1. Suppose the actual diameters $x$ in a batch of steel cylinders are normally distributed with mean $\mu_x = 2.5025$ inch and standard deviation $\sigma_x = 0.004$ inch. Suppose further, that associated with the measurement of any cylinder is a measurement error $\epsilon$ independent of (a randomly selected) $x$ and normal with mean 0 and standard deviation $\sigma_{\text{device}} = 0.003$ inch. ($\epsilon$ is perhaps produced by small inconsistencies in how the measuring device is used and unavoidable/unpredictable small mechanical and electrical effects in the device.)

a) Modeling a measurement $y$ (of a randomly selected cylinder) as

$$y = x + \epsilon$$

exactly what probability distribution is appropriate for $y$? (Name the distribution and give numerical values for its parameters.)

- $y$ is normal with $\mu_y = \mu_x + \mu_\epsilon = 2.5025 + 0 = 2.5025$ and
- standard deviation $\sigma_y = \sqrt{\sigma_x^2 + \sigma_\epsilon^2} = \sqrt{(0.004)^2 + (0.003)^2} = 0.005$ (units are inches)

b) The modeling in a) (and, actually, all we've done in IE 361) ignores the fact that real measurement is digital (measurements are made only "to the nearest something"). Sometimes (not always) this is OK. But more careful modeling treats a digital measurement $y^*$ derived from $y$ by "rounding to the nearest something" as discrete. In the present context, measurements to the nearest .01 inch might be described using the pmf below. Show how the probability for $y^* = 2.50$ was derived. Then compare the mean and standard deviation of $y^*$ to those of $y$ (from a)) and $x$.

<table>
<thead>
<tr>
<th>$y^*$</th>
<th>$f(y^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.52</td>
<td>.0062</td>
</tr>
<tr>
<td>2.51</td>
<td>.3023</td>
</tr>
<tr>
<td>2.50</td>
<td>.6247</td>
</tr>
<tr>
<td>2.49</td>
<td>.0666</td>
</tr>
<tr>
<td>2.48</td>
<td>.0002</td>
</tr>
</tbody>
</table>

$$P[y^* = 2.50] = P[2.495 < y < 2.505]$$

$$= P\left[\frac{2.495 - 2.5025}{0.005} < Z < \frac{2.5025 - 2.5025}{0.005}\right]$$

$$= P[-1.5 < Z < 1.5] = \Phi(1.5) - \Phi(-1.5)$$

$$= 0.9332 - 0.0668 = 0.8664$$

$$Ey^* = 2.52(.0062) + (2.51)(.3023) + (2.50)(.6247) + (2.49)(.0666) + 2.48(.0002) \approx 2.50248 \text{ inch}$$

$$Ey^* = (2.52)^2(.0062) + \cdots + (2.48)^2(.0002) \approx 6.262424$$

So $\text{Var}y^* = Ey^*^2 - (Ey^*)^2 \approx 6.262424 - (2.50248)^2 \approx 3.331 \times 10^{-5}$

and $\sqrt{\text{Var}y^*} \approx 0.058 \text{ inch}$

BTW: These show that digital measurements do not have exactly the same characteristics as one might expect. The difference here between $\mu_x = \mu_y$ and $Ey^* \approx 2.50248$ bigger than $\sigma_y$ (which is not so big (it can be worse in other cases)) but $\sigma_y$ is clearly bigger than $\sigma_y^*$ (which in turn is bigger than $\sigma_x$). Digitalization typically increases one's perception of the size of variability.!!!
2. See NQT Reports #1 attached at the end of this exam. They concern some measurements made on the machined inside diameter of a single steel forklift wheel (units are inches) using a single gauge. Use the information on those reports in answering the following questions. (The 18 measurements together have sample mean 2.04748 inch and sample standard deviation .00343 inch.)

a) Consider first only the measurements made by Operators #1 and #2. Give 95% confidence limits for the difference in measurement biases for the two operators.

Use \[ \bar{x}_1 - \bar{x}_2 \pm t \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \] i.e.

Use d.f. = min\((n_1-1), n_2-1\)

\[ 2.0432 \pm 2.0539 \pm 4.303 \sqrt{\frac{(0.005122)^2}{3} + \frac{(0.0002867)^2}{3}} \]

i.e. \[ -0.0106 \pm 0.0015 \text{ inch} \]

b) What would your interval from a) estimate if every measurement had been made on a different wheel produced by a stable production process (assuming that the two "gauge-operator" combinations constitute linear measurement devices)?

It would still estimate the difference in operator biases.

