

IE 361 Module 10

Introduction to Shewhart Control Charting

(Statistical Process Control, or More Helpfully: Statistical
Process *Monitoring*)

Prof.s Stephen B. Vardeman and Max D. Morris

Reading: Section 3.1, *Statistical Quality Assurance Methods for Engineers*

Generalities About Shewhart Control Charting

SPC (SPM) is process watching for purposes of change detection.

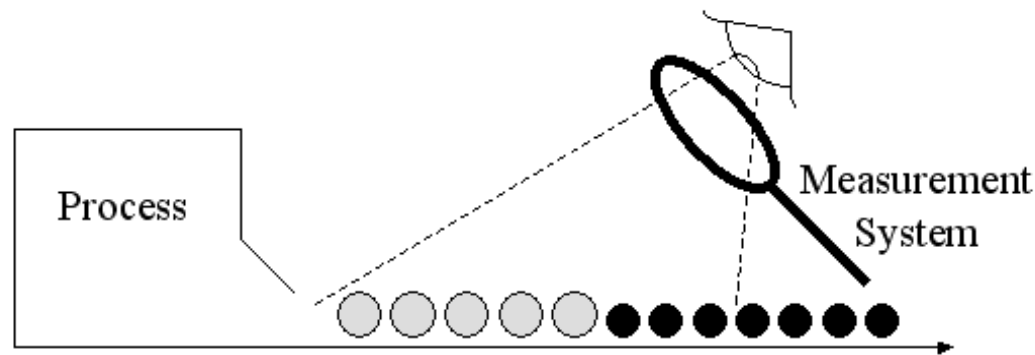


Figure 1: SPC (SPM) is About "Process Watching"

One famous statistician has called it "organized attention to process data."

Walter Shewhart, working at Bell Labs in the late 20's and early 30's reasoned that while some variation is inevitable in any real process, the variation seen in data taken on a process can be decomposed as

$$\text{overall observed variation} = \text{baseline variation} \\ + \text{variation that can be eliminated}$$

baseline	that which can be eliminated
inherent to a system configuration	
due to system/common/universal causes	due to special/assignable causes
random	nonrandom
short term	long term

If one accepts Shewhart's conceptualization how is one to detect the presence of the second kind of variation so that appropriate steps can be taken to eliminate it? The hope is to leave behind a process that might be termed physically stable (not without variation, but consistent in its pattern of variation). The point of Shewhart control charting is to provide a detection tool.

Shewhart's charting idea was to periodically take a sample from a process and

- compute the value of a statistic meant to summarize process behavior at the period in question
- plot against time order of observation
- compare to so-called **control limits**

For a generic statistic, Q , this looks like

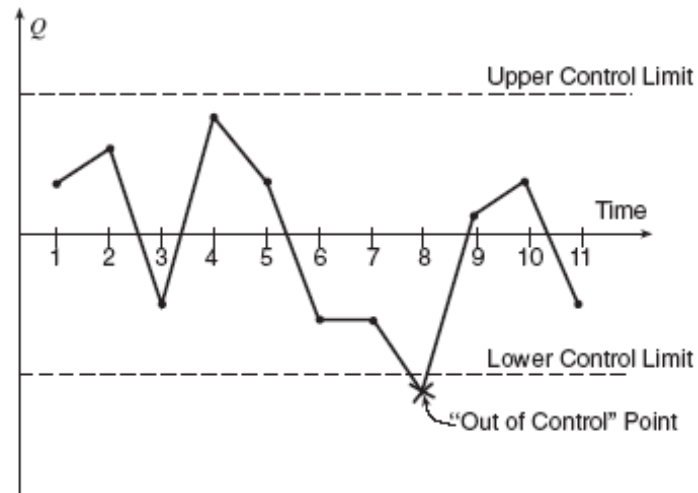


Figure 2: A Generic Shewhart Control Chart

Points plotting "out of control" indicate process change, i.e. the presence of variation of the 2nd kind, and signal the need for intervention (of some unspecified type) to find and take action on the physical source of any assignable cause.

There are many different kinds of Shewhart charts, corresponding to various choices of the plotted statistic, Q .

