IE 316 Exam 1
Fall 2012

I have neither given nor received unauthorized assistance on this exam.

________________________________________________________
Name Signed                Date

_________________________________________________________
Name Printed
1. Here are 10 True-False questions worth 2 points each. Write (very legibly) "T" or "F" next to each one. If I can't tell whether you intend a "T" or an "F" I'll count the answer wrong.

- [F] The width of a confidence interval for a standard deviation primarily is the primary indicator of how large I believe the corresponding population variation to be (the wider the interval the more variable the population being sampled).

- [F] A 95% confidence interval for $\mu_1 - \mu_2$ is $(-3, -1)$. Based on this, I really can't say which of the means (i.e. which of $\mu_1, \mu_2$) is larger.

- [T] Bias in a linear measurement device does not impact the comparison of two measurands.

- [F] If measurement is destructive, it is impossible to compare the biases of two linear devices.

- [F] Worker training aimed at improving measurement technique consistency between/among workers is aimed at the reduction of repeatability variation.

- [T] The major reason for using range-based calculations in 21st century quality assurance contexts is simply their historical momentum, i.e. people are familiar with them.

- [T] Essentially the same kinds of statistical analyses (data collection and calculations) can be used to separate "process variation from measurement variation" and to separate "repeatability variation from reproducibility variation."

- [T] "Validity" of measurement is more less like "shooting at the right target" in target shooting.

- [F] A calibration study is intended to improve precision of measurement.

- [T] An accurate measurement system need not be an adequate measurement system for some purposes.
2. 8 different students measure the width of a single "binder clip" (a small steel spring made for holding sheets of paper together) with a Vernier micrometer. These measurements have standard deviation .0365 mm. Subsequently, another student measures the clip 10 times with the same micrometer. These measurements have standard deviation .0052 mm.

**10pts a)** Give a single number estimate of the measurement reproducibility standard deviation, \( \sigma_{\text{reproducibility}} \).

\[
\hat{\sigma} = \sqrt{(\text{std dev from multiple operators})^2 - (\text{std dev from single operator})^2} = \sqrt{(0.0365)^2 - (0.0052)^2}
\]

\[= 0.0361 \, \text{mm}\]

**10pts b)** Find approximate 95% confidence limits for the reproducibility standard deviation.

We need d.f.

\[
\frac{(0.0361)^4}{(0.0365)^4} + \frac{(0.0052)^4}{10-1} = 6.70
\]

Round down to d.f. 6. Then use

\[
\hat{\sigma} \sqrt{\frac{\frac{\hat{\sigma}}{X^2}}{X^2}} \quad \text{to} \quad \hat{\sigma} \sqrt{\frac{\frac{\hat{\sigma}}{X^2}}{X^2}}
\]

Here this is

\[0.0361 \sqrt{\frac{6}{14.449}} \quad \text{to} \quad 0.0361 \sqrt{\frac{6}{1.237}}\]

\[0.0233 \, \text{mm} \quad \text{to} \quad 0.0795 \, \text{mm}\]
c) Would you judge the Vernier micrometer referred to above adequate to judge conformance to ±15 mm width specifications? Explain, phrasing your answer in terms of 95% confidence limits for a gauge capability ratio that you provide here.

\[ GCR = \frac{6.0365}{2(1.5)} \]

So 95% confidence limits for this are

\[ 6.0365 \left( \frac{8-1}{1.690} \right) \]

\[ 0.30 \]

i.e.,

\[ 0.48 \] to \[ 1.49 \]

We want a GCR .1 to .01. What we've got is clearly much bigger than this. NO this micrometer is not adequate to check conformance to ±15 mm specifications.

d) In fact, the actual width of the binder clip is 31.950 mm and the 9th (last) student's sample mean measurement is \( \bar{y} = 31.960 \) mm. Find 95% confidence limits for the student's measurement bias. Is it plausible that the student is completely accurate? Explain.

95% confidence limits for the student's mean measurement are

\[ \bar{y} \pm t_{\frac{0.05}{\sqrt{n}}} \]

i.e.,

\[ 31.960 \pm 2.262 \frac{0.052}{\sqrt{10}} \]

i.e.,

\[ 31.960 \pm .0037 \] mm

So 95% limits for the bias are

\[ 31.960 - 31.950 \pm .0037 \]

\[ .0100 \pm .0037 \] mm

This interval does not cover 0. So we have conclusive evidence that the student's bias is not 0. It is not possible that the student is completely accurate.
e) Suppose that in a subsequent study, the same 10 students measure the sizes of 3 binder clips, each student measuring each clip 5 times. This results in a $3 \times 10$ table containing 30 sample means and 30 sample ranges. The 30 sample ranges average to .0127 mm. The ranges of the 10 sample means in given rows are as below.

