}}, \quad \hat{\sigma}_x \sqrt{\frac{2}{\chi^2_{\nu, \text{lower}}}}
\]

i.e. \[.106 \sqrt{\frac{1}{5.024}} \quad \text{and} \quad .106 \sqrt{\frac{1}{0.087}} \]

i.e. \[(.0473, 3.35) \text{ sec}\]

So limits for $C_p = \frac{U-L}{6\bar{x}}$ are

\[
\frac{.6}{6(3.35)} = .03 \quad \text{and} \quad \frac{.6}{6(.0473)} = 2.11
\]

we know very little about the real value of $C_p$.
A sample \( n = 20 \) of so-called bottle "pre-forms" from one cavity of a plastic injection molding machine have mean 22.34 g and standard deviation .008 g. The smallest weight observed is 22.1 g and the largest is 22.5 g.

What confidence do you have that the interval from 22.1 g to 22.5 g contains 90% of all future pre-form weights from this cavity?

What confidence level does \((\min \bar{x}_i, \max \bar{x}_i)\) have as a tolerance interval for 90% of all weights? This is

\[
1 - p^n - n(1-p) p^{n-1}
\]

which here is

\[
1 - (.9)^{20} - 20(.1)(.9)^{19} = .6083
\]

This is a 61% tolerance interval.

Give a normal distribution 99% lower bound for 95% of all future pre-form weights from this cavity. (Plug in completely, but there is no need to simplify.)

Use \( \bar{x} - T_s \) s. Here this is

\[
22.34 - 2.808(.008) = 22.3175 \text{ g}
\]
IE 361 Exam 3
Fall 2010

I have neither given nor received unauthorized assistance on this exam.

KEY

Name Signed                      Date

Name Printed
A relatively small ball of radius $R_1$ plus a volume $V$ of liquid are placed into a circular-cylinder-shaped container of radius $R_2$ and height $H$. The quantity

$$ U = \pi R_2^2 H - V - \frac{4}{3} \pi R_1^3 $$

is how much unused space there remains in the container. Simple calculus easily establishes that

$$ \frac{\partial U}{\partial R_1} = -4 \pi R_1^2, \quad \frac{\partial U}{\partial V} = -1, \quad \frac{\partial U}{\partial R_2} = 2 \pi R_2 H, \quad \frac{\partial U}{\partial H} = \pi R_2^2 $$

Suppose that the variables $R_1, V, R_2,$ and $H$ are in fact random with means (that are target values for the variables) and standard deviations as in the table below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$R_1$</th>
<th>$V$</th>
<th>$R_2$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>10 mm</td>
<td>1000 mm$^3$</td>
<td>20 mm</td>
<td>50 mm</td>
</tr>
<tr>
<td>Std Dev</td>
<td>.2 mm</td>
<td>2 mm$^3$</td>
<td>.2 mm</td>
<td>.5 mm</td>
</tr>
</tbody>
</table>

**a)** Approximate the mean and standard deviation of $U$, $\mu_U$, and $\sigma_U$.

$$ \sigma_U^2 \approx ( -4 \pi (10)^2 )^2 (.2)^2 + ( -1 )^2 (2)^2 + ( 2 \pi (20) 50 )^2 (.2)^2 + ( \pi (20)^2 ) (.5)^2 $$

$$ \approx 63.165 + 4 + 1579.163 + 394.784 $$

So $\sigma_U \approx \sqrt{4506.2} = 1.418 \text{ mm}^3$

$$ \mu_U \approx \pi (20)^2 (50) - 1000 - \frac{4}{3} \pi (10)^3 = 57,643 \text{ mm}^3 $$

$$ \mu_U \approx \frac{57,643 \text{ mm}^3}{2} \quad \sigma_U \approx 1.418 \text{ mm}^3 $$

**b)** Identify the input variable that is "the biggest contributor to the variability in $U$." Explain your choice.

The biggest term above is the one related to variation in $R_2$, $(\frac{\partial U}{\partial R_2})^2 \text{ Var} R_2$. $R_2$ is the biggest contributor.
2. Consider a $2^{7-3}$ fractional factorial experiment (with Factors A,B,C,D,E,F, and G) based on generators

$$E \leftrightarrow ABC, F \leftrightarrow BCD \text{ and } G \leftrightarrow ACD$$

a) This experimental plan will involve collecting data from 16 different combinations of levels of the 7 factors. Identify 4 of these 16 by finishing filling in the table below.

<table>
<thead>
<tr>
<th>A Level</th>
<th>B Level</th>
<th>C Level</th>
<th>D Level</th>
<th>E Level</th>
<th>F Level</th>
<th>G Level</th>
<th>$2^7$ Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(1)</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>cde</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>abfg</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>abcdefg</td>
</tr>
</tbody>
</table>

b) Write out the defining relation for this study, and then show what effects are aliased with the D main effect, $\delta_D$.

$$I \leftrightarrow ABCDE \leftrightarrow BCF \leftrightarrow AC \leftrightarrow AEF \leftrightarrow BEG \leftrightarrow ABDFG \leftrightarrow CDEFG$$

so that

$$D \leftrightarrow ABCDE \leftrightarrow BCF \leftrightarrow AC \leftrightarrow AEF \leftrightarrow BEG \leftrightarrow ABDFG \leftrightarrow CDEFG$$

c) Suppose that the 4-cycle Yates algorithm applied to the 16 sample means from the study (listed in Yates order as regards levels of factors A,B,C, and D) produces the pattern of magnitudes below in the "(Cycle #4)/16" column. What is the simplest interpretation of the pattern?