A basic question is how to set the control limits, UCL_Q and LCL_Q . Shewhart's answer was essentially:

If one models process output (individual measurements from the process) under stable conditions as random draws from a fixed distribution, then probability theory can often be invoked to produce a distribution

for Q and corresponding mean μ_Q and standard deviation σ_Q . For many distributions, most of the probability is within three standard deviations of the mean. So, if μ_Q and σ_Q are respectively a "stable process" mean and standard deviation for Q , common generic control limits are

$$UCL_Q = \mu_Q + 3\sigma_Q \quad \text{and} \quad LCL_Q = \mu_Q - 3\sigma_Q.$$

and it is common to draw a "center line" on a Shewhart control chart at

$$CL_Q = \mu_Q.$$

To make this concrete, consider the sample mean of n individual measurements, $Q = \bar{x}$. If individuals can be modeled random draws from a **process** distribution with mean μ and standard deviation σ , elementary probability implies that

$Q = \bar{x}$ has a distribution with mean $\mu_Q = \mu_{\bar{x}} = \mu$ and standard deviation $\sigma_Q = \sigma_{\bar{x}} = \sigma/\sqrt{n}$. It follows that typical control limits for \bar{x} are

$$UCL_{\bar{x}} = \mu + 3\frac{\sigma}{\sqrt{n}} \text{ and } LCL_{\bar{x}} = \mu - 3\frac{\sigma}{\sqrt{n}}$$

with a center line drawn at μ .

This illustrates that very often process parameters appear in formulas for control limits ... and values for them must come from somewhere. There are two possibilities:

- Sometimes past experience with a process, engineering standards, or other considerations made prior to a particular application specify what values should be used. (This is a **standards given** situation.)

- In other circumstances, one has no information on a process outside a series of samples that are presented along with the question "Is it plausible that the process was physically stable over the period represented by these data?" This is sometimes called an **as-past-data** or **retrospective** scenario. Here all that one can do is
 - tentatively assume that, in fact, the process was stable
 - make provisional estimates of process parameters and plug them into formulas for control limits, and
 - apply those limits to the data in hand as a means of criticizing the provisional assumption of stability

In a standards given context, with each new sample one faces the question

"Are process parameters currently at their standard values?"

In a retrospective context, one can only wait until a number of samples have been collected (often, using data from a minimum of 20–25 time periods is recommended) and then looking back over the data ask the question

"Are these data consistent with any fixed set of process parameters?"

The greatest importance of control charts is in standards given applications to real-time process monitoring. Textbooks, however, are full of problems involving retrospective charts. (Of course, what other kinds of data sets could be in a textbook???)

Example 10-1 (The "Brown Bag" Process) The population of numbers on washers used in the "Deming Drama" in IE 361 Labs is (approximately) Normal with mean $\mu = 5$ and standard deviation $\sigma = 1.715$. If (as in the Deming Drama) one is sampling $n = 5$ washers per period and doing process monitoring with the hope of maintaining the "brown bag parameters," appropriate control limits for \bar{x} are

$$UCL_{\bar{x}} = 5 + 3\frac{1.715\sigma}{\sqrt{5}} = 7.3 \text{ and } LCL_{\bar{x}} = 5 - 3\frac{1.715}{\sqrt{5}} = 2.7$$

and these are to be applied in an "on-line" fashion to sample means as they are observed.

As a first illustration of retrospective calculations for Shewhart charts, the following table shows sample means and ranges for 15 samples collected in a Deming Drama. The first 12 of those samples were drawn from the brown bag and the last 3 from a much different population.

Sample	1	2	3	4	5	6	7	8	9
\bar{x}	5.4	6.2	6.0	5.6	4.4	3.4	4.0	4.8	4.4
R	3	5	5	6	6	5	2	2	3

Sample	10	11	12		13	14	15
\bar{x}	6.2	5.6	4.8		8.0	9.6	12.8
R	2	6	4		10	9	5

Note, by the way, that all 12 of the samples actually drawn from the brown bag have sample means inside the control limits for \bar{x} . Only after the process change between samples 12 and 13 (indicated by the vertical double line in the table) do the \bar{x} 's begin to plot outside the standards given control limits 2.7 and 7.3.

But what could one do in the way of control charting if the brown bag parameters were not provided? Is it possible to look at the 13 sample means and ranges

in the table above and conclude that there was clearly some kind of process change over the period of data collection? One might reason (retrospectively) as follows.