Now consider the measurements from all 6 operators (on the single wheel).

c) Give 95% confidence limits for a repeatability standard deviation.

\[ \sqrt{7.2541 \times 10^{-6}} \text{ and } \sqrt{3.8452 \times 10^{-7}} \]

i.e. \[ 0.00027 \text{ inch} \] \[ 0.00062 \text{ inch} \]

d) Give 95% confidence limits for the standard deviation of operator biases.

\[ 0 \text{ and } \sqrt{0.0000256} \]

i.e. \[ 0 \text{ inch and } 0.0054 \text{ inch} \]
e) Under the one-way random effects model, the Operator-sample-means are from a normal distribution with standard deviation \( \sqrt{\frac{\sigma^2\text{reproducibility}}{n} + \frac{1}{3}\frac{\sigma^2\text{repeatability}}{m}} \). If one computes the sample standard deviation of the operator-sample-means here, one gets the value .003633 inch. Use this to make 95% confidence limits for \( \sqrt{\frac{\sigma^2\text{reproducibility}}{n} + \frac{1}{3}\frac{\sigma^2\text{repeatability}}{m}} \). Say why your limits are (or are not) consistent with your answer to d).

\[
\text{Use } \pm \sqrt{\frac{n-1}{X^2_n}} \text{ and } \pm \sqrt{\frac{m-1}{X^2_m}}. \text{ Here, with } \frac{n}{m} = 6,
\]

This is \( .003633 \sqrt{\frac{5}{12.933}} \) and \( .003633 \sqrt{\frac{5}{.931}} \)

i.e. \( .0023 \) inch and \( .0083 \) inch. These are more or less consistent with d). They estimate something a bit bigger than the answer from d)... but in light of c), not a whole lot bigger (\( \frac{1}{3}\sigma^2\text{repeatability} \) is probably not so big compared to \( \sigma^2\text{reproducibility} \)).

3. Suppose that in the context of Problem 2, the \( J = 6 \) operators all measure a total of \( I = 4 \) wheels \( m = 3 \) times each and that NQT produces \( SS\text{Operator} = 66.0, \ SS\text{Part} \times \text{Operator} = 45.0 \) and \( SSE = 8.1 \), all in units of \( 10^{-6} \text{ inch}^2 \).

a) Find approximate 95% confidence limits for the standard deviation of measurements that would be produced by a large number of operators each measuring the same wheel once each. (You may take as given that appropriate approximate degrees of freedom are \( \nu = 13 \).)
b) If engineering specifications on the machined diameter are $2.0474 \pm 0.005$ inch, is the gauge in question adequate to check conformance to these specifications? **Explain** in terms of a GCR.

99% confidence limits for a GCR are

$$\frac{6(1.02 \times 10^{-3})}{2(0.05)} \quad \text{and} \quad \frac{6(2.26 \times 10^{-3})}{2(0.05)}$$

i.e. .6 and 1.4

We want GCR's of .1 or .01. This one is **way** too big. No, this gauge is not adequate for checking conformance to these specs.

---

5 pts

5 pts

c) If it were possible to completely eliminate operator-to-operator variability in measurement, **would** the gauge in question be adequate to check conformance to the $2.0474 \pm 0.005$ inch specifications? **Explain** very carefully.

Well, this would mean that only **repeatability** needs considering.

Limits on Repeatability are

$$\hat{\text{Repe}} \frac{\sqrt{15(n-1)}}{X^2_n} \quad \text{and} \quad \hat{\text{Repe}} \frac{\sqrt{15(n-1)}}{X^2_L}$$

for 95%

$$\sqrt{8.1} \frac{\sqrt{4.6(2)}}{6(4)(2)} \quad \text{and} \quad \sqrt{8.1} \frac{\sqrt{4.6(2)}}{6(4)(2)} \frac{30.7545}{30.7545}$$

.34 x 10^{-3} inch and .51 x 10^{-3} inch

which translates to limits for the GCR of

$$\frac{6(0.34) \times 10^{-3}}{.01} \quad \text{and} \quad \frac{6(0.51) \times 10^{-3}}{.01} = .3$$

A

I still estimate that the GCR would be at least .2. This is still not small enough to make the gauge adequate for this purpose.
4. Visual inspection of garments is less than perfectly consistent. A company has many inspectors that work at removing imperfect clothing items from a production stream and classifying them as "seconds." To study inspector consistency, a particular test garment marked with ink that can be seen under ultraviolet light is repeatedly included in a stream of items submitted to the inspectors to see if it passes inspection. Results from 10 inspections by each of 5 inspectors (in terms of the fraction of times the garment was identified a second) are below. Use this information to answer the following questions.

<table>
<thead>
<tr>
<th>Inspector</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{p} )</td>
<td>.1</td>
<td>.3</td>
<td>.4</td>
<td>.0</td>
<td>.2</td>
</tr>
</tbody>
</table>

\[ \overline{\hat{p}} = .2 \]

7 pts a) Is there clear evidence in these values of a consistent difference in how Inspectors #1 and #2 classify this garment? (Provide an argument based on an appropriate 95% confidence interval.)

Use

\[ \hat{p}_1 - \hat{p}_2 \pm z \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \]

\[ (-.2) \pm .391 \]

This interval covers 0. There is no conclusive evidence that inspectors #1 and #2 call this part consistently differently.

