

October 9, 2002

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1. The viscosity of paint is measured with a "viscometer" in units of "Krebs."

First, a standard liquid of "known" viscosity 92 Krebs is tested with a company viscometer (by a single employee).

a) 10 measured values have a mean of 93.6 Krebs and standard deviation of .5 Krebs. What do the 95% "t" confidence limits  $93.6 \pm 2.262(.5/\sqrt{10})$  then estimate? (Use 10 words or less.)

b) Give 95% confidence limits for the bias of this viscometer. (No need to simplify.)

c) Give 95% confidence limits for the "repeatability" standard deviation of this viscometer. (No need to simplify.)

d) Suppose that the "reproducibility" standard deviation for use of the viscometer by company employees is about .8 Krebs. What would you expect for a standard deviation of measured viscosities if many different employees measure the viscosity of the standard liquid once each?

5 samples of paint dipped from different locations in single vat have measured viscosities with mean 91.6 Krebs and standard deviation 1.1 Krebs. (A single employee does all measuring.)

e) Using both sets of measurements, estimate a standard deviation of actual viscosities (not including measurement error) of such samples dipped from this vat of paint.

f) A standard error for the estimate in e) is approximately .44 Krebs. What does this figure indicate about the reliability of your answer to e)? (Use 10 words or less.)

2. Kumala, Nithang, Pramadi and Simpson worked with a company on improving a paperwork process. They developed a method of scoring the correctness of employee handling of a particular kind of transaction on a scale from 0 to 100. A large number of transactions were scored by this method and initial standards of  $\mu = 70$  and  $\sigma = 22$  described the strongly left-skewed distribution of scores at the beginning of their involvement.

a) What probability fact from Stat 231 suggests that (if the sample size is not too small) despite the fact that the distribution of scores was skewed, "usual" control limits may be used for  $\bar{x}$ ? (Use 10 words or less.)

b) Suppose that each month,  $n = 50$  of these transactions will be sampled and scored. Find standards given control limits for the sample mean score.

$$LCL_{\bar{x}} = \underline{\hspace{2cm}}$$

$$UCL_{\bar{x}} = \underline{\hspace{2cm}}$$

c) Management clearly wanted high scores, and in particular individual scores below 90 were considered "blunders." Your lower control limit in b) should be below 90 and you should have an upper control limit that is less than 100. BRIEFLY say why such limits are not incompatible with management's goals for transaction scores.

3. Olberding, Paulsen, Schreiner and Williams worked with a company on the milling of a high precision metal part. Below are means and ranges for a critical dimension of this part from periodic samples of size  $n = 4$ . The units are .0001 inch above .9490 inch.

sample	1	2	3	4	5	6	7	8	9	10	
$\bar{x}$	7.25	11.50	13.75	6.50	9.50	7.25	9.50	13.50	11.25	7.25	$\sum \bar{x} = 97.25$
$R$	6	9	7	15	9	9	12	16	15	10	$\sum R = 108$

a) Find retrospective control limits for both sample means and ranges.

For  $\bar{x}$ :

For  $R$  :

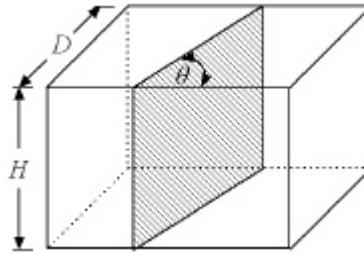


b) Is there any evidence of process instability in the values given in the table? Explain.

c) Supposing the process to be stable over the period of study, give estimates of the process mean and standard deviation.

d) Engineering specifications on the dimension being monitored were  $-10$  to  $30$  (in the units above). Use this fact, the estimates from c) and a normal distribution assumption to approximate the fraction of parts meeting engineering specifications.

1. Bar stock of height  $H$  and depth  $D$  is crosscut at an angle  $\theta$  to the front face as pictured below.



The area of the face produced by the cut is

$$A = \frac{HD}{\sin\theta}$$

Suppose that  $H$ ,  $D$  and  $\theta$  vary with means  $\mu_H = 1.00$  in,  $\mu_D = 1.00$  in and  $\mu_\theta = \frac{\pi}{2}$  radians and standard deviations  $\sigma_H = .01$  in,  $\sigma_D = .01$  in and  $\sigma_\theta = .02$  radians. Find an approximate mean and standard deviation for  $A$  (assuming that  $H$ ,  $D$  and  $\theta$  are independent). (Plug in completely, but no need to simplify.)

2. In a real application of control charting with attributes data, a company monitored the number of customer complaints received on their "customer complaint hotline" for each of its food products.

a) The "standard" complaint rate for a particular product is 3 per week. What then are sensible control limits for the number of complaints on this product received in a 13 week "quarter"?

b) A company executive argues that "the correct" way to monitor complaints is in terms of "complaints per unit sold" rather than "complaints per unit time." Last year, roughly 1,000,000 units of a product were sold and 500 complaints received. Using the executive's idea, what are sensible control limits for the number of complaints in a month where 100,000 units of this product are sold?

