

Spring 2000 IE 361 Exam Solutions

Exam 1

1.

(a) A calibration experiment is done to determine how to correct for instrument bias, by figuring out how to translate easily available measurements to estimated "gold standard" measurements.

An R&R study is done to determine the precision of measurement equipment, specifically to assess the size of "intrinsic"/repeatability measurement variation and the size of "operator-to-operator" measurement variation.

2.

(a) Use $\hat{x}_{n+1} \pm t_{n-2} \frac{\sqrt{MSE}}{|b_1|} \sqrt{1 + \frac{1}{n} + \frac{(\hat{x}_{n+1} - \bar{x}_n)^2}{\sum_{i=1}^n (x_i - \bar{x}_n)^2}}$.

$\hat{x}_{n+1} = \frac{1}{b_1}(y_{n+1} - b_0) = \frac{1}{.99}(105 - .01) = 106.05$. So limits are $106.5 \pm 1.860 \frac{5}{|.99|} \sqrt{1 + \frac{1}{10} + \frac{(106.05 - 100)^2}{9(20)^2}}$.

3.

(a) $\hat{\sigma}_{\text{repeatability}} = \frac{\bar{R}}{d_2(m)} = \frac{.0033}{1.128} = .0029$ inch. This is an estimate of the standard deviation a single operator would experience measuring the same disk many times.

(b) Use display (2.11). $\hat{\sigma}_{\text{reproducibility}} = \sqrt{\max\left(0, \left(\frac{.0092}{1.693}\right)^2 - \frac{1}{2}\left(\frac{.0033}{1.128}\right)^2\right)} = .0050$ inch.

Yes, there is important operator-to-operator variability here. This standard deviation is 1.7 times as large as the repeatability standard deviation.

(c) $G\hat{C}R = \frac{6 \hat{\sigma}_{\text{R\&R}}}{U-L} = \frac{6 \sqrt{\hat{\sigma}_{\text{repeatability}}^2 + \hat{\sigma}_{\text{reproducibility}}^2}}{2(.0625)} = \frac{\sqrt{(.0029)^2 + (.0050)^2}}{2(.0625)} = .28$. This is a pretty big/unsatisfactory estimated GCR . We'd like a value on the order of .1 to .01. This is at least 2.8 times that size.

4.

(a) These limits don't allow for any real "process variation" but only for measurement variation. They are set up as if one could produce consistently perfect diameters.

(b) NO. Doing things this way would be setting oneself up to have stratification hide consistent differences between heads. The 3 heads should be monitored separately.

(c) $\bar{R} = \frac{18.40}{10} = 18.4$. Use display (3.18). $UCL_R = 2.115(18.4) = 3.89$ with no LCL_R . There are no "out-of-control" ranges. There is no evidence in the ranges of physical change/instability in the process.

(d) $\hat{\sigma} = \frac{\bar{R}}{d_2(n)} = \frac{1.84}{2.326} = .79 \times 10^{-2}$ inch (Note, by the way, that consistent with problems 3a and 4a, this is larger than $\hat{\sigma}_{\text{repeatability}}$, reflecting *process variation* in addition to the measurement/repeatability variation.)

(e) Supposing the mean measured diameter to be at the ideal, the specifications are at $z = \frac{.0625}{.0079} = 7.911$ and $z = -7.911$. Since $P[|Z| < 7.911] \approx 1$ my projection for measured diameters should be 1.00. Since the actual diameters should be *less variable* than the measured diameters, if this weren't already essentially 1.00 we could expect the best possible fraction of actual diameters in spec.s to exceed the fraction for the measured ones.

(f) Use display (3.7), i.e. $.962 \pm .577(1.84)$. That is $UCL_{\bar{x}} = 2.024$ and $LCL_{\bar{x}} = -.100$ in the units of the table. There are no \bar{x} 's outside these limits and therefore no indication of physical instability.

5.

(a) Use display (3.33). When $UCL_{\hat{p}} = .001 + 3\sqrt{\frac{.001(.999)}{200}} = .0077$ and since $.001 - 3\sqrt{\frac{.001(.999)}{200}} < 0$, no lower control limit is appropriate.

(b) $q = P[\text{the first } \hat{p} \text{ plots outside limits}] = P[\hat{p} > .0077] = P[X > 1.54]$ for X a binomial random variable with $n = 200$ and $p = .005$. This is $1 - f(0) - f(1) = .264$. So $ARL = 1/q = 1/.264 = 3.78$.

