

Practice Problems

Problem 1. Consider an n -firm homogenous product industry in which the i th firm produces output q_i with marginal cost c and faces inverse demand $p = \alpha - \beta Q$, where $Q = q_1 + \dots + q_n$.

1(a) Find the Cournot equilibrium. What happens as $n \rightarrow \infty$?

1(b) Would two of the n firms agree to merge and form a single firm with marginal cost c ?

From now on assume that $n = 4$. Also, a firm that produces positive output incurs both the marginal cost c of producing output and a fixed cost $F = \frac{2}{9}(\alpha - c)^2/\beta$, i.e. its total costs are $c_i(q_i) = F + cq_i$.

1(c) What are the pure-strategy Cournot equilibria? *Hint: There are two kinds. Do not just consider symmetric equilibria.*

1(d) Are there any mixed strategy equilibria?

1(e) * Find all the mixed strategy equilibria.

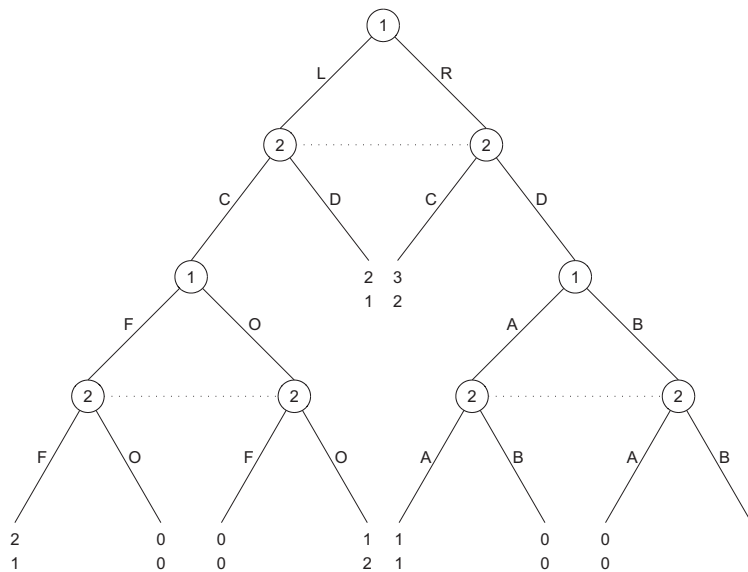
Problem 2. Consider the following two player game of incomplete information played by friends A and B on the telephone who live twenty miles away from each other. The radio correctly predicts that it will rain on at least one house. It will either rain on both houses (chance $1/3$), on just A's house (chance $1/3$), or just B's house (chance $1/3$). Each player observes the weather outside his door (rainy or dry), but has no other information about his neighbour's weather. They then play the game of chicken below.

	c	t
c	5,5	2,6
t	6,2	1,1

2(a) Prove: It is a Bayesian Nash equilibrium for each player to play chicken when it rains, and play tough otherwise.

2(b) Show that the expected value of the sum of both players' payoffs in this BNE is greater than the best Nash equilibrium payoff.

Problem 3. Find *all* SPE in the following extensive form game.



Problem 4. This question is about venture capital funding. A risk neutral entrepreneur E knows the value V of a potential project. Specifically, E learns either $V = L$ or $V = H$, with $H > L$. The chance $p \in (0, 1)$ of $V = H$ is common knowledge. The project needs a seed money investment of $I > 0$ dollars. A risk-neutral venture capitalist C has the I dollars, and wishes to invest wisely. After learning his type, E 's action is the equity stake $e \in [0, 1]$ to give to C . Then C either declines to fund the project, and earns a return i on his money, or accepts the project. Thus the payoffs to (E, C) are $((1 - e)V, eV - I)$ if C accepts, and if the project pays out V . If C declines, the payoffs are $(0, (1 + i)I)$.

4(a) Carefully depict this as an extensive form.

4(b) Suppose that when C moves, his posterior on $V = H$ is q . What is the condition on q required for him to accept offer e ?

4(c) Assume $L - I > (1 + i)I$. Find all Perfect Bayesian equilibria.