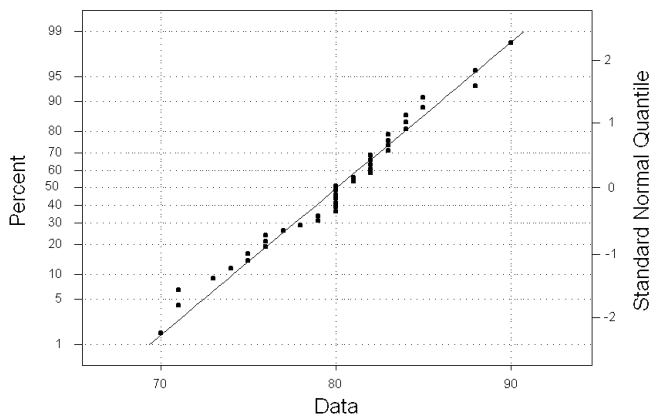
3. IE 361 students Olberding, Paulsen, Schreiner and Williams studied the performance of a CNC machining process. A sample of  $n = 40$  parts was produced and a particular critical dimension measured on each. Summary statistics for these (in units of .0001 inches above .2080 inch) were  $\bar{x} = 80.05$  and  $s = 4.65$ . In these units, engineering specifications on this dimension were  $80 \pm 20$ .

a) Give limits that you are 95% sure contain 99% of such dimensions. (No need to simplify.)

b) Give a 95% lower confidence bound for a process capability index that measures current performance (not potential). (Plug in, but again you need not simplify.)

c) Here is a normal plot of the students' data. Say what it indicates about the practical usefulness of the answers to a) and b) above and use it to explain the "process capability" to a non-quantitative person.

Normal Probability Plot



4. A (real) US government (FDA) draft guideline prescribing the monitoring of the concentration of white blood cells in certain filtered blood products calls for the weekly testing of  $n = 5$  units. "Lack of control" is to be declared if any of these units has more than a specified concentration of white cells. Supposedly, a company is in compliance with government guidelines if 5% or less of all units produced have more than the specified concentration of white blood cells.

A certain producer of these blood products is in fact consistently producing only 5% of units that are unacceptable in terms of the white cell concentration (and is therefore actually "in compliance" with FDA guidelines). What is the mean number of weeks the supplier will test until first having an "out of control" result?

5. Ideally, two variables  $x_1$  and  $x_2$  measured on a widget have means  $\mu_1 = 5$  and  $\mu_2 = 7$ , standard deviations  $\sigma_1 = 2$  and  $\sigma_2 = 2$  and correlation  $\rho_{12} = -.25$ . Process monitoring will be done based on samples of only  $n = 1$  widget.

a) How do you suggest monitoring  $x_1$  alone? (Say exactly what you would plot and what limits you would use.)

b) How do you suggest monitoring both  $x_1$  and  $x_2$  if the relationship between the two is physically important? (Say exactly what you would plot and what limits you would use.)

1. (Motivated by “Reduction of Testing Time of a Chemical Parameter Using Design of Experiments” by Rao and Roshan, *Quality Engineering*, 14(2), 2001-2002.) In the production of a rayon-grade pulp in the poly-fiber industry, measuring pulp solubility in 7.14% caustic is important. Two steps (A and B) in the standard measurement process take respectively 120 minutes and 180 minutes. An experiment is done to see if these times can be reduced by 30 minutes each without ruining measurement consistency. 9 pulp samples are prepared and split. Solubility of one part is measured via the standard method, and in a second measurement one or both of times for A and B are reduced. For each (split) pulp sample the response

$$y = \frac{\text{measured solubility under the reduced-time regimen}}{\text{measured solubility under the standard regimen}}$$

is recorded. Below are summary statistics for the study.

<u>Reduced A Time Only</u>	<u>Reduced B Time Only</u>	<u>Reduced A and B Times</u>
$n_1 = 3$	$n_2 = 3$	$n_3 = 3$
$\bar{y}_1 = .9765$	$\bar{y}_2 = .9603$	$\bar{y}_3 = .9209$
$s_1 = .0033$	$s_2 = .0287$	$s_3 = .0244$

Begin by assuming the relevance of the one-way normal model for the  $r = 3$  alternative measurement regimens.