<table>
<thead>
<tr>
<th>Row</th>
<th>Range of Sample Means in the Row</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.0525 mm</td>
</tr>
<tr>
<td>2</td>
<td>.0625 mm</td>
</tr>
<tr>
<td>3</td>
<td>.0462 mm</td>
</tr>
</tbody>
</table>

Estimate both $\sigma_{\text{repeatability}}$ and $\sigma_{\text{reproducibility}}$ on the basis of these values.

$$\sigma_{\text{repeatability}} = \frac{R}{d_2(m)} = \frac{.0127}{2.326} = .0055 \text{ mm}$$

$$\sigma_{\text{reproducibility}} = \sqrt{\left(\frac{.0175}{2}\right)^2 - \frac{1}{5} \left(\frac{.0055}{2}\right)^2} = .0173 \text{ mm}$$

3. In a visual inspection context, "good versus bad" call rates will vary part-to-part. In routine end-of-the-line inspection, two inspectors (unbeknownst to them) inspect 5 parts 10 times each with the results below. Is there evidence (based on appropriate 95% confidence limits) of a systematic difference in "bad" call rates for the two inspectors?

<table>
<thead>
<tr>
<th>Part</th>
<th>Inspector #1 &quot;bad&quot; call rate</th>
<th>Inspector #2 &quot;bad&quot; call rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.1</td>
<td>.2</td>
</tr>
<tr>
<td>2</td>
<td>.3</td>
<td>.2</td>
</tr>
<tr>
<td>3</td>
<td>.9</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>.2</td>
</tr>
</tbody>
</table>

Use $\bar{d} \pm \frac{3d}{\sqrt{n}}$ i.e. $-.06 \pm 2.776 \frac{.140}{\sqrt{5}}$ i.e. $-.06 \pm .14$

There is not evidence of a consistent difference in call rates. (The interval coins 0.)
4. The book *Statistics and Chemometrics for Analytical Chemistry* by J. N. Miller and J. C. Miller presents a calibration data set for fluorescence spectrophotometry, where known concentrations $x$ of a chemical (in pg/ml) produce measured fluorescence intensities $y$ (in machine units). The data are below along with a JMP report based on the data (with 95% prediction limits shown on the plot).

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2.1</td>
<td>5.0</td>
<td>9.0</td>
<td>12.6</td>
<td>17.3</td>
<td>21.0</td>
<td>24.7</td>
</tr>
</tbody>
</table>

**a)** A read intensity of 10 corresponds to what single-number estimate of concentration? (Give a number correct to several decimal places.)

\[ 10 = 1.5178571 + 1.9303571 \times x \]

\[ x = \frac{10 - 1.5178571}{1.9303571} = 4.394 \text{ pg} \]

---

**b)** What are 95% confidence limits for the concentration corresponding to a read intensity of 10? Indicate how you produced these limits.

Reading from above, the limits are about 3.8 pg to 5.1 pg.
IE 361 Exam 2
Fall 2012

I have neither given nor received unauthorized assistance on this exam.

KEY

Name Signed                                     Date

Name Printed
1. Over 40 shifts of production, a total of 5470 LED displays were made and inspected. A total of 430 nonconformities were identified on those 5470 displays, and 194 of the displays were judged to be nonconforming. A retrospective analysis of the data from the 40 shifts found no evidence of process instability over that period, and so these historical rates will here be taken as standard values.

