

IE 361 Lab#11/ Analysis of 2^p Factorial and 2^{p-q} Fractional Factorial Data

(Name)

An example of a partially replicated 2^3 factorial experiment due originally to R. Snee and appearing in *Engineering Statistics* by Hogg and Ledolter concerns the effects of the three two-level factors

Factor	Levels
A-Polymer Type	Standard (-) vs New (But Expensive) (+)
B-Polymer Concentration	.01% (-) vs .04% (+)
C-Amount of an Additive	2 lb (-) vs 12 lb (+)

on the response variable

$$y = \text{percentage impurity produced}$$

by a chemical process. Below are Snee's data.

Combination	y	Combination	y
(1)	1.0	c	.9,.7
a	1.0,1.2	ac	1.1
b	.2	bc	.2,.3
ab	.5	abc	.5

a) Compute and record the sample means and sample standard deviations for the 8 samples.

Combination	\bar{y}	s
(1)		
a		
b		
ab		

Combination	\bar{y}	s
c		
ac		
bc		
abc		

b) What is an estimate of the standard deviation of percentage impurity that should be associated with any single process set-up (combination of levels of the 3 factors) and its associated degrees of freedom?

$s_{\text{pooled}} =$ _____ $d.f. =$ _____

c) Use the Yates algorithm and compute fitted factorial effects corresponding to the "all high" treatment combination.

d) There is replication here, so that the statistical detectability of the factorial effects can be judged using "margins of error" derived from confidence limits for effects. Using 95% two-sided confidence limits, which of the 2^3 factorial effects can be judged as "clearly larger than background variation"?

e) Supposing only the grand mean and those effects you identified in part d) are important, what do you estimate to be the mean percentage impurity produced when first all factors are at their high levels (combination abc is used in the running of the process), and then when all factors are at their low levels (combination (1) is used in the running of the process)?

$$\hat{y}_{abc} = \underline{\hspace{10em}}$$

$$\hat{y}_{(1)} = \underline{\hspace{10em}}$$

Suppose now, for sake of illustration, that because of resource constraints only combinations a,b,c, and abc had been run in the original study (this is a half fraction of the full factorial structure). This would have produced data as below. Copy the corresponding sample means and standard deviations from page 1 into the table below.

Combination	y	\bar{y}	s
c	.9,.7		
a	1.0,1.2		
b	.2		
abc	.5		

f) Based on these 4 samples, what does one have for an estimate of the standard deviation of percentage impurity that should be associated with any single process set-up (combination of levels of the 3 factors) and its associated degrees of freedom?

$s_{\text{pooled}} =$ _____ d.f. = _____

g) Note that if one ignores which level of Factor C is involved, the combinations in the table above are listed in Yates standard order as regards factors A and B. Do the (2-cycle) Yates algorithm on the 4 sample means here to get 4 estimated sums of effects.

h) Find a "margin of error" based on 95% confidence limits to apply to the 4 values you computed in g). By this standard, which of the 4 values computed in g) are "clearly bigger than experimental noise"?

The "generator" used to pick the 4 combinations a,b,c, and abc was in fact

$$C \leftrightarrow AB$$

i) Verify the above statement by writing out the 4 combinations of plus and minus signs specifying all 4 combinations of levels of Factors A and B and "multiplying the A and B signs" to get a sign for a level of C. For each row of your table, give the "special 2^3 factorial name" for the combination indicated by the signs in the row.

A sign	B sign	AB Product (C sign)	2^3 Combination Name

j) Multiply the "generator" for this 2^{3-1} study through by C to get the so called "defining relation" for this half fraction of the 2^3 (that indicates what is "aliased with" the grand mean). Use this to say what effects are aliased with the grand mean, the A main effect, the B main effect, and the C main effect.

aliased with the grand mean _____ aliased with the A main effect _____

aliased with the B main effect _____ aliased with the C main effect _____

k) What, in light of your answer to j) is the simplest possible interpretation of your results in h)? Is this interpretation the same as your answer to part d)?