IF in fact there had been no process change, all 15 of the sample means \bar{x} would be estimates of μ (that we know to be 5 for the first 12 samples). So a reasonable empirical replacement for μ in control limit formulas might be

$$\bar{\bar{x}} = \frac{5.4 + 6.2 + \cdots + 12.8}{15} = 6.08$$

Further (in a manner similar to what we did in the range-based estimation of $\sigma_{\text{repeatability}}$ in Gauge R&R analyses), one might invent an estimate of σ from each of the 15 ranges in the table, as $\hat{\sigma} = R/d_2$, where d_2 is based on the sample size of $n = 5$. Supposing again that there had been no process change in the data collection, these might be averaged across the 15 samples

to produce

$$\frac{\bar{R}}{d_2} = \frac{(3 + 5 + \cdots + 9 + 5) / 15}{2.326} = 2.092$$

as a potential empirical replacement for σ in control limit formulas. That is, as-past-data control limits for \bar{x} in the Deming Drama might have been

$$CL_{\bar{x}} = \hat{\mu} = 6.08$$

$$\begin{aligned} UCL_{\bar{x}} &= \hat{\mu} + 3 \frac{\hat{\sigma}}{\sqrt{n}} \\ &= 6.08 + 3 \frac{2.092}{\sqrt{5}} = 8.9 \end{aligned}$$

and

$$LCL_{\bar{x}} = 6.08 - 3 \frac{2.092}{\sqrt{5}} = 3.3$$

If these limits are applied to the 15 sample means in the table, we see that (although the change between samples 12 and 13 is not caught until sample 14) even though there are 3 samples from a process different from the brown bag process that "contaminate" our estimates of the brown bag parameters, this method of retrospective calculation does detect the fact that there was some kind of process change in the period of data collection.

A matter that needs some general introductory discussion is that of **rational subgrouping** or rational sampling. It is essential that anything one calls a single "sample" be collected over a short enough period that there is little question that the process was "physically stable" during that period. It must be clear that a "random draws from a fixed distribution" model is appropriate for describing data in a given sample. This is because the variation within such a sample essentially specifies the level of "background noise" against which one looks for process changes. If what one calls "samples" often contain data from

genuinely different process conditions, the level of background noise will be so large as to mask important process changes.

In high-volume manufacturing applications of control charts, single samples (rational subgroups) typically consist of n consecutive items taken from a production line. On the other hand, in extremely low-volume (or batch) operations, where one unit of product might take many hours to produce and there is significant opportunity for real process change between consecutive units, the only natural samples may be of size $n = 1$.

An unfortunate and persistent type of confusion concerns the *much different* concepts of control limits and engineering specifications.

Control Limits	Specifications
have to do with process stability	have to do with product acceptability
apply to \bar{Q}	apply to individuals, x
usually derived from process data	derived from performance requirements

A real process can be stable without being acceptable and vice versa. There is not a direct link between being "in control" and producing acceptable results.

Example 10-1 continued The Deming Drama as typically run in an IE 361 Lab uses the understanding that washers from $L = 3$ to $U = 7$ are "in specifications"/functional, while those outside those limits are not. This has nothing whatsoever to do with either the truth about the brown bag process or any data collected in the Drama. In fact, using the process parameters one might compute

$$z_1 = \frac{U - \mu}{\sigma} = \frac{7 - 5}{1.715} = 1.17$$

and

$$z_2 = \frac{L - \mu}{\sigma} = \frac{3 - 5}{1.715} = -1.17$$

and note that the fact that

$$P[-1.17 \leq \text{a standard normal random variable} \leq 1.17] \approx .76$$

suggests that only about 76% of the brown bag process actually meets those specifications/requirements. (We are here ignoring the fact that the basic discreteness of what is drawn from the brown bag serves to make this estimate lower than the truth.) And, of course, lacking known values for μ and σ , one could use past data to estimate μ and σ and then this kind of *fraction conforming* for the brown bag process.

Control charting can signal the need for process intervention and can keep one from ill-advised and detrimental over-adjustment of a process that is behaving in a stable fashion. But in doing so, what is achieved is simply reducing variation to the minimum possible for a given system configuration (in terms of equipment, methods of operation, methods of measurement, etc.). That achieving of

process stability provides a solid background against which to evaluate possible innovations and fundamental/order-of-magnitude improvements in production methods.