14 pts a) In a subsequent shift, 200 displays are produced. A total of 18 nonconformities are identified on these, and 13 of the displays are judged to be nonconforming. Do either of these counts provide clear evidence of process change? (Compare the two counts to appropriate control limits and interpret your result.)

\[
\lambda = \frac{430 \text{ nonconformities}}{5470 \text{ units}} = 0.076 \text{ nonconformities per unit}
\]

\[
\lambda (200 \text{ units}) = 15.72 \text{ nonconformities}
\]

Here \( c \) chart limits will be

\[
\begin{align*}
\text{UCL}_c &= 15.72 + 3\sqrt{15.72} = 27.62 \\
\text{LCL}_c &= 15.72 - 3\sqrt{15.72} = 3.83
\end{align*}
\]

18 non-conformities is inside These limits

Interpretation: neither of these counts provides evidence of process change

b) Suppose that at some point in the future, the standard rate of nonconforming displays is reduced, production is increased to 300 displays per shift, and a process monitoring system is adopted that signals "out of control"/"intervention warranted" if there are 3 or more nonconforming displays produced. If, in fact, a fraction .005 of all displays produced are nonconforming, what is the probability that the next shift will have an "out of control" signal?

\[
p = 0.005 \quad X = \text{# nonconforming in the 300 displays produced}
\]

\[
X \sim \text{Binomial} \quad n = 300 \quad p = 0.005
\]

\[
P\left[X \geq 3\right] = 1 - P\left[X \leq 2\right] = 1 - f(0) - f(1) - f(2)
\]

\[
= 1 - \binom{300}{0}(0.005)^0(0.995)^{300} - \binom{300}{1}(0.005)^1(0.995)^{299} - \binom{300}{2}(0.005)^2(0.995)^{298}
\]

\[
= 1 - (0.995)^{300} - 300(0.005)(0.995)^{299} - \frac{(300)(0.995)^{299}}{2} \cdot (0.005)^2(0.995)^{298}
\]

\[
= 1 - 0.2223 - 0.3351 - 0.2518 = 0.19
\]
2. Lengths of steel shafts are measured by a laser gauge that produces a coded voltage proportional to shaft length (above some reference length). In the units produced by the gauge, shafts of a particular type have length specifications $2300 \pm 20$. Below are measured lengths of $n = 8$ such shafts.

2298, 2289, 2291, 2290, 2290, 2287, 2280, 2289

Assume that these shafts were produced by a physically stable process.

6 pts  a) How confident are you that an 9th shaft produced by this process would measure at least 2280? (Give a number.)

The prediction confidence is $\frac{8}{8+1} = \frac{8}{9} = .89$

Henceforth assume that it is sensible to model shaft length as normally distributed. For the lengths listed above, $\bar{x} = 2289.3$ and $s = 4.9$.

7 pts  b) Give 90% two-sided confidence limits for a process capability index that measures process potential. (Plug in completely, but you need not simplify.)

We want confidence limits for $CP = \frac{U-L}{6s}$ Thus we

$\frac{U-L}{6s} \leq \frac{\chi^2_{\text{lower}}}{n-1}$ and $\frac{U-L}{6s} \geq \frac{\chi^2_{\text{upper}}}{n-1}$

$\frac{40}{6(4.9)} \sqrt{\frac{2.167}{8-1}}$ and $\frac{40}{6(4.9)} \sqrt{\frac{14.067}{8-1}}$

i.e. .757 and 1.929

7 pts  c) Give a 95% lower confidence bound for a process capability index that measures current process performance. (Plug in completely, but you need not simplify.)

Use $\hat{C}_{pk} = z \sqrt{\frac{1}{3n} + \frac{Cpk^2}{2n-2}}$ Here $\hat{C}_{pk} = \min \left[ \frac{2320 - 2289.3}{3(4.9)}, \frac{2289.3 - 2260}{3(4.9)} \right]$

$= \min (2.088, .6327)$

So use $.6327 - 1.645 \left( \frac{1}{3(4.9)} + \frac{(6327)^2}{16 - 2} \right)$

i.e. .2937
3. IE 361 students did a class project with a company that cuts rolled paper into sheets. The students periodically sampled \( n = 10 \) consecutive sheets as they were cut and measured their lengths. 15 samples are summarized below. (The measurements were in 64ths of an inch above nominal.)

<table>
<thead>
<tr>
<th>Sample</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>∑x</th>
<th>∑s</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>12.2</td>
<td>11.2</td>
<td>10.6</td>
<td>12.2</td>
<td>12.6</td>
<td>12.2</td>
<td>13.6</td>
<td>12.2</td>
<td>10.4</td>
<td>10.4</td>
<td>10.6</td>
<td>10.6</td>
<td>12.0</td>
<td>11.2</td>
<td>171</td>
<td>26.18</td>
<td></td>
</tr>
<tr>
<td>s</td>
<td>.84</td>
<td>1.64</td>
<td>2.07</td>
<td>2.49</td>
<td>.84</td>
<td>1.82</td>
<td>2.95</td>
<td>1.67</td>
<td>1.30</td>
<td>1.52</td>
<td>1.95</td>
<td>1.67</td>
<td>1.67</td>
<td>2.91</td>
<td>.84</td>
<td>26.18</td>
<td></td>
</tr>
</tbody>
</table>

