I have neither given nor received unauthorized assistance on this exam.

________________________________________________________
Name Signed                      Date

________________________________________________________
Name Printed
This exam consists of 20 multiple choice questions. There is a single best answer for each question. **Circle EXACTLY ONE response** for each question on this answer sheet.

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E
1. The method of "propagation of errors" or "statistical/probabilistic tolerancing" applied to
\[ U = g(X, Y, ..., Z) \]
- is useful for finding an approximate theoretical mean and an approximate theoretical standard deviation for \( U \) based on the form of the function \( g \) and means and standard deviations for the inputs \( X, Y, ..., \) and \( Z \).
- is an important data analysis method for decomposing observed variation in the output, \( U \), into parts attributable to several factors.
- can be a design tool for setting engineering tolerances on various inputs that impact a critical product characteristic.
- can help identify which input \( X, Y, ..., \) or \( Z \) is the most important contributor to variation in the output \( U \).

a) None of the bullets above is a correct completion of the sentence.
b) Exactly 1 of the bullets above is a correct completion of the sentence.
c) Exactly 2 of the bullets above is a correct completion of the sentence.
d) Exactly 3 of the bullets above is a correct completion of the sentence.
e) All 4 of the bullets above are correct completions of the sentence.

2. Below are some statements about sample sizes in experimentation. How many of them are true?
- Large sample sizes provide the ability to see small experimental effects.
- "Balanced data" are data where sample sizes are all the same across the whole set of experimental conditions.
- It is very important that some samples sizes(s), \( n_i \), in an experiment be larger than 1.
- Constant sample sizes across the set of experimental conditions provide relatively simple/clean formulas for data analysis, but are in no other way essential to good experimentation.

a) None are true,
b) Exactly 1 is true
c) Exactly 2 are true.
d) Exactly 3 are true.
e) All 4 are true.

3. In an experimental study where \( n \) observations are made overall for the comparison of \( r \) different experimental conditions, the pooled sample standard deviation, \( s_{\text{pooled}} \)
- serves as a measure of "background noise" for the study and a basic ingredient of measures against which observed differences in responses for the experimental conditions can be judged.
- has \( n - 1 \) associated degrees of freedom.
- is unaffected by measurement noise and quantifies only item-to-item variation.
- is an estimate of the variability in response for any fixed one of the experimental conditions.

a) None of the bullets above is a correct completion of the sentence.
b) Exactly 1 of the bullets above is a correct completion of the sentence.
c) Exactly 2 of the bullets above is a correct completion of the sentence.
d) Exactly 3 of the bullets above is a correct completion of the sentence.
e) All 4 of the bullets above are correct completions of the sentence.
4. Below are some statements about experimental design and analysis. How many of them are true?

- A standard gauge R&R study is a balanced $I \times J$ two-way factorial study.
- A $3^4$ factorial experiment studies 3 factors, each at 4 levels.
- Interactions in a two-way factorial study are measures of lack of parallelism for plots of means versus level of Factor A (with means for a given level of B connected by line segments).
- Full factorial experiments are a practical way of learning about the impact of many factors on a response variable, $y$.

a) None are true,

b) Exactly 1 is true

c) Exactly 2 are true.

d) Exactly 3 are true.

e) All 4 are true.

5. Below are some statements about factorial experimental design and analysis. How many of them are true?

- Two-way interactions in a two-way factorial measure what part of mean response cannot be accounted for by an overall mean and factors A and B operating separately on the response variable.
- Two-way $A \times B$ interactions in a three-way factorial are averages across levels of C of what one would call $A \times B$ interactions looking at one level of C at a time.
- Two-way $A \times B$ interactions in a three-way factorial are two-way interactions computed on a two-way table of means produced by averaging mean responses across levels of C.
- Three-way interactions in a three-way factorial measure differences in what one would call $A \times B$ interactions looking at one level of C at a time.

a) None are true,

b) Exactly 1 is true

c) Exactly 2 are true.

d) Exactly 3 are true.

e) All 4 are true.