<table>
<thead>
<tr>
<th>(Cycle #4)/16</th>
<th>Huge (and mean + aliases)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Huge</td>
<td>$a_2 + \text{aliases}$</td>
</tr>
<tr>
<td>Big</td>
<td>$a_2 + \text{aliases}$</td>
</tr>
<tr>
<td>Small</td>
<td>$b_{22} + \text{aliases}$</td>
</tr>
<tr>
<td>Small</td>
<td>$a_{22} + \text{aliases}$</td>
</tr>
<tr>
<td>Big</td>
<td>$a_{22} + \text{aliases}$</td>
</tr>
<tr>
<td>Big</td>
<td>$a_{22} + \text{aliases}$</td>
</tr>
</tbody>
</table>

The simplest possible interpretation is that only the 2 factors A and E impact response and there is an important interaction effect between them.
3. Below are some summary statistics for a study of the effects of 3 sealing process factors on

\[ y = \text{seal strength of a particular food wrapping material (g/in)} \]

Factor A is a seal temperature, Factor B is a cooling bar temperature, and Factor C is a polyethylene content.

<table>
<thead>
<tr>
<th>Combination</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>n</th>
<th>y</th>
<th>( \sigma^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>6.6</td>
<td>.4</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>6.9</td>
<td>.1</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>4</td>
<td>7.9</td>
<td>1.3</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>2</td>
<td>6.1</td>
<td>.1</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>4</td>
<td>9.2</td>
<td>2.2</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>2</td>
<td>6.8</td>
<td>.1</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>2</td>
<td>10.4</td>
<td>5.0</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>4</td>
<td>7.3</td>
<td>3.7</td>
</tr>
</tbody>
</table>

\[ \frac{1.4 + 3.1 + 5.1.3 + 1.1 + 3.2.2 + 1.1 + 1.5 + 3.3.7}{4(2-1) + 4(9-1)} = 1.7188 \]

so \( \hat{\sigma}_{\text{pooled}} = 1.311 \text{ g/in} \)

Use limits \( \leq \frac{df}{X^2} \) and \( \leq \frac{df}{X^2} \)

\[ 1.311 \sqrt{\frac{16}{28.845}} \text{ and } 1.311 \sqrt{\frac{16}{6608}} \]

\( \leq .98 \text{ g/in and } 2.00 \text{ g/in} \)

b) What is a "margin of error" (based on two-sided 95% confidence limits) that can be applied to sample means for combinations 1, 4, 6, and 7 (as representing their respective long run mean seal strengths)?

This is \( \pm \frac{\hat{\sigma}_{\text{pooled}}}{\sqrt{n_i}} \).

Here this is \( 2.120 \frac{1.311}{\sqrt{2}} = 1.57 \)

(Units are g/in)

(Note that for samples 2, 3, 5, and 8 this is \( 2.120 \frac{1.311}{\sqrt{4}} = 1.39 \text{ g/in} \))
c) Consider those sample means that correspond to combinations 5 through 8 (those with C at its high level). Plot them below on a kind of "interaction plot" and mark 95% error bars on the plot (derived as in part e) but noting that sample sizes are not constant. For this plot alone, which effects seem largest: A Main Effects, B Main Effects, or AB Interactions?

\[ \pm 1.96 \text{ error bars} \]

The biggest thing going on here are the left-to-right differences (A main effects) not low-high-B differences or lack of parallelism (interactions).

\[ \pm 1.39 \text{ error bars} \]

\[ \text{high B} \]

\[ \text{low B} \]

\[ \text{Factor A} \]

---

d) Find 95% two-sided confidence limits for the difference in mean strengths under combinations/conditions #1 and #2.

Use

\[ \overline{y}_1 - \overline{y}_2 \pm t_{spooled} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \]

\[ (6.6 - 6.3) \pm 2.120 (1.31) \sqrt{\frac{1}{2} + \frac{1}{4}} \]

\[ -0.3 \pm 2.41 \text{ g/in} \]
e) Fitted $2^3$ effects from the Yates algorithm applied to the 8 sample means on page 4 are 
\[ y_i = 7.65, \hat{a}_2 = -0.875, \hat{b}_2 = 0.275, \hat{a}_2b_2 = -0.350, \]
\[ \hat{c}_2 = 0.775, \hat{a}_2c_2 = -0.500, \hat{b}_2c_2 = 0.150, \hat{a}_2b_2c_2 = 0.175. \]

Find the 95% "margin of error" for any of these effects, and say which if any of the above are statistically significant and make a qualitative interpretation of your outcome.

This is: \[ t_{S_p} \text{ split} \frac{1}{2} \sqrt{\text{sum of reciprocal sample sizes}} \]
\[ 2.120 \left( 1.311 \right) \frac{1}{8} \sqrt{4(\frac{1}{2})+4(\frac{1}{4})} = 0.6017 \text{ g/in} \]

Statistically significant effects: The fitted A main effect and fitted C main effect are bigger in magnitude than the above margin of error.

Interpretation: Seal temperature and polyethylene content impact seal strength in a detectable manner. They act "separately" on seal strength.

f) Assuming that only the statistically detectable effects from e) are employed in modeling seal strength, find predicted/fitted seal strengths for the "all low" and "all high" treatment combinations. (Produce two values $\hat{y}$.) (Plug in completely, but you need not do arithmetic.) If no effects are statistically significant, say how $\hat{y}$ values would differ if there WERE any that were significant.

All Low: \[ \hat{y}_{(i)} = \overline{y} + a_1 + c_1 = 7.65 + (-0.875) + (0.775) = 7.75 \]

All High: \[ \hat{y}_{(i)} = \overline{y} + a_2 + c_2 = 7.65 + (-0.875) + (0.775) = 7.55 \]