

IE 361 Module 6

Gauge R&R Studies Part 2: Two-Way ANOVA and Corresponding Estimates for R&R Studies

Reading: Section 2.2 *Statistical Quality Assurance for Engineers*
(Section 2.4 of Revised *SQAME*)

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ANOVA and R&R Analysis

The range-based Gauge R&R estimates of *SQAME* are fairly simple and serve the purpose of helping make the analysis goals easy to understand. But we have no good handle on how reliable these estimates are. In order to 1) produce Gauge R&R estimates that are typically better than range-based ones, and 2) produce confidence limits, we must instead use "ANOVA-based" estimates.

A careful treatment of ANOVA would require its own course. We'll simply make use of its main "output" and direct the interested student to books on engineering statistics (like Vardeman's *Statistics for Engineering Problem Solving*) for more details. The fact is that an $I \times J \times m$ data set of y_{ijk} 's like that produced in a typical Gauge R&R study is often summarized in a so-called ANOVA table. A generic version of such a table is

ANOVA and R&R Analysis

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Part	<i>SSA</i>	$I - 1$	$MSA = SSA / (I - 1)$
Operator	<i>SSB</i>	$J - 1$	$MSB = SSB / (J - 1)$
Part \times Operator	<i>SSAB</i>	$(I - 1)(J - 1)$	$MSAB = SSAB / (I - 1)(J - 1)$
Error	<i>SSE</i>	$IJ(m - 1)$	$MSE = SSE / IJ(m - 1)$
Total	<i>SSTot</i>	$IJm - 1$	

Any decent statistical package (and even EXCEL) will process a Gauge R&R data set and produce such a summary table. In this table the "mean squares" are essentially sample variances (squares of sample standard deviations). (*MSA* is essentially a sample variance of part averages, *MSB* is essentially a sample variance of operator averages, *MSE* is an average of within cell-sample variances, "*MSTot*" isn't typically calculated, but is a grand sample variance of all observations, ...) The mean squares indicate how much of the overall variability is accounted for by the various sources.

ANOVA and R&R Analysis

Example 6-1

We'll use the data set with $I = 4$, $J = 3$, $m = 2$ from the in-class R&R study (used as a numerical example in Module 5) to illustrate. The JMP data table and some screen shots for using the program to get the sums of squares follow.

	Part	Operator	Peanut Measurement
1	1	1	0.52
2	1	1	0.52
3	1	2	0.54
4	1	2	0.53
5	1	3	0.55
6	1	3	0.55
7	2	1	0.56
8	2	1	0.55
9	2	2	0.54
10	2	2	0.54
11	2	3	0.55
12	2	3	0.56
13	3	1	0.57
14	3	1	0.56
15	3	2	0.55
16	3	2	0.56
17	3	3	0.57
18	3	3	0.57
19	4	1	0.55
20	4	1	0.55
21	4	2	0.54
22	4	2	0.55
23	4	3	0.56
24	4	3	0.55

Figure: JMP Data Sheet for the In-Class R&R Study

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Example 6-1

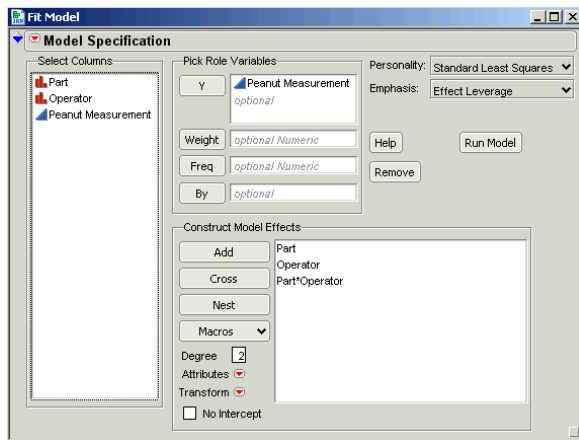


Figure: JMP Dialogue Box for Fit Model Two-Way ANOVA on the Gauge R&R Data

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Example 6-1

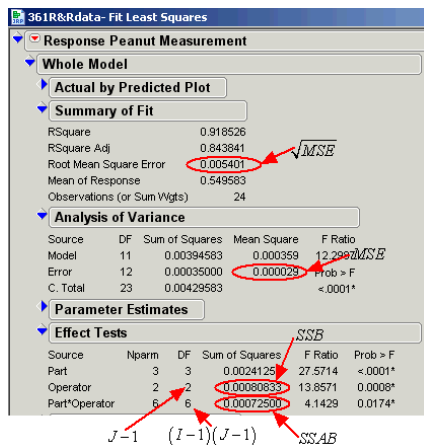


Figure: JMP Two-Way ANOVA Report for the In-Class Gauge R&R Study

ANOVA and R&R Analysis

Example 6-1

Although we certainly **don't** recommend using EXCEL (a spreadsheet is no substitute for a statistical package and, besides, EXCEL has terribly unreliable numerical analysis) we found instructions on using the program's two-way ANOVA plug-in at http://www.cvgs.k12.va.us/digstats/main/Guides/g_2anovx.html. Two screen shots from using these instructions on the in-class two-way data follow.