9 pts b) What fraction of the variance in "perfect-vs-second" calls on this garment appears to be attributable to consistent differences between inspectors?

\[ \hat{\sigma}^2_{\text{repro}} = \hat{\sigma}^2_{R+R} - \hat{\sigma}^2_{\text{repeat}} = \hat{\sigma}^2(1-\overline{\hat{p}}) - \frac{\hat{\sigma}^2}{n} \]

\[ = (.2)(.8) - \frac{1}{5} (.1)(.5) + (.3)(.7) + (.4)(.6) + 0 \]

\[ = .16 - \frac{1}{5} (.09 + .21 + .24 + 0 + .16) = .16 - .14 = .02 \]

\[ \frac{\hat{\sigma}^2_{\text{repro}}}{\hat{\sigma}^2_{R+R}} = \frac{.02}{.16} = .125 \]
5. See NQT Reports #2 and #3 attached at the end of this exam. These concern regression analyses of some chemical analysis data taken from https://facultystaff.richmond.edu/~cstevens/301/Calibration6.html and listed below.

<table>
<thead>
<tr>
<th>Concentration</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>.006</td>
<td>.077</td>
<td>.138</td>
<td>.199</td>
<td>.253</td>
<td>.309</td>
<td>.356</td>
</tr>
</tbody>
</table>

The data are (real) graphite furnace atomic absorption spectroscopy data concerning lead content of water samples. (The units of concentration were ppb and those of the signal were A-s.) The plotted limits are 95% prediction limits for the signal at each concentration.

Subsequent to the calibration study represented by the data above, a tap water sample is measured as having signal value .278.

8 pts  a) First use Report #2 and provide a single-number estimate of the lead content of the tap water and also 95% confidence limits for that measurand.

\[
\hat{x}_{\text{new}} = \frac{y_{\text{new}} - b_0}{b_1} = \frac{.278 - 0.166071}{.0058179} = 44.93
\]

Read the limits from the plot. They are roughly 42 - 48.

5 pts  b) Report #3 is based on a quadratic (rather than simple linear) regression of signal on lead content. What single-number estimate of the lead content of the tap water sample does it provide? Does this differ substantially from your answer to a)?

Reading from the graph \( \hat{x} \approx 45 \) just as in a). The single number prediction is not much different than in a).

5 pts  c) In what practical way does the analysis on Report #3 differ from that on Report #2? (Do not say "one is linear and the other is quadratic." Instead tell me how the difference would affect what a user believes concerning lead content measurements.) What basic weakness of the data collection makes it impossible to tell which analysis actually makes the most sense here?

The estimate of \( \hat{x} \) is far smaller (.001803 vs .007924) and correspondingly, the prediction limits are much tighter around the fitted curve. In the linear fit, all deviation from linearity of the fit is charged as "noise," while in the quadratic fit, much of it is charged as "curvature" and absorbed into the estimated mean function. (The chemists like the 2nd fit.) Because there is no replication (an inexcusable weakness) in the data collection plan, there is no absolutely definitive way to tell which analysis is most appropriate.
Report #2

**Bivariate Fit of Signal By Pb Conc**

**Linear Fit**

Signal = 0.0166071 + 0.0058179*Pb Conc

**Summary of Fit**

| Term            | Estimate | Std Error | t Ratio | Prob>|t| |
|-----------------|----------|-----------|---------|------|---|
| Intercept       | 0.0166071| 0.005399  | 3.08    | 0.0276* |
| Pb Conc         | 0.0058179| 0.00015   | 38.85   | <.0001* |
Bivariate Fit of Signal By Pb Conc

Polynomial Fit Degree=2

Signal = 0.0071429 + 0.0069536*Pb Conc - 1.8929e-5*Pb Conc^2

Summary of Fit
- RSquare: 0.999863
- RSquare Adj: 0.999795
- Root Mean Square Error: 0.001803
- Mean of Response: 0.191143
- Observations (or Sum Wgts): 7

Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Squares</th>
<th>Mean Square</th>
<th>F Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>2</td>
<td>0.09507386</td>
<td>0.047537</td>
<td>14628.75</td>
</tr>
<tr>
<td>Error</td>
<td>4</td>
<td>0.00001300</td>
<td>3.25e-6</td>
<td>Prob &gt; F</td>
</tr>
<tr>
<td>C. Total</td>
<td>6</td>
<td>0.09508686</td>
<td>&lt;.0001*</td>
<td></td>
</tr>
</tbody>
</table>

Parameter Estimates

| Term       | Estimate | Std Error | t Ratio | Prob>|t| |
|------------|----------|-----------|---------|------|---|
| Intercept  | 0.0071429| 0.0001374 | 4.04    | 0.010*|
| Pb Conc    | 0.0069536| 0.000123  | 56.61   | <.0001*|
| Pb Conc^2  | -0.000019| 1.967e-6  | -9.62   | 0.007*|
IE 361 Exam 2
Fall 2011

I have neither given nor received unauthorized assistance on this exam.