Exam 2

Quality Culture Questions

1. Malcom Baldrige
2. ?Quality Function Deployment? and Robust Design
3. 6 Sigma-Motorola Corp., Malcom Baldrige-National Institute for Standards and Technology, Quality Function Deployment-Yoji Akao, Robust Design-Genichi Taguchi, ISO 9000-International Organization for Standards, Total Quality Management-W. Edwards Deming
4. There is typically no way to provide a useful operational definition of a company "using" a method. Does that mean that at least one person in the company attended a seminar or what? There is similarly no obvious way to invent a valid measure of dollars saved. How could I ever compare what happened to what might have happened? There is also the likely problem of bias/inaccuracy in that people providing these figures (if there really are any) are presumably disciples of those making the claims.
5. Much of the available material comes from consultants wishing to sell their advice/services. One should expect their claims to be overblown and self-serving. Information from academics is sometimes (not always) more dispassionate/"objective." (Academic payoff is more often in terms of ego/reputation than dollars, but the impulse to be self-serving remains.)
6. If their methodology is truly effective, it constitutes a "strategic advantage" and the company is likely to consider it proprietary, not something to be "given away" fairly cheaply on the open market.

Technical Questions

1.

(a) $\mu_z \approx \mu_x - 3\cot(\mu\theta) = 2 - 3(0) = 2$ and

$\sigma_z^2 \approx \left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial \theta}\right)^2 \sigma_\theta^2 = 1(.01)^2 + \left(\frac{-3}{(0-1)}\right)^2 (.02)^2 = .0037$ so that

$\sigma_z \approx \sqrt{.0037} = .061$ cm.

(b) (I got the correlation wrong originally. It should be $\rho = .164$.) Use a multivariate chart with $n = 3$, $\mu_x = \mu_z = 2$, $\sigma_x = .01$, $\sigma_z = .061$ and $\rho = .164$. Plot

$$X^2 = 3(\bar{x} - 2, \bar{z} - 2) \begin{pmatrix} (.01)^2 & .164(.01)(.061) \\ .164(.01)(.061) & (.061)^2 \end{pmatrix}^{-1} \begin{pmatrix} \bar{x} - 2 \\ \bar{z} - 2 \end{pmatrix}$$

with control limit $UCL_{X^2} = 2 + 3\sqrt{2(2)} = 8$

2. SPC is process watching for purposes of change-detection. EFC is on-line process knob-turning for purposes of process guidance/creation of process stability. These are

different but complimentary activities that both can contribute to reduction in overall process variation.

3.

(a) This plot is remarkably (almost unbelievably!) linear. It indicates that a normal distribution model for coating weight is a sensible one.

(b) Use equation (5.5). Here this gives the interval $\left(6(.0148)\sqrt{\frac{30-1}{42.557}}, 6(.0148)\sqrt{\frac{30-1}{17.708}}\right)$ i.e. (.073, .114).

(c) Use equations (5.9) and (5.10). Here this is $\hat{C}_{pk} = \frac{.3500-.3210}{3(.0148)} = .653$ and the lower confidence bound is then $.653 - 1.645\sqrt{\frac{1}{9(30)} + \frac{(.653)^2}{2(30)-2}} = .480$.

(d) Use equation (5.13). Here this is $.3210 \pm 3.355(.0148)$ i.e. $.3210 \pm .0497$

(e) (CUSUM) $k_1^{\text{opt}} = .3210 + \frac{.0074}{2} = .3247$ and $k_2^{\text{opt}} = .3210 - \frac{.0074}{2} = .3173$ (using equations (4.12)). Use $U_0 = L_0 = 0$. Then apply display (4.13) to enter Table 4.6.

$\mathcal{K} = \frac{(.0074)/2}{(.0148)/\sqrt{4}} = .5$ and using display (4.14) $h = (4.77)\frac{.0148}{\sqrt{4}} = .0353$.

(EWMA) Using display (4.4) $shift = \frac{.0074}{.0148/\sqrt{4}} = 1$ and $\lambda^{\text{opt}} = .14$ Round this to .1 so we can use Table 4.3 to get $\mathcal{K} = 2.70$. Then use

$\lambda = .1, EWM A_0 = .3210, UCL_{EWMA} = .3210 + 2.70\frac{.0148}{\sqrt{4}}\sqrt{\frac{.1}{(2-.1)}} = .3210 + .00458$
and $LCL_{EWMA} = .3210 - .00458$.

(f) (CUSUM) $D^* = \frac{|.3100-.3210|}{.0200/\sqrt{4}} = 1.1$ from display (4.18). $\mathcal{K}^* = \frac{2(.0074)/2}{.0200/\sqrt{4}} = .74$ (from display (4.19)) and $\mathcal{H}^* = \frac{.0353}{.0200/\sqrt{4}} = 3.53$ (from display (4.15)). Now go to Table A.5.

? $ARL \approx 10$?

(EWMA) $D^* = \frac{|.3100-.3210|}{.0200/\sqrt{4}} = 1.1$ from display (4.6). $\mathcal{K}^* = \frac{2(.00458)}{2(.0200)/\sqrt{4}}\sqrt{\frac{(2-.1)}{.1}} = 1.996$ (from display (4.7)). Now go to Table A.3. ? $ARL \approx 6$?