ANOVA and R&R Analysis

Example 6-1

The screenshot displays a Microsoft Excel spreadsheet with a data table and an open dialog box. The data table is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3			Operator 1	Operator 2	Operator 3							
4		Part 1	0.52	0.54	0.55							
5			0.52	0.53	0.55							
6		Part 2	0.56	0.54	0.55							
7			0.55	0.54	0.56							
8		Part 3	0.57	0.55	0.57							
9			0.56	0.56	0.57							
10		Part 4	0.55	0.54	0.56							
11			0.55	0.55	0.55							
12												
13												
14												
15												
16												

The 'Anova: Two-Factor With Replication' dialog box is open, showing the following settings:

- Input Range: R3C2:R11C5
- Rows per sample: 2
- Alpha: 0.05
- Output options: Output Range: R15C2, New Worksheet Ply, New Workbook

Figure: EXCEL Two-Way Data Spreadsheet for the In-Class R&R Study

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ANOVA						
Source of Variation	SS	df	MS	F	P-value	F crit
Sample	0.002412	3	0.000804	27.57143	1.15E-05	3.4903
Columns	0.000808	2	0.000404	13.85714	0.000761	3.88529
Interaction	0.000725	6	0.000121	4.142857	0.017388	2.996117
Within	0.00035	12	2.92E-05			
Total	0.004296	23				

Figure: EXCEL Two-Way Data Spreadsheet for the In-Class R&R Study

For our present purposes, we will take mean squares and degrees of freedom out of such an ANOVA table and make Gauge R&R estimates based on them. Point estimators for the quantities of most interest in a Gauge R&R study are partially summarized on the bottom of page 27 in *SQAME*. These are

$$\hat{\sigma}_{\text{repeatability}} = \hat{\sigma} = \sqrt{MSE}$$

and

$$\hat{\sigma}_{\text{reproducibility}} = \sqrt{\max\left(0, \frac{MSB}{ml} + \frac{(l-1)}{ml}MSAB - \frac{1}{m}MSE\right)}$$

Although it is not presented in *SQAME*, an appropriate estimator for $\sigma_{\text{R\&R}} = \sqrt{\sigma_{\gamma}^2 + \sigma_{\alpha\gamma}^2 + \sigma^2}$ (that is called σ_{overall} in *SQAME*) is

$$\hat{\sigma}_{\text{R\&R}} = \sqrt{\frac{1}{ml}MSB + \frac{l-1}{ml}MSAB + \frac{m-1}{m}MSE}$$

ANOVA and R&R Analysis

It is further possible to use these estimates to make an exact confidence interval for $\sigma_{\text{repeatability}} = \sigma$ and Satterthwaite approximate confidence limits for $\sigma_{\text{reproducibility}}$ and $\sigma_{\text{R\&R}}$. Let

$$v_{\text{repeatability}} = IJ(m - 1)$$

Then, confidence limits for $\sigma_{\text{repeatability}}$ are

$$\hat{\sigma}_{\text{repeatability}} \sqrt{\frac{v_{\text{repeatability}}}{\chi_{v_{\text{repeatability}}, \text{upper}}^2}} \quad \text{and} \quad \hat{\sigma}_{\text{repeatability}} \sqrt{\frac{v_{\text{repeatability}}}{\chi_{v_{\text{repeatability}}, \text{lower}}^2}}$$

For estimating $\sigma_{\text{reproducibility}}$, let

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$$\begin{aligned}\hat{\sigma}_{\text{reproducibility}} &= \frac{\hat{\sigma}_{\text{reproducibility}}^4}{\frac{\left(\frac{MSB}{ml}\right)^2}{J-1} + \frac{\left(\frac{(I-1)MSAB}{ml}\right)^2}{(I-1)(J-1)} + \frac{\left(\frac{MSE}{m}\right)^2}{IJ(m-1)}} \\ &= \frac{\hat{\sigma}_{\text{reproducibility}}^4}{\frac{1}{m^2} \left(\frac{MSB^2}{I^2(J-1)} + \frac{(I-1)MSAB^2}{I^2(J-1)} + \frac{MSE^2}{IJ(m-1)} \right)}\end{aligned}$$