_______  ______
Name          Date

There are 12 questions here, each worth 10 points. Answer *at least 10* of them. I will count your best 10 scores.
1. Thicknesses of $n = 6$ sheets of wallboard before and after drying during manufacture, corresponding differences, and some summary statistics are below. (Units are inches.) Use these data in what follows.

<table>
<thead>
<tr>
<th>Sheet</th>
<th>Before $(x)$</th>
<th>After $(y)$</th>
<th>Difference $(d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.514</td>
<td>.510</td>
<td>.004</td>
</tr>
<tr>
<td>2</td>
<td>.505</td>
<td>.502</td>
<td>.003</td>
</tr>
<tr>
<td>3</td>
<td>.500</td>
<td>.493</td>
<td>.007</td>
</tr>
<tr>
<td>4</td>
<td>.490</td>
<td>.486</td>
<td>.004</td>
</tr>
<tr>
<td>5</td>
<td>.503</td>
<td>.497</td>
<td>.006</td>
</tr>
<tr>
<td>6</td>
<td>.500</td>
<td>.494</td>
<td>.006</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std dev</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.5020</td>
<td>.0078</td>
</tr>
<tr>
<td></td>
<td>.4970</td>
<td>.0082</td>
</tr>
</tbody>
</table>

| a) What 6 ordered pairs would be plotted on ordinary graph paper to make a normal plot for the "Before" data? (List the 6 numerical ordered pairs.)

<table>
<thead>
<tr>
<th>$i$</th>
<th>6 ordered data value</th>
<th>$z$ value from $(i - .5)/6$</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.0833</td>
<td>-.138</td>
<td>(.500, -.67)</td>
</tr>
<tr>
<td>2</td>
<td>.25</td>
<td>-.067</td>
<td>(.500, -.21)</td>
</tr>
<tr>
<td>3</td>
<td>.4167</td>
<td>-.21</td>
<td>(.503, .21)</td>
</tr>
<tr>
<td>4</td>
<td>.5833</td>
<td>.21</td>
<td>(.505, .67)</td>
</tr>
<tr>
<td>5</td>
<td>.75</td>
<td>.67</td>
<td>(.514, 1.38)</td>
</tr>
<tr>
<td>6</td>
<td>.5167</td>
<td>1.38</td>
<td></td>
</tr>
</tbody>
</table>

| b) Give two-sided limits that you are "95% sure" bracket the next reduction-in-thickness-by-drying of one of these sheets made on the day of data collection. (Plug in completely, but you don't need to simplify.)

Use $\bar{x} \pm t_{\alpha} \sqrt{\frac{1}{n}}$ with $\alpha = n - 1 = 5$

$.005 \pm 2.571 (.0015) \sqrt{1 + \frac{1}{6}}$
c) Give 99% two-sided limits for 95% of the "After" thicknesses of these sheets of wallboard on the day of data collection. (Plug in completely, but you don't need to simplify.)

\[ \bar{x} \pm t_{2 \alpha / 2} s \]

\[ 0.4970 \pm (6.373)(0.0082) \]

(from Table A9.9)

---

d) If specifications on the "After" thickness of these sheets are \( .5 \pm .01 \) inch, give 95% confidence limits for a measure of process capability that quantifies potential (as opposed to current performance). (Plug in completely, but you don't need to simplify.)

\[ \text{Use } \frac{U-L}{6s \sqrt{n-1/X_{L}^2}} \text{ and } \frac{U-L}{6s \sqrt{n-1/X_{U}^2}} \]

\[ \text{Use } \frac{.02}{6(0.0082) \sqrt{.831}} \text{ and } \frac{.02}{6(0.0082) \sqrt{12.883}} \]

( limits for $C_p$)

---

10 pts
e) These sheets of wallboard can be inspected for surface blemishes. Suppose that a standard rate of occurrence for important blemishes is .03 per sheet. If 50 sheets are inspected per day and $X = \text{total number of important blemishes observed in a day}$, what are appropriate control limits for $X$?

\[
\lambda = .03 \text{ blemishes/sheet} \quad \text{This is } \bar{x} = \frac{50\lambda}{50} = 1.5 \text{ blemishes per 50 sheets}
\]

This is a $c$-chart problem and we thus want

\[
UCL_X = \bar{c} + 3\sqrt{\bar{c}} \quad \text{and} \quad LCL_X = \bar{c} - 3\sqrt{\bar{c}}
\]

Here this is $UCL_X = 1.5 + 3\sqrt{1.5} = 5.2$ with no $LCL_X$.