**14 pts**

a) What do these values indicate about the stability of the cutting process? (Show work and consider both process mean and standard deviation.)

\[
\bar{x} = \frac{171}{15} = 11.4 \quad \bar{s} = 1.7453 \\
\text{consider retrospective control limits.}
\]

For \( \bar{x} \): control limits are

\[
\text{UCL } \bar{x} = \bar{x} + A_2 \bar{s} = 11.4 + \cdot 905 (1.7453) = 13.10 \\
\text{LCL } \bar{x} = - \quad - = 9.70
\]

For \( s \): control limits are

\[
\text{UCL } s = B_4 \bar{s} = 1.716 (1.7453) = 3.00 \\
\text{LCL } s = B_3 \bar{s} = .284 (1.7453) = .50
\]

The 8th \( \bar{x} \) is outside retrospective limits. (All other sample means and standard deviations are inside control limits.) This is evidence of process instability.

**6 pts**

b) If one judges the process to be stable and sheet length to be normally distributed, it is possible to estimate the fraction of sheets below nominal in length. Do this. (Show your work.)

\[
\bar{x} = 11.4 = \mu \\
\bar{s} = \frac{1.7453}{.9727} = 1.7943 = \sigma
\]

"Nominal" line is 0 (64ths above nominal).

\[
\sigma = 1.7943 \\
Z = \frac{0 - 11.4}{1.7943} = -6.35
\]

\[
P(Z < -6.35) \approx 0
\]
c) Each inch that the cutting process mean is above nominal represents a $200,000/year loss to the company from product "given away." On the other hand, the company wants to be sure that essentially no sheets are produced with below-nominal lengths. With this in mind, what adjustment in mean length do you suggest, and what yearly savings or additional costs do you project assuming this adjustment is made?

One could set $\mu$ at only 3\sigma above nominal. This is setting $\mu$ at

$$3(1.7943) = 5.38$$

instead of at 11.4. This is a reduction of $11.4 - 5.38 = 6.02$ (64ths of an inch) — so we expect to save

$$\frac{6.02}{64}(200,000) = 18,812 \leq 0$$


\begin{align*}
\text{c) } & E \text{ chart} \\
& UCL_E = 5.38 + 3 \frac{1.7943}{\sqrt{5}} \\
& LCL_E = 5.38 - 3 \frac{1.7943}{\sqrt{5}}
\end{align*}

\begin{align*}
\text{d) } & S \text{ chart} \\
& UCL_s = B_6 \sigma = 1.864(1.7943) \\
& LCL_s = B_3 \sigma = \text{ now}
\end{align*}
4. Short Answers. (5 points apiece.) Write only a sentence or two (no more than 25 words) for each.

a) State the goal of control charting and appropriate action on out-of-control signals.

Control charting concerns process monitoring for purposes of change detection. Change detection plus intervention is meant to eliminate change and produce stability/consistency.

b) Why might it be argued that the name "control" chart invites confusion?

Engineering "control" typically refers to an online adjustment/process tinkering methodology rather than a monitoring/detection methodology. These are different methods.

c) Although statistical quality control tools have been applied primarily in manufacturing industries, they also have important applications in service industries. Which step in the process improvement cycle seems relatively most difficult in service contexts? Explain.

The formulation of appropriate measures of process performance is typically far more difficult in service contexts than in manufacturing situations.

d) Distinguish between "quality of design" and "quality of conformance." With which of these is control charting concerned?

Quality of design is appropriateness of features. Quality of conformance is small deviation from an ideal design. Control charting (indirectly) concerns the end of these.
IE 361 Exam 3
Fall 2012

I have neither given nor received unauthorized assistance on this exam.