Then approximate confidence limits for $\sigma_{\text{reproducibility}}$ are

$$\hat{\sigma}_{\text{reproducibility}} \sqrt{\frac{\hat{\sigma}_{\text{reproducibility}}}{\chi_{\hat{\sigma}_{\text{reproducibility}}, \text{upper}}^2}} \quad \text{and} \quad \hat{\sigma}_{\text{reproducibility}} \sqrt{\frac{\hat{\sigma}_{\text{reproducibility}}}{\chi_{\hat{\sigma}_{\text{reproducibility}}, \text{lower}}^2}}$$

For estimating $\sigma_{\text{R\&R}}$, let

$$\hat{\nu}_{R\&R} = \frac{\hat{\sigma}_{R\&R}^4}{\frac{\left(\frac{MSB}{ml}\right)^2}{J-1} + \frac{\left(\frac{(I-1)MSAB}{ml}\right)^2}{(I-1)(J-1)} + \frac{\left(\frac{(m-1)MSE}{m}\right)^2}{IJ(m-1)}}$$

$$= \frac{\hat{\sigma}_{R\&R}^4}{\frac{1}{m^2} \left(\frac{MSB^2}{I^2(J-1)} + \frac{(I-1)MSAB^2}{I^2(J-1)} + \frac{(m-1)MSE^2}{IJ} \right)}$$

then approximate confidence limits for $\sigma_{R\&R}$ are

$$\hat{\sigma}_{R\&R} \sqrt{\frac{\hat{\nu}_{R\&R}}{\chi_{\hat{\nu}_{R\&R}, \text{upper}}^2}} \quad \text{and} \quad \hat{\sigma}_{R\&R} \sqrt{\frac{\hat{\nu}_{R\&R}}{\chi_{\hat{\nu}_{R\&R}, \text{lower}}^2}}$$

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These formulas are tedious (but hardly impossible) to use with a pocket calculator. Or a very small program could be written to evaluate the estimates of standard deviations and approximate degrees of freedom. Vardeman uses a simple MathCAD worksheet to do the computing. The following figure illustrates the use of that worksheet beginning from SSB , $SSAB$, SSE , I , J , and m for the in-class R&R study.

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Example 6-1

$$I := 4$$

$$J := 3$$

$$m := 2$$

$$SSB := .00080833$$

$$SSAB := .000725$$

$$SSE := .00035$$

$$MSB := \frac{SSB}{(J-1)}$$

$$MSAB := \frac{SSAB}{(I-1) \cdot (J-1)}$$

$$MSE := \frac{SSE}{I \cdot J \cdot (m-1)}$$

$$a := \frac{MSB}{m \cdot I} + \frac{(I-1) \cdot MSAB}{m \cdot I} - \frac{MSE}{m}$$

$$\text{sighatrepro} := \sqrt{\left(\frac{a}{|a|} + 1\right) \cdot \frac{a}{2}}$$

$$\text{sighatRnR} := \sqrt{\frac{MSB}{(m \cdot I)} + \frac{(I-1) \cdot MSAB}{(m \cdot I)} + \frac{(m-1) \cdot MSE}{m}}$$

$$\text{nuhatrepro} := \frac{\text{sighatrepro}^4 \cdot m^2}{\frac{MSB^2}{I^2 \cdot (J-1)} + \frac{(I-1) \cdot MSAB^2}{I^2 \cdot (J-1)} + \frac{MSE^2}{I \cdot J \cdot (m-1)}}$$

$$\text{nuhatRnR} := \frac{\text{sighatRnR}^4 \cdot m^2}{\frac{MSB^2}{I^2 \cdot (J-1)} + \frac{(I-1) \cdot MSAB^2}{I^2 \cdot (J-1)} + \frac{MSE^2 \cdot (m-1)}{I \cdot J}}$$

$$MSB = 4.042 \times 10^{-4}$$

$$MSAB = 1.208 \times 10^{-4}$$

$$MSE = 2.917 \times 10^{-5}$$

$$\sqrt{MSE} = 5.401 \times 10^{-3}$$

$$\text{sighatrepro} = 9.014 \times 10^{-3}$$

$$\text{sighatRnR} = 0.011$$

$$\text{nuhatrepro} = 4.035$$

$$\text{nuhatRnR} = 7.452$$

Figure: MathCAD Worksheet for Example 6-1

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Example 6-1

The results in panels 6,9, and 15 show that 95% confidence limits for $\sigma_{\text{repeatability}}$ are

$$\hat{\sigma}_{\text{repeatability}} \sqrt{\frac{\nu_{\text{repeatability}}}{\chi_{\nu_{\text{repeatability}}, \text{upper}}^2}} \quad \text{and} \quad \hat{\sigma}_{\text{repeatability}} \sqrt{\frac{\nu_{\text{repeatability}}}{\chi_{\nu_{\text{repeatability}}, \text{lower}}^2}}$$

i.e.