f) Daily production of another kind of wallboard (5/8 inch thickness) is 1000 sheets, and the company keeps track of daily scrap rates (out of 1000 sheets). Below are fractions of daily production that had to be scrapped. Assuming that sheets are (independently) scrap with a fixed probability, find appropriate retrospective control limits for these values. Is there any evidence of process instability in these values? Explain.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scrap Rate</td>
<td>.003</td>
<td>.001</td>
<td>.002</td>
<td>.005</td>
<td>.006</td>
<td>.004</td>
<td>.002</td>
<td>.005</td>
<td>.001</td>
<td>.005</td>
<td>.034</td>
</tr>
</tbody>
</table>

This is a retrospective $p$-chart problem - $\bar{p}_{\text{prod}} = \frac{.034}{10} = .0034$

\[
UCL_p = p + 3\sqrt{\frac{p(1-p)}{n}} \quad \text{and} \quad LCL_p = p - 3\sqrt{\frac{p(1-p)}{n}}
\]

Substituting $\bar{p}_{\text{prod}}$ for $p$ and plugging in $n=1000$ we get

\[
UCL_p = .0034 + 3\sqrt{\frac{(.0034)(.9966)}{1000}} = .0034 + .0055 = .0089 \quad \text{with no $LCL_p$ - All $p$'s are inside control limits - There is thus no evidence of process instability.}
\]
g) If one is not willing to adopt the assumption that sheets within a day are independently scrapped with a fixed probability, one might instead view daily scrap rates as single daily "measurements" (rather than as count data). Treat the values in f) above this way and make estimates of process mean and standard deviation of scrap rate that are affected as little as possible by potential process changes across days.

\[
\bar{X} = \frac{0.034}{10} = 0.0034 = \bar{\mu} \quad \text{(estimated process mean)}
\]

\[
\bar{MR} = \frac{0.022}{6} \quad \text{and} \quad \bar{\sigma} = \frac{\bar{MR}}{1.128} = 0.022 \quad \text{(estimated process std dev)}
\]

2. Suppose that for some reason I decide to use "2 sigma" control limits for an \( \bar{X} \) chart. If, in fact, process parameters remain at their standard values how many periods will pass on average through the first out-of-control-signal (false alarm)?

Control limits are now \( \text{UCL}_{\bar{X}} = \mu + 2 \frac{\sigma}{\sqrt{n}} \) and \( \text{LCL}_{\bar{X}} = \mu - 2 \frac{\sigma}{\sqrt{n}} \)

These (of course) have \( Z \)-values (assuming nothing changed)

\[
z_1 = \frac{\mu + 2 \frac{\sigma}{\sqrt{n}} - \mu}{\sigma/\sqrt{n}} = 2 \quad \text{and} \quad z_2 = \frac{\mu - 2 \frac{\sigma}{\sqrt{n}} - \mu}{\sigma/\sqrt{n}} = -2
\]

So \( P \left( z < -2 \text{ or } z > 2 \right) \approx 2 \left( 0.0228 \right) = 0.0456 \)

\[
\text{ARL} = \frac{1}{0.0456} \approx 21.8 \text{ periods... pretty small!}
\]
3. Below are some means and ranges for samples of size \( n = 4 \) measured part diameters for pieces turned on a CNC lathe in a particular machine shop.  (Units are 10\(^{-4}\) inch above 1.1800 inch .)

<table>
<thead>
<tr>
<th>Sample</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} )</td>
<td>5.25</td>
<td>5.75</td>
<td>19.50</td>
<td>10.00</td>
<td>9.50</td>
<td>9.50</td>
<td>9.75</td>
<td>12.25</td>
<td>12.75</td>
<td>14.50</td>
</tr>
<tr>
<td>( R )</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>9</td>
<td>2</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

\[ \sum \bar{x} = 108.75 \]
\[ \sum R = 40 \]

Engineering specifications for this diameter are 9 \( \pm 4 \) in the units specified above.

**a)** Based on computation of appropriate control limits for both \( \bar{x} \)'s and \( R \)'s above, is there evidence of process instability in these values?  **SHOW YOUR COMPUTATIONS** for the control limits and say clearly (yes or no) whether there is instability and why you believe there is or is not evidence of such.

\[ UCL_R = D_4 R = 2.282 (4) = 9.13 \quad \text{with no LCL_R} \quad \text{According to these, no ranges are outside control limits} \]

\[ UCL_\bar{x} = \bar{x} + A_2 R = 10.875 + (.729) (4) = 10.875 + 2.916 = 13.791 \]
\[ LCL_\bar{x} = \bar{x} - A_2 R = 10.875 - (.729) (4) = 10.875 - 2.916 = 7.959 \]

There are 4 (circled) \( \bar{x} \)'s outside these retrospective control limits.