6. Below are some statements about fractional factorial experimental design and analysis. How many of them are true?

- A virtue of good fractional factorial experimentation is that it guarantees that high-order interactions will not be present in a many-factor system.
- A virtue of good fractional factorial experimentation is that it allows at least some information to be gathered on the pattern of joint action of many factors on a response variable.
- In "many-factor" contexts, small fractions of full factorials almost always produce complete understanding of dependence of system response on the many factors.
- Data from essentially any set of half of all $2^p$ combinations of levels of $p$ two-level factors will provide the same level of insight into system reaction to changes in levels of the factors.

a) None are true,

b) Exactly 1 is true

c) Exactly 2 are true.

d) Exactly 3 are true.

e) All 4 are true.
An optical measuring system makes use of "triangulation" to determine distances. A schematic for using triangulation to determine the 2-d distance of a point to a line is below.

Simple trigonometry shows that the perpendicular distance \( D \) from the point C to the line through points A and B is (in terms of the distance \( L \) from A to B and the positive angles \( \alpha \) and \( \beta \))

\[
D = L \cdot \frac{(\sin \alpha)(\sin \beta)}{\sin(\alpha + \beta)}
\]

Suppose that the distance between A and B can be measured with standard deviation \( 10^{-3} \) meter, and angles can be measured with standard deviation \( .01 \) radian. As it turns out

\[
\frac{\partial D}{\partial \alpha} = L \cdot \frac{\sin^2(\beta)}{\sin^2(\alpha + \beta)} \quad \text{and} \quad \frac{\partial D}{\partial \beta} = L \cdot \frac{\sin^2(\alpha)}{\sin^2(\alpha + \beta)}
\]

7. For an instance of the use of such an optical measuring device (with the indicated precisions of measurement of distance A to B and angles) the standard deviation associated with measurement of \( D \) if the distance from A to B is \( L = 1 \) meter and \( \alpha = \pi / 4 \) and \( \beta = \pi / 4 \)
   a) is \( .00005 \) meter.
   b) is \( .00709 \) meter.
   c) is \( .01003 \) meter.
   d) is \( .01005 \) meter.
   e) None of a) through d) is a correct completion of the sentence.

8. For a set-up of an optical measuring device where the distance from A to B is \( L = 1 \) meter and in fact \( D = .5 \) meter, consider the two possibilities that
   1. C is horizontally positioned half way between A and B (the perpendicular from C to the line through A and B bisects the line segment \( \overline{AB} \))
   2. C is horizontally positioned as opposite B (the perpendicular from C to the line through A and B intersects the line at point B)

For which of the possibilities is measurement of \( D \) most precise?
   a) Measurement of \( D \) is most precise under possibility 1.
   b) Measurement of \( D \) is most precise under possibility 2.
   c) Measurement of \( D \) is equally precise under the two possibilities.
   d) It is not possible to say which possibility gives the most precise measurement of \( D \) based on the given information.
An example in the book *Testing 1,2,3: Experimental Design in Marketing and Service Operations* by Ledolter and Swersey concerns a study made to identify causes of cracking in ceramic flower pots produced by Company X. Three factors potentially affecting both the fraction of pots that crack in firing and the cost of making those pots were

<table>
<thead>
<tr>
<th>A- Cooling Rate</th>
<th>B- Peak Kiln Temperature</th>
<th>C- Clay Mix</th>
</tr>
</thead>
</table>

The following series of questions is not necessarily true to the scenario developed in the book, but take their motivation from the example and data there.

A company standard production method and 3 others were suggested for study by a Company X analyst (who had not taken IE 361). The 3 new methods involved changing level of only one of the three factors A, B, or C. Two production runs were made for each method and the response variable \( y \) = percentage of pots produced that were cracked was observed for each. Some summary statistics for these production runs are below.