KEY

________________________________________________________
Name                Date
1. I wish to measure the density of a small rock. My method is to read the volume of water in a graduated cylinder before and after putting the rock into the cylinder, producing

\[ v_1 = \text{volume read without the rock} \]

and

\[ v_2 = \text{volume read with the rock} \]

and then to measure the mass, \( m \), of the rock on a precision balance. Ultimately, I will use the relationship

\[ d = \frac{m}{v_2 - v_1} \]

to compute density. Suppose that \( v_1 \approx 65 \text{ ml} \) and \( v_2 \approx 75 \text{ ml} \) and that volumes can be determined with a standard deviation around .5 ml. Further, \( m = 27.2 \text{ g} \) and mass can be determined with a standard deviation around .1 g.

a) What are an approximate mean and standard deviation of measured density? By the way, partial derivatives of \( d \) are:

\[
\frac{\partial d}{\partial m} = \frac{1}{v_2 - v_1}, \quad \frac{\partial d}{\partial v_2} = \frac{-m}{(v_2 - v_1)^2}, \quad \text{and} \quad \frac{\partial d}{\partial v_1} = \frac{m}{(v_2 - v_1)^2}
\]

\[
E(d) \approx \frac{m}{v_2 - v_1} = \frac{27.2}{75 - 65} = 2.72 \text{ g/ml}
\]

\[
\text{Var}(d) \approx \left(\frac{1}{75 - 65}\right)^2 (1)^2 + \left(-\frac{27.2}{(75 - 65)^2}\right)^2 (0.5)^2 + \left(\frac{27.2}{(75 - 65)^2}\right)^2 (0.5)^2
\]

\[
= 0.0001 + 0.0185 + 0.0185
\]

\[
\sqrt{\text{Var}(d)} = \sqrt{0.0371} = 0.193
\]

mean: 2.72 g/ml \quad standard deviation: 0.193 g/ml

b) For measuring the density of this particular rock, is the imprecision of the graduated cylinder more or less important than the imprecision of the balance (in terms of producing uncertainty in the measured density)? Explain.

most important (circle one): cylinder/balance

Explaination: The contributions to \( \text{Var}(d) \) related to volume measurements are .0185 + .0185, while the contribution related to mass measurement is .0001. Clearly .0185 + .0185 is much larger than .0001.
2. The rest of this exam concerns analysis of some data taken from the NIST website, [http://www.itl.nist.gov/div898/handbook/toolaids/pff/pri.pdf](http://www.itl.nist.gov/div898/handbook/toolaids/pff/pri.pdf) The data are from an experiment conducted on the operation of a toy catapult used to launch golf balls. Here's a picture of the device (also take from the NIST website).

The response variable was

\[ y = \text{measured golf ball flight distance (inches)} \]

and the effects of 5 physical parameters were studied. The variables considered were:

- **A**--band height (height of pivot point for bands above the surface of the device (inches))
- **B**--start angle (location of the arm at the beginning of launch (degrees from horizontal))
- **C**--rubber bands (number of bands used)
- **D**--arm length (distance an "extension" setting on the arm (inches)), and
- **E**--stop angle (the angle at which rotation of the arm is stopped (degrees from horizontal))

The last page of the exam provides a printout of a JMP data table giving all the data collected in the experiment. We will use various parts of the data set as we go through this question.

10pts **a)** Among the 20 trials there were 18 different set-ups of the catapult. Runs #2 and #13 were from the same set-up and produced measured distances of 99.0 and 84.5 inches, while runs #7 and #19 were from a different single set-up and produced measured distances of 45 and 37.5 inches. Give a single number estimate of the standard deviation of measured flight distance for any fixed set-up and its degrees of freedom.

\[ \text{Run } #2 \text{ and } #13 \text{ produce } s = 10.253 \quad \text{while runs } #7 \text{ and } #19 \text{ produce } s = 5.3033. \]

\[
\begin{align*}
\text{Pooled } s & = \sqrt{\frac{1(10.253)^2 + 1(5.3033)^2}{1+1}} = 66.625 \\
\text{Pooled } s & = \sqrt{66.625} = 8.1629
\end{align*}
\]

\[ \text{estimate: } 8.1624 \text{ inches} \quad \text{degrees of freedom: } 2 \]
b) Give 95% confidence limits for the standard deviation of measured flight distance for any fixed set-up. (Plug in completely, but you need not simplify.)

\[ s_{\text{pooled}} \sqrt{\frac{df}{\chi^2_{0.025}}} \quad \text{and} \quad s_{\text{pooled}} \sqrt{\frac{df}{\chi^2_{0.975}}} \]

\[ 8.16 \sqrt{7.378} \quad \text{and} \quad 8.16 \sqrt{0.051} \]