This is evidence of process instability.  **YES**

**b)** If one judged the short-term-process-variation (as quantified by \( \sigma \)) to be constant across time, what estimate would you make of it based on the information above? (Give a number.)  In rough terms does this machining process then seem capable of meeting the engineering requirements (if properly aimed and monitored)?  Explain.

\[ \text{I'd use } \sigma = \frac{R}{d_2} = \frac{4.0}{2.059} = 1.943 \quad (10^{-4} \text{ inch}) \]

\[ \text{Specs are } \pm 4. \ 4 \text{ is about } 2.06 \bar{x}, \text{ or differently put} \]
\[ \frac{U-L}{6\sigma} = \frac{8}{6(1.943)} \approx .7. \text{ This is not a good estimated } C_p \]
\( C_p \text{ can never exceed } C_p, \text{ so } C_p \text{ will also be bad.} \)  **No, this process doesn't seem to be *capable.***
4. Below are a number of statements that might be made about various kinds of numerical "limits" met in a course like IE 361, each statement labeled with a letter (A through F). For each type of limit, list all of statements A through F that describe it. (I'll score this as 10 points minus 1 point for every letter written that should not be written and every missing letter that should be written … ;+}. There are at least a couple that could go either way, depending upon one's interpretation.)

A- Such limits often are used to draw conclusions about process parameters.
B- Such limits are used to assess process stability.
C- Such limits are typically used to judge to individual measurements from a process.
D- Such limits are made on the basis of an implicit assumption of process stability.
E- Such limits are derived from functionality requirements for individual process outcomes.
F- Such limits are potentially and naturally involved when one wants to assess the "capability" of a process to perform adequately.

Confidence limits ____________________
Prediction limits _____________________
Tolerance limits _____________________
Control limits _______________________
Engineering specification limits ______________

5. Your home furnace has a control mechanism on it that automatically turns it on when your room temperature gets too low and turns it off when the temperature gets too high. What, if any, role might there be for improving the operation of your home heating system using an automated "control chart" in your home heating system? (You could suggest replacement or augmentation of the current controller. If you do that be very careful to say exactly how what you suggest might help.) Does the possibility of central control of not just a home furnace but rather a large physical plant (like that of the university) make the possibility of automated control charting any more relevant? Explain.

Control charting would only be relevant for purposes of confirming that your furnace control was behaving consistently. (It is no substitute for an engineering controller.)

In the context of running the university system, the small central HVAC group absolutely needs some automated way of "watching" all (the huge number) of heating control loops and detecting when/if they start to behave badly. Here "control charting" could be a huge help.
I have neither given nor received unauthorized assistance on this exam.

_________________________________________  ____________________________
Name                                               Date
1. An old-style (before lasers, GPS, etc.) land survey located the four corners of a land parcel by measuring distances \(a, b, \) and \(c\) and angles \(\alpha\) and \(\beta\) as pictured below.

![Diagram of a land parcel with labeled dimensions and angles.]

Of primary interest is the area, \(A\), of the land parcel. Simple geometry/trigonometry shows that
\[
\frac{\partial A}{\partial \alpha} = \frac{1}{2} b \sin (\beta - \alpha) = \frac{1}{2} \left( \frac{500 \text{ ft}}{1000 \text{ ft}} \right) = 530.3
\]
\[
A = \frac{1}{2} (ab \sin \alpha + bc \sin (\beta - \alpha)) \Rightarrow \frac{\partial A}{\partial b} = \frac{1}{2} (a \sin \alpha + c \sin (\beta - \alpha))
\]
Suppose that in fact \(a = c = 1000 \text{ ft}, b = 1500 \text{ ft}, \alpha = \frac{\pi}{4}\) and \(\beta = \frac{\pi}{2}\) (radians), the lengths can be measured with standard deviation .5 ft and the angles can be measured with standard deviation .02 radians.

5 pts  a) Finish filling in the table below (partial derivatives are evaluated at mean inputs).

<table>
<thead>
<tr>
<th>Input</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\partial A}{\partial \text{Input}})</td>
<td>530.3</td>
<td>707.1</td>
<td>530.3</td>
<td>0</td>
<td>530,330</td>
</tr>
<tr>
<td>(\sigma_{\text{Input}})</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
<td>.02</td>
<td>.02</td>
</tr>
</tbody>
</table>

10 pts  b) Use the table above and find an approximate standard deviation to associate with a value of \(A\).
\[
\sigma_A^2 \approx (.5(530.3))^2 + (.5(707.1))^2 + (.5(530.3))^2 + 0 + (.02(530.330))^2
\]
\[
\sigma_A = \sqrt{\sigma_A^2} \approx 10.619 \text{ ft}^2
\]

5 pts  c) Uncertainty in which measured variable is the largest contributor to uncertainty in \(A\)? Explain.

Biggest contributor: \(\beta\)  Explanation: Compare the summands above. The largest of them is that, which is \(\left( \frac{\partial A}{\partial \beta} \right)^2 \sigma_\beta^2\).
2. This question is based on the article "Results May Not Vary" by Johnson and McNeilley that appeared in Quality Progress in May 2011. The article describes an experiment done to determine how 3 processing factors, **A-Ammonium**, **B-Stir Rate**, and **C-Reaction Temperature** affect the physical properties of high purity silver powder made for use in the electronics industry. (Many details of the problem, including the units for B were suppressed in the article for reason of corporate security.) Experimental response variables were

\[ y = \text{density \ (g/cm}^3) \]
\[ z = \text{surface area \ (cm}^2 / \text{g}) \]

Every process set-up was run \( m = 2 \) times. The \( 2^3 + 1 = 9 \) process set-ups, sample means, and sample standard deviations from the 2 runs of each type are in the table below.