<table>
<thead>
<tr>
<th>Standard Method (i = 1)</th>
<th>New Cooling Rate (i = 2)</th>
<th>New Temp (i = 3)</th>
<th>New Clay Mix (i = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_i = 2 )</td>
<td>( n_2 = 2 )</td>
<td>( n_3 = 2 )</td>
<td>( n_4 = 2 )</td>
</tr>
<tr>
<td>( \bar{y}_1 = 16 )</td>
<td>( \bar{y}_2 = 34 )</td>
<td>( \bar{y}_3 = 14 )</td>
<td>( \bar{y}_4 = 6 )</td>
</tr>
<tr>
<td>( s_i = \sqrt{18} )</td>
<td>( s_2 = \sqrt{8} )</td>
<td>( s_3 = \sqrt{8} )</td>
<td>( s_4 = \sqrt{8} )</td>
</tr>
</tbody>
</table>

(We will follow Ledolter and Swersey and treat the response variable as an instance of "variables" data rather than as an instance of "attributes" data, and use the one-way normal model rather than trying to employ Binomial analyses.)

9. 95% confidence limits for the standard deviation of lot percentages of cracked pots are
   a) 6.29 and 30.19.
   b) 3.77 and 86.78.
   c) 1.94 and 9.32.
   d) 1.91 and 9.15.
   e) None of a) through d) is a correct completion of the sentence.

10. A margin of error applicable to any one of the sample means, \( \bar{y}_i \), as representing a corresponding long run percentage of cracked pots \( \mu_i \), based on two-sided 95% confidence limits is
    a) impossible to determine based on the given information.
    b) 20.61
    c) 9.00
    d) 6.36
    e) possible to determine, but not close to any of b) through d).
11. The data summarized on page 6 produce \( t \cdot s_{\text{pooled}} \sqrt{\frac{1}{n_i} + \frac{1}{n_j}} = 9.0 \). This indicates

a) that exactly one of the new production methods clearly reduces pot cracking.
b) that exactly one of the new production methods clearly increases pot cracking.
c) that exactly one of the new methods clearly increases pot cracking and exactly one clearly reduces pot cracking.
d) that the results of the study are completely inconclusive.
e) None of a) through d) is a correct completion of the sentence.

Thinking of the 2-level factors A, B, and C as potentially providing a \( 2^3 \) factorial structure for a more complete study of the pot production process, we might call the current levels of the factors the low (−) levels and the new ones the high (+) levels. As it turns out,
- the new/high (+) level of Cooling Rate (A) is faster and cheaper than the present one,
- the new/high (+) level of Temperature (B) is physically higher and more expensive than the present one, and
- the new/high (+) level of Clay Mix is simply a different clay mix available at the same price as the one presently in use.

A full \( 2^3 \) factorial would involve all 8 different possible combinations of levels of the three factors.

12. From this factorial point of view, the "one-factor-at-a-time" experimental plan consisting of the four combinations represented in the table on page 6

- is a half fraction of the full \( 2^3 \) factorial.
- provides for estimation of the main effects of all three factors if there are no two or three factor interaction effects on \( y \).
- fails to be as good as a "standard half fraction" plan because it doesn't provide full \( 2 \times 2 \times 2 \) factorials in all pairs of factors.
- fails to have a simple way to understand its associated "alias/confounding structure."

a) None of the bullets above is a correct completion of the sentence.
b) Exactly 1 of the bullets above is a correct completion of the sentence.
c) Exactly 2 of the bullets above is a correct completion of the sentence.
d) Exactly 3 of the bullets above is a correct completion of the sentence.
e) All 4 of the bullets above are correct completions of the sentence.

Suppose that a replicated balanced full \( 2^3 \) factorial study with all sample sizes \( m = 3 \) is done on the flower pot production process, producing the 8 sample means listed on the next page and \( s_{\text{pooled}} = 2.0 \).