4.25 inch to 51.12 inch

c) Two of the sample sizes in the study were \( n_i = 2 \) and the other 16 were \( n_i = 1 \). Thus there are two different (95%) "margins of error" that might be associated with any one of the 18 different "sample means" from the study. Give their values.

\[ n_i = 2 \text{ margin of error for } \bar{y}_i \]

\[ s_{\text{pooled}} \sqrt{\frac{1}{2}} \]

This is 4.303 8.16 \[ \sqrt{2} \]

11 24.8 inch

\[ n_i = 1 \text{ margin of error for } y_i \]

\[ s_{\text{pooled}} \sqrt{\frac{1}{1}} \]

11 35.1 inch

d) The quantity \( \bar{y}_{00+00} - \bar{y}_{00-00} \) (the sample average for runs #2 and #13 minus the sample average for runs #7 and #19) is a measure of the effect of moving from 1 band to 2 bands (with all other variables held fixed at their "center" experimental values). Give 95% confidence limits based on this difference (note that \( \bar{y}_{00+00} - \bar{y}_{00-00} = 50.5 \)) for the difference in population means. Does this give clear evidence of a difference in mean flight distance with the addition of the second band?

\[ \bar{y}_1 - \bar{y}_2 \pm t_{\text{pooled}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \]

\[ 50.5 \pm 4.303 (8.1624) \sqrt{\frac{1}{2} + \frac{1}{2}} \]

50.5 ± 35.1

limits: 50.5 ± 35.1 detectable difference? yes

(\text{the interval doesn't cover 0})
e) If one considers only 1 band and arm length 0, the 4 values below were obtained (written in Yates order for Factors A and E). Use the Yates algorithm and find 4 fitted effects

<table>
<thead>
<tr>
<th>A-band height</th>
<th>E-stop angle</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>(−) (3.25)</td>
<td>(−) (45)</td>
<td>8</td>
</tr>
<tr>
<td>(+) (4.75)</td>
<td>(−) (45)</td>
<td>35</td>
</tr>
<tr>
<td>(−) (3.25)</td>
<td>(+) (80)</td>
<td>28</td>
</tr>
<tr>
<td>(+) (4.75)</td>
<td>(+) (80)</td>
<td>28.25</td>
</tr>
</tbody>
</table>

\[ \text{cycle } \#1 \] \[ \text{cycle } \#2 \] \[ \text{cycle } \#2/4 \]

\[ 43 \] \[ 56.25 \] \[ 29.8125 \]
\[ 39.25 \] \[ 27.25 \] \[ 6.8125 \] \[ A+BE \]
\[ 27 \] \[ 13.25 \] \[ 3.3125 \] \[ E+AB \]
\[ -2.25 \] \[ -26.75 \] \[ -6.6875 \] \[ B+AE \]

f) Notice that the start angles (levels of Factor B) corresponding to the y’s above are (in the order the observations are written top to bottom) \((+,−,−,+); i.e., (20,0,0,20)\). In light of this fact, what are two plausible interpretations of the set of values you produced in part e) for how the catapult operates (with 1 band and arm length set at 0)?

Interpretation 1: A and B main effects are the largest/most important effects.

Interpretation 2: The A main effects and AE interactions are the largest/most important effects.

g) The 16 set-ups in the experiment excluding runs 2,7,13,19 constitute a standard half fraction of a full \(2^5\) factorial. If one were to run the corresponding responses through the Yates algorithm to produce fitted sums of effects, \(\hat{E}\), how big would one of those need to be in order to represent a statistically detectable sum? (Make use of your estimate from part a) and 95% confidence.)

The margin of error will be

\[
\text{ls pooled } \frac{1}{2} \frac{1}{5-1} \sqrt{\frac{1}{4} + \frac{1}{4} + \cdots + \frac{1}{4}}
\]

\[
= 4.303 (8.1624) \frac{1}{2^4} \sqrt{16} = \frac{4.303 (8.1624)}{4} \approx 8.78 \text{ inches}
\]
In light of your answer to g), the $16 = 2^4$ fitted effects in the JMP report below, and the fact that half-fraction data are being analyzed here, interpret the results of the $2^{5-1}$ fractional factorial part of this data set. Say clearly what assumptions you are making about likely simplicity of the physical situation.

The simplest interpretation here is that all main effects are statistically detectable and important. This assumes that the estimates underlined primarily represent main effects and not aliased 4-factor interactions.

NIST Catapult Data

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