<table>
<thead>
<tr>
<th>A (% )</th>
<th>B</th>
<th>C (°C)</th>
<th>( \bar{y} )</th>
<th>( s_y )</th>
<th>( \bar{z} )</th>
<th>( s_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>100</td>
<td>8</td>
<td>14.930</td>
<td>.364</td>
<td>.415</td>
<td>.021</td>
</tr>
<tr>
<td>30</td>
<td>100</td>
<td>8</td>
<td>16.300</td>
<td>1.669</td>
<td>.415</td>
<td>.007</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>8</td>
<td>7.100</td>
<td>.622</td>
<td>.680</td>
<td>.014</td>
</tr>
<tr>
<td>30</td>
<td>150</td>
<td>8</td>
<td>12.540</td>
<td>.113</td>
<td>.440</td>
<td>.113</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>40</td>
<td>14.315</td>
<td>4.759</td>
<td>.505</td>
<td>.106</td>
</tr>
<tr>
<td>30</td>
<td>100</td>
<td>40</td>
<td>14.305</td>
<td>2.341</td>
<td>.555</td>
<td>.021</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>40</td>
<td>8.435</td>
<td>.573</td>
<td>.715</td>
<td>.049</td>
</tr>
<tr>
<td>30</td>
<td>150</td>
<td>40</td>
<td>14.960</td>
<td>0</td>
<td>.410</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \begin{array}{ccc}
16 & 125 & 24 \\
\end{array} \]

13.120 \hspace{1cm} .339 \hspace{1cm} .480 \hspace{1cm} .014

7 pts  a) Making use of data from all 9 process set-ups, find an estimate for the standard deviation of density \( (y) \) associated with any fixed process set-up. What are the associated degrees of freedom?

\[
\begin{align*}
S_{\text{pooled}}^2 &= \frac{(2-1)(.364)^2 + (2-1)(1.669)^2 + \ldots + (2-1)(.335)^2}{(2-1) + (2-1) + \ldots + (2-1)} \\
&= 3.543 \\
S_{\text{pooled}} &= \sqrt{S_{\text{pooled}}^2} \\
&= 1.882
\end{align*}
\]

estimated standard deviation = 1.882  
\( \text{d.f.} = n-r = 18-9 = 9 \)

7 pts  b) Give 95% confidence limits for the standard deviation of density associated with any fixed process set-up based on your answer to a). (Plug in completely but don't simplify.)

\[
\begin{align*}
&\text{Use } S_{\text{pooled}} \sqrt{\frac{n-r}{\chi^2_{\alpha}}} \text{ and } S_{\text{pooled}} \sqrt{\frac{n-r}{\chi^2_{1-\alpha}}} \\
&1.882 \sqrt{\frac{9}{19.023}} \quad 1.882 \sqrt{\frac{9}{2.700}}
\end{align*}
\]
7 pts  c) What "margin of error" (based on two-sided 95% confidence limits) might be associated with any one of the density sample means in the table on page 3?

\[ t_{(\alpha/2)} \frac{S_{\text{pooled}}}{\sqrt{n}} = 2.262 \frac{1.882}{\sqrt{12}} = 3.01 \]

7 pts  d) Based on two-sided 95% confidence limits, by how much must two of the sample mean densities in the table on page 3 differ before that difference can be called statistically detectable?

\[ t_{(\alpha/2)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 2.262 (1.882) \sqrt{\frac{1}{12} + \frac{1}{12}} \]
\[ = 4.26 \]

6 pts  e) Temporarily consider only the 4 process set-ups with reaction temperature 8°C. Make an interaction plot WITH (95%) ERROR BARS on the axes below. DO you expect A x B interactions to be statistically detectable if attention is restricted to this temperature? EXPLAIN.

\[ \text{I don't really expect these to be detectable. One can imagine sliding the endpoints of the line segments around inside the error bars and getting the plot to exhibit parallelism.} \]
\[ \text{If they would be detectable, they will be only barely so.} \]
8 pts  f) In a context like the present one, where 2-level factors are quantitative and a "full 2\textsuperscript{r}" factorial plus center point" experiment is run, the linear combination of means

$$L = \text{average of the } 2^r \text{ "corner" means} - \text{center point mean}$$

is sometimes used as a single measure of "curvature in response"/interaction. Considering again density (\(y\)), give 95\% two-sided confidence limits for this quantity. (Notice, this involves mean responses for all \(2^3 + 1 = 9\) setups.) Based on your answer, is \(L\) detectably different from 0?