Also listed on the next page are the corresponding "cycle #2" values from application of the Yates algorithm. Complete the Yates algorithm and use your results as appropriate in what follows.
<table>
<thead>
<tr>
<th>combination</th>
<th>$\bar{y}_{\text{comb}}$</th>
<th>cycle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>16</td>
<td>98</td>
</tr>
<tr>
<td>a</td>
<td>34</td>
<td>40</td>
</tr>
<tr>
<td>b</td>
<td>14</td>
<td>38</td>
</tr>
<tr>
<td>ab</td>
<td>34</td>
<td>16</td>
</tr>
<tr>
<td>c</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>ac</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>bc</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>abc</td>
<td>16</td>
<td>4</td>
</tr>
</tbody>
</table>

13. A "% margin of error" to associate with any of the sample means in the table above based on 95% confidence limits
   a) is .87.
   b) is 2.45.
   c) is 4.24.
   d) is smaller in magnitude than a) above.
   e) is larger in magnitude than c) above.

14. The "% margin of error" for any of the fitted effects you computed in the Yates algorithm based on 95% two-sided confidence limits
   a) is .87.
   b) is 2.45.
   c) is 4.24.
   d) is smaller than a) above.
   e) is larger than c) above.

Regardless of how you answered 14, apply a ±1.5 margin of error criterion, and judge which $2^3$ factorial effects are detectable. Use your results in answering questions 15 through 17.

15. The sign of $b_2$ produced by the Yates algorithm
   a) is positive, but this has no statistical significance. If it had statistical significance, it would indicate that the new temperature increases the percentage of cracked pots.
   b) is negative, but this has no statistical significance. If it had statistical significance, it would indicate that the new temperature decreases the percentage of cracked pots.
   c) is positive, and this is statistically significant. It indicates that the new temperature increases the percentage of cracked pots.
   d) is negative, and this is statistically significant. It indicates that the new temperature decreases the percentage of cracked pots.
   e) None of responses a) through d) is correct.
16. Taking account of which effects are statistically detectable and the context of the problem given in the reading before question 12, Company X will almost surely wish to henceforth use
   a) process conditions represented by combination (1).
   b) process conditions represented by combination c.
   c) process conditions represented by combination bc.
   d) either process conditions represented by combination c or conditions represented by combination bc (they are equally attractive).
   e) None of responses a) through d) is correct.

17. Using a fitted model that includes only statistically detectable effects, a fitted or predicted percentage of cracked pots associated with the current method of process operation is closest to
   a) 12
   b) 13
   c) 14
   d) 15
   e) 16

Consider now an hypothetical context where 7 two-level factors A,B,C,D,E,F, and G are to be studied. \(2^7 = 128\) and rather than a full factorial, a 1/4 fraction study is contemplated (requiring data collection for 32 combinations of levels of the 7 factors). The set of generators 
   \[ F \leftrightarrow ABCD \text{ and } G \leftrightarrow BCDE \]
   is under consideration for choosing a fractional factorial design.

18. For this choice of generators, what levels of F and G will be used when all of factors A,B,C,D, and E are at their low levels?
   a) low F and low G
   b) low F and high G
   c) high F and low G
   d) high F and high G
   e) It is impossible to tell from the given information.

19. For this choice of generators
   a) the lowest-order/simplest effect aliased with any 2-factor interaction is a 3-factor interaction.
   b) the lowest-order/simplest effect aliased with any 2-factor interaction is another 2-factor interaction.
   c) the lowest-order/simplest effect aliased with any 2-factor interaction is a main effect.
   d) the lowest-order/simplest effect aliased with any 2-factor interaction can not be determined until data are collected and analyzed.
   e) None of a) through d) is a correct completion of the sentence.
20. Data analysis for such an experiment
   • would involve a 5-cycle Yates algorithm.
   • would recognize that every effect has 3 aliases.
   • may leave some ambiguities of interpretation that require subsequent additional data collection to resolve.
   • lacking any replication in the data, would require normal plotting of the last 31 fitted sums of effects produced by the Yates algorithm.

a) None of the bullets above is a correct completion of the sentence.
b) Exactly 1 of the bullets above is a correct completion of the sentence.
c) Exactly 2 of the bullets above is a correct completion of the sentence.
d) Exactly 3 of the bullets above is a correct completion of the sentence.
e) All 4 of the bullets above are correct completions of the sentence.