$$.005401 \sqrt{\frac{4 \cdot 3 \cdot (2 - 1)}{23.337}} \quad \text{and} \quad .005401 \sqrt{\frac{4 \cdot 3 \cdot (2 - 1)}{4.404}}$$

i.e.

$$.0039 \text{ in} \quad \text{and} \quad .0089 \text{ in}$$

Similarly, approximate 95% confidence limits for $\sigma_{\text{reproducibility}}$ are

$$\hat{\sigma}_{\text{reproducibility}} \sqrt{\frac{\hat{\nu}_{\text{reproducibility}}}{\chi_{\hat{\nu}_{\text{reproducibility}}, \text{upper}}^2}} \quad \text{and} \quad \hat{\sigma}_{\text{reproducibility}} \sqrt{\frac{\hat{\nu}_{\text{reproducibility}}}{\chi_{\hat{\nu}_{\text{reproducibility}}, \text{lower}}^2}}$$

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Example 6-1

i.e.

$$.009014\sqrt{\frac{4}{11.143}} \quad \text{and} \quad .009014\sqrt{\frac{4}{.484}}$$

i.e.

$$.0054 \text{ in} \quad \text{and} \quad .0259 \text{ in}$$

And finally, approximate 95% confidence limits for $\sigma_{R\&R}$ are

$$\hat{\sigma}_{R\&R}\sqrt{\frac{\hat{v}_{R\&R}}{\chi_{\hat{v}_{R\&R}, \text{upper}}^2}} \quad \text{and} \quad \hat{\sigma}_{R\&R}\sqrt{\frac{\hat{v}_{R\&R}}{\chi_{\hat{v}_{R\&R}, \text{lower}}^2}}$$

i.e.

$$.011\sqrt{\frac{7}{16.013}} \quad \text{and} \quad .011\sqrt{\frac{7}{1.690}}$$

i.e.

$$.0073 \text{ in} \quad \text{and} \quad .0224 \text{ in}$$

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Example 6-1

These intervals show that none of these standard deviations are terribly well-determined (degrees of freedom are small and intervals are wide). If better information is needed, more data would have to be collected. But there is at least some indication that $\sigma_{\text{repeatability}}$ and $\sigma_{\text{reproducibility}}$ are roughly of the same order of magnitude. The caliper used to make the measurements was a fairly crude one, *and* there were detectable differences in the way the student operators used that caliper.

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Quite often industrial Gauge R&R studies are done to investigate the adequacy of a measuring device (and the operators that use it) to check conformance of items produced to *engineering specifications* (values that delineate limits of what is required of the item for it to be *functional*). Suppose that some feature of a product needs to have a value, x , that is at least L and no more than U in order for it to be functional. (L is the *lower specification* for x and U is the *upper specification*.) In this context, it is common to want to compare one's perception of the size of $\sigma_{R\&R}$ to "how tight L and U are." (For example, trying to compare x that can be seen only through a large amount of measurement noise to very tight specifications is a hopeless task.)

A way of quantifying this kind of comparison is this. If one thinks of measurement error as normally distributed, in the absence of (average across operators) measurement bias ($\mu_\delta = 0$), a measurement y made by a "randomly selected" operator in some sense represents x to within

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$\pm 3\sigma_{R\&R}$ and so $6\sigma_{R\&R}$ might be taken as a kind of measurement uncertainty. The difference $U - L$ represents the allowable variation in x . So the ratio

$$GCR = \frac{6\sigma_{R\&R}}{U - L}$$

is sometimes called a **Gauge Capability Ratio** or a **Precision to Tolerance Ratio** and used as an index of the adequacy of a measurement system to verify the functionality of product. Of course, this can only be estimated using the output of a Gauge R&R study, so an estimated version of this is

$$\widehat{GCR} = \frac{6\hat{\sigma}_{R\&R}}{U - L}$$

Notice too that having computed confidence limits for $\sigma_{R\&R}$, one needs only multiply these by

$$\frac{6}{U - L}$$

in order to produce confidence limits for GCR .

A common rule of thumb is that one needs to be fairly sure that $GCR < .1$ (and preferably that $GCR < .01$) before a gauge can be considered adequate for the purpose of checking conformance of x to specifications L and U .