\[
\hat{L} = \text{average of } 2^3 \text{ means} - \text{center mean} = 12.860 - 13.120 = -0.26
\]

Limits are

\[
\hat{L} \pm t_{s_p} \sqrt{\frac{2 \cdot \frac{\hat{L}^2}{n_c}}{n_c}}
\]

\[
2.262 (1.882) \sqrt{\frac{1}{2} \left[ 8 \left(\frac{1}{8}\right)^2 + (-1)^2 \right]}
\]

\[
= 3.19
\]

\(L \neq 0\?:\) Yes/No (circle only one)

8 pts  g) Suppose that one applies the Yates algorithm to the \(2^3\) density means to produce fitted factorial effects. What margin of error based on two-sided 95\% confidence limits would you associate with the values produced by the algorithm? (You don't need to compute a standard deviation from only the 8 samples. Instead, use the one from part a.)

Limits will be

\[
\hat{\beta} \pm t_{s_p} \sqrt{\frac{1}{2^3} \left[ \frac{1}{n_{c1}} + \frac{1}{n_c} + \ldots \right]}
\]

\[
2.262 (1.882) \frac{1}{8} \sqrt{8 \left(\frac{1}{2}\right)}
\]

\[
= 1.064
\]

margin of error
Here are the results of applying the Yates Algorithm to both the \( y \) and \( z \) sample means for the \( 2^3 \) part of the data set are below.

<table>
<thead>
<tr>
<th>Fitted Effect</th>
<th>Density (( y ))</th>
<th>Surface Area (( z ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>12.861</td>
<td>.517</td>
</tr>
<tr>
<td>( a )</td>
<td>1.666</td>
<td>.062</td>
</tr>
<tr>
<td>( b )</td>
<td>-2.102</td>
<td>.044</td>
</tr>
<tr>
<td>( ab )</td>
<td>1.326</td>
<td>-.074</td>
</tr>
<tr>
<td>( c )</td>
<td>.143</td>
<td>.029</td>
</tr>
<tr>
<td>( ac )</td>
<td>-.037</td>
<td>-.002</td>
</tr>
<tr>
<td>( bc )</td>
<td>.796</td>
<td>-.028</td>
</tr>
<tr>
<td>( abc )</td>
<td>.308</td>
<td>-.014</td>
</tr>
</tbody>
</table>

6 pts **h)** Use your answer to part **g)** and a corresponding value of .031 for a margin of error for the surface area (\( z \)) effects to say what factors in this study have statistically detectable effects.

Only \( A \) and \( B \) have (main and \( 2 \) factor interaction) effects clearly larger than background noise / that are statistically detectable.

6 pts **i)** Specifications on density (\( y \)) were that it be below 14 \( g/cm^2 \) and specifications on surface area were that it be between .3 \( cm^2/g \) and .6 \( cm^2/g \). Based on your answer to **h)**, does any set of the \( 2^3 \) levels of factors \( A, B, \) and \( C \) have mean responses meeting these specifications? Based on the original data set, do you see any way to potentially meet these specifications? Explain.

Compute \( \eta \) and \( \xi \) values:

\[
\begin{align*}
A & & B \\
- & - & 12.861 - 1.666 + 2.102 + 1.326 = 14.62 \\
+ & - & 12.861 + 1.666 + 2.102 - 1.326 = 15.30 \\
- & + & - - - = 7.77 \\
+ & + & + - + = 13.75
\end{align*}
\]

It looks like the high \( A \) and high \( B \) combination with any level of \( C \) will at least have mean response inside specifications (for individuals?!) The entire point of the plan also seems to have mean response inside specifications for individuals.
3. Below is a normal plot of 15 fitted sums of effects from a $2^{6-2}$ fractional factorial experiment whose generators were

$$E \leftrightarrow ABC \text{ and } F \leftrightarrow CD$$

6 pts a) How many different combinations of levels of the factors A, B, C, D, E, and F (how many different process set-ups) were involved in this study? Give levels of the factors E and F used with the indicated levels of A, B, C, and D for 4 of the combinations that were involved.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>+</td>
<td>−</td>
<td>+</td>
<td>−</td>
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<td>−</td>
<td>+</td>
<td>+</td>
<td>−</td>
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<tr>
<td>−</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
</tbody>
</table>

The number of combinations is 16.

6 pts b) Write out the defining relation for the experimental plan used in this study. Based on your answer, what is the simplest possible interpretation of the plot at the top of this page?

Interpretation:

I $\leftrightarrow$ ABCE $\leftrightarrow$ CDF $\leftrightarrow$ ABDEF

The main effect and EF 2 factor interaction are detectable.

6 pts c) Suppose that the average $\bar{y}$ in the experiment was 15. In light of the plot above, give a combination of levels of Factors A, B, C, D, E, and F that you expect to have maximum mean response. What mean response do you predict for that combination?

$\bar{y} = 15 + 20 + 25 = 60$