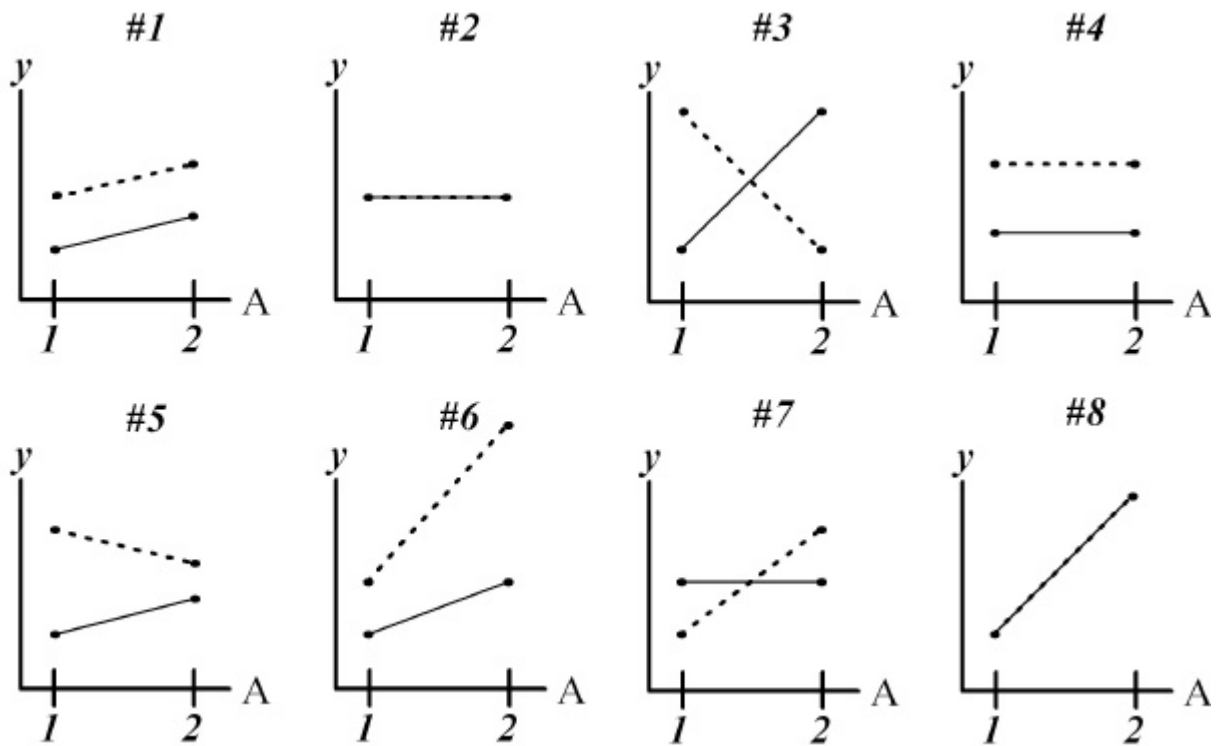
a) Give an estimate of  $\mathbf{S}$  in this model and say what it represents (interpret it!).

b) Note that  $\mathbf{m}_2 = 1$  would mean that the “Reduced B Time Only” regimen produces values like the standard regimen. Based on appropriate 95% confidence limits, is this plausible?

c) Do the first and third regimens produce detectably different mean values of  $y$ ? (Refer to appropriate 95% limits.)

d) The summary statistics suggest that the usual one way model may not be appropriate. More importantly, what do these suggest about the possibility of measuring solubility with one of the reduced-time methods and then “correcting back” to the standard regimen by dividing by an estimate of  $\mathbf{m}$ ? (Which of the  $r = 3$  alternatives seems best for this? Why?)

$B = 1$
$B = 2$



2. Assume that each of the interaction plots above represents a different  $2 \times 2$  factorial study. Fill in each empty cell in the table below with the number of a **single plot** that has the indicated effects.

A main effects present?	B main effects present?	AB interactions present?	Plot Number
No	No	No	
Yes	No	No	
No	Yes	No	
Yes	Yes	No	
No	No	Yes	
Yes	No	Yes	
No	Yes	Yes	
Yes	Yes	Yes	

3. Now consider pairs of interaction plots: first #1 and #2, then #3 and #4. Suppose that the first plot in each pair represents system response at the low level of factor C and the second represents system response at the high level of factor C. Do the pairs represent situations where there is non-zero ABC 3-factor interaction?

#1 and #2? Yes or No (circle one)      #3 and #4? Yes or No (circle one)

4. In a  $2^3$  full factorial study, four samples of size 3 and four of size 2 produce a pooled sample standard deviation of  $s_p = 3.2$ . How large does a fitted effect from the Yates algorithm have to be (in absolute value) in order to be judged “statistically detectable”? (You may use 95% confidence.)

5. Below are 8 sample means listed in Yates standard order for factors A, B and C.

a) Find the grand mean and fitted effects for the “all high” treatment combination ( $a_2, b_2, ab_{22}, c_2, ac_{22}, bc_{22}, abc_{222}$ ).

Mean

8

4

0

12

9

5

1

13

b) If the means above come from a complete  $2^3$  factorial study and only effects larger in magnitude than 1.5 are big enough to be considered to be statistically detectable, what do you estimate as a mean response for a situation where A is high, B is low, and C is high?

c) Suppose now that in fact the means above come from a  $2^{6-3}$  fractional factorial with generators  $D \longleftrightarrow AB, E \longleftrightarrow BC$  and  $F \longleftrightarrow AC$ . What is the simplest possible interpretation of the results of your calculations in a) in light of the criterion in b) that fitted effects larger in magnitude than 1.5 represent detectable (sums of) effects?

d) Write out the whole defining relation for the fractional factorial plan referred to in part c).

I  $\longleftrightarrow$        $\longleftrightarrow$        $\longleftrightarrow$        $\longleftrightarrow$        $\longleftrightarrow$        $\longleftrightarrow$        $\longleftrightarrow$