Exam 3

1. Here $\bar{y}_1 = 2.580267$ and $\bar{y}_2 = 2.572267$ (these are needed in part (f)).

(a) No, I am not surprised. I expect to capture both measurement variation *and* copy to copy variation. The preliminary study captured only measurement variation.

(b) $2.014 - 2.0158 = -.0018$, $2.020 - 2.0158 = .0042$, $2.017 - 2.0158 = .0012$,
 $2.015 - 2.0158 = -.0008$ and $2.013 - 2.0158 = -.0028$

(c) That normal plot is very linear. It indicates no obvious problems with the "normal distributions with a common standard deviation" model here. There are no obvious problems with using the standard analysis.

(d) Use $\Delta = t \frac{sp}{\sqrt{n_{ij}}} = 2.0281 \frac{.0017}{\sqrt{5}} = .0015$

(e) Use $\bar{y}_{ij} - \bar{y}_{i'j'} \pm t_{sp} \sqrt{\frac{1}{n_{ij}} + \frac{1}{n_{i'j'}}$ This is $2.0158 - 2.0134 \pm 2.0281(.0017) \sqrt{\frac{1}{5} + \frac{1}{5}}$
 or $.0024 \pm .0022$.

(f) Note that $a_1 - a_2 = \bar{y}_1 - \bar{y}_2$. So use $\bar{y}_1 - \bar{y}_2 \pm t_{sp} \sqrt{\frac{2}{mJ}}$ This is
 $(2.580267 - 2.572267) \pm 2.0281(.0017) \sqrt{\frac{2}{5(3)}}$ or $.0080 \pm .0013$.

(g) Fitted interactions add to 0 across rows and down columns, so they are:

-.00296	-.00142	.00438
.00264	.00038	-.00302
.00032	.00104	-.00136

Confidence intervals for the $\alpha\beta_{ij}$ are $ab_{ij} \pm t_{sp} \sqrt{\frac{(I-1)(J-1)}{mIJ}}$ So for 95% confidence the
 "plus or minus part" of this is $2.0281(.0017) \sqrt{\frac{4}{45}} = .001028$. So only $\alpha\beta_{22}$ and $\alpha\beta_{31}$ fail
 to be detectable. YES there are detectable interactions.

2.

(a) Use the Yates algorithm.

$\bar{y}_{...} = 118.125$, $a_2 = 11.375$, $b_2 = -14.625$, $ab_{22} = -6.875$, $c_2 = 55.125$,
 $ac_{22} = -3.125$, $bc_{22} = 1.375$, $abc_{22} = 4.625$

combination	\bar{y}	cycle #1	cycle #2	cycle #3	cycle #3/8
(1)	53	158	252	945	118.125
a	105	94	693	91	11.375
b	44	373	58	- 117	- 14.625
ab	50	320	33	- 55	- 6.875
c	176	52	- 64	441	55.125
ac	197	6	- 53	- 25	- 3.125
bc	154	21	- 46	11	1.375
abc	166	12	- 9	37	4.625

(b) Confidence limits for effects are $\hat{E} \pm t_{SP} \frac{1}{2^3} \sqrt{\frac{8}{m}}$ and $t(7.2) \frac{1}{2^3} \sqrt{\frac{8}{8}} = .9t$. For reasonable confidence levels, t is between, say, 1.5 and 2.5. By this standard all effects except the BC 2 factor interactions are detectable.

(c) A high, B low and C high will make all of the fitted main effects and the fitted AB interaction positive. $\hat{y} = 118.125 + 11.375 + 14.625 + 55.125 + 6.875 = 206.125$. (Note that this is substantially larger than the estimated mean given in the table.)

3.

(a) # possible = $2^9 = 512$, # run = 16, fraction = $\frac{1}{32}$

(b)

A	B	C	D	BC	BD	ACD	AD	AB
E	F	G	H	J				
+	+	-	-	-	-	+	-	+
+	-	-	+	+	-	-	+	-

(c) 31

(d) For example, from the last 3 generators, the CDG 3 factor interaction, the DH 2 factor interaction and the BJ 2 factor interaction.

(e) Note that $- 6.38$ estimates the D main effect plus aliases and that $- 10.13$ estimates the AD 2 factor interaction plus aliases. From the generators, the simplest possible interpretation of $- 10.13$ is as an H main effect (the negative sign indicating that that "Existing" is best) and the simplest possible interpretation of $- 6.38$ is as a D main effect (the negative sign indicating that "75°C" is best).

(f) Use any setting for A, B, C, E, F, G, J and the settings of D and H indicated above. Then $\hat{y} = 92 + (- 10.13) + (- 6.38) = 75.49$.

(g) Depending upon how one "eye-balls" a line on this plot, either no effects are statistically detectable, or perhaps the $(- 10.13)$ and $(- 6.38)$ effects are "bigger than noise."