

IE 361 Lab#3/ ANOVA-Based Gauge R&R Calculations

_____ (Name)

Attached to this lab is a JMP report for a Fit Model two-way ANOVA analysis of a standard "Gauge R&R" data set. (The data come from an IE 361 measurement study that was done using the same calipers and Styrofoam peanuts you used in Labs #1 and #2.) Use it and the Formula Sheet also provided to complete the calculations in this lab (covering the content of Module 6). **After the in-class lab period, run JMP and get your own two-way ANOVA analysis and staple it to this lab before turning the lab in.**

How many peanuts were used in this study? $I =$ _____

How many operators were involved in the study? $J =$ _____

How many times did each operator measure each part? $m =$ _____

The units of measurement were (as in Labs #1 and #2) *inches*. Be sure to attach units to answers that you give in this lab.

Give a single-number estimate of the standard deviation that would be experienced by a single operator repeatedly remeasuring the same peanut.

What degrees of freedom should be associated with the estimate above?

Use the previous two answers and make 95% two-sided confidence limits for the standard deviation that would be experienced by a single operator repeatedly remeasuring the same peanut.

Give a single-number estimate of the standard deviation that would be experienced by many operators measuring the same peanut once each *in the absence of repeatability variation*.

What approximate degrees of freedom should be associated with the estimate above?

Use the previous two answers and make 95% two-sided confidence limits for the standard deviation that would be experienced by many operators measuring the same peanut once each *in the absence of repeatability variation*.

Give a single-number estimate of the standard deviation that would be experienced by many operators measuring the same peanut once each.

What approximate degrees of freedom should be associated with the estimate above?

Use the previous two answers and make 95% two-sided confidence limits for the standard deviation that would be experienced by many operators measuring the same peanut once each.

In light of the inferences on pages 1 through 3, how do the "repeatability" and "reproducibility" components of measurement variability seem to compare? If this were a real industrial situation and you were responsible for improving measurement consistency, what would you do first here?

Suppose for sake of illustration that engineering specifications on the actual sizes of these peanuts are from .49 inch to .51 inch.

Based on your first answer on page 3, what do you estimate to be the gauge capability ratio for calipers used in this study for purposes of checking conformance to these specifications. That is, estimate $GCR = 6\sigma_{R\&R} / (U - L)$.

Use your last answer on page 3 and make approximate 95% confidence limits for the gauge capability ratio (the "precision to tolerance" ratio).

If this were a real industrial situation, would you consider the calipers adequate to check conformance of the peanuts to the engineering specifications? Explain.

	Part	Operator	Measurement
1	1	1	0.53
2	1	1	0.51
3	1	2	0.52
4	1	2	0.53
5	1	3	0.56
6	1	3	0.55
7	2	1	0.55
8	2	1	0.55
9	2	2	0.57
10	2	2	0.57
11	2	3	0.58
12	2	3	0.58
13	3	1	0.58
14	3	1	0.57
15	3	2	0.58
16	3	2	0.58
17	3	3	0.64
18	3	3	0.62
19	4	1	0.59
20	4	1	0.58
21	4	2	0.57
22	4	2	0.58
23	4	3	0.59
24	4	3	0.57

361GR&RS04 - Fit Least Squares			
Response Measurement			
Whole Model			
Actual by Predicted Plot			
Summary of Fit			
RSquare	0.956326		
RSquare Adj	0.916292		
Root Mean Square Error	0.008416		
Mean of Response	0.56875		
Observations (or Sum Wgts)	24		
Analysis of Variance			
Source	DF	Sum of Squares	Mean Square
Model	11	0.01861250	0.001692
Error	12	0.00085000	0.000071
C. Total	23	0.01946250	
			F Ratio
			23.8877
			Prob > F
			<.0001*
Parameter Estimates			
Effect Tests			
Source	Nparm	DF	Sum of Squares
Part	3	3	0.01244583
Operator	2	2	0.00377500
Part*Operator	6	6	0.00239167
			F Ratio
			58.5686
			26.6471
			5.6275
			Prob > F
			<.0001*
			<.0001*
			0.0055*
Residual by Predicted Plot			

Formulas for ANOVA-Based Gauge R&R Estimation

We consider a standard Gauge R&R data set consisting of m (repeat) measurements of I different parts by each of J operators. This sheet of formulas summarizes the computations discussed on the slides for IE 361 Module 6. The two-way random effects model is

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}$$

and the quantities of most obvious interest are

$$\sigma_{\text{repeatability}} = \sigma$$

$$\sigma_{\text{reproducibility}} = \sqrt{\sigma_{\beta}^2 + \sigma_{\alpha\beta}^2}$$

and

$$\sigma_{\text{R\&R}} = \sqrt{\sigma_{\text{reproducibility}}^2 + \sigma^2} = \sqrt{\sigma_{\beta}^2 + \sigma_{\alpha\beta}^2 + \sigma^2}$$

Estimates of these are:

$$\hat{\sigma}_{\text{repeatability}} = \hat{\sigma} = \sqrt{MSE}$$

$$\hat{\sigma}_{\text{reproducibility}} = \sqrt{\max\left(0, \frac{MSB}{mI} + \frac{(I-1)MSAB}{mI} - \frac{1}{m}MSE\right)}$$

and

$$\hat{\sigma}_{\text{R\&R}} = \sqrt{\frac{1}{mI}MSB + \frac{I-1}{mI}MSAB + \frac{m-1}{m}MSE}$$

Corresponding (exact and approximate) degrees of freedom are

$$\nu_{\text{repeatability}} = IJ(m-1)$$

$$\hat{\nu}_{\text{reproducibility}} = \frac{\hat{\sigma}_{\text{reproducibility}}^4}{\frac{1}{m^2} \left(\frac{MSB^2}{I^2(J-1)} + \frac{(I-1)MSAB^2}{I^2(J-1)} + \frac{MSE^2}{IJ(m-1)} \right)}$$

and

$$\hat{\nu}_{\text{R\&R}} = \frac{\hat{\sigma}_{\text{R\&R}}^4}{\frac{1}{m^2} \left(\frac{MSB^2}{I^2(J-1)} + \frac{(I-1)MSAB^2}{I^2(J-1)} + \frac{(m-1)MSE^2}{IJ} \right)}$$

to be used with the basic formulas for confidence limits

$$\hat{\sigma} \sqrt{\frac{\nu}{\chi_{\text{upper}}^2}} \quad \text{and} \quad \hat{\sigma} \sqrt{\frac{\nu}{\chi_{\text{lower}}^2}}$$