Solutions to Practice Problems

Problem 1.

1(a) The Cournot equilibrium is $q_i = \frac{\alpha - c}{(n+1)\beta}$ and firm profits are $\pi_i = \frac{(\alpha-c)^2}{(n+1)^2\beta}$. As $n \to \infty$ the equilibrium prices tends to the marginal cost c, and firm and industry profits tend to 0.

1(b) Firms would only want to merge in the case $n = 2$. Thus a partial monopoly is not attractive reason to merge.

1(c) The hint was a bit misleading because the exercise had a typo. It should have read $F = \left[\frac{2}{9}(\alpha - c)\right]^2/\beta$. In this case there are pure equilibria with one and two firms active. Because I put the bracket incorrectly, there are only equilibria with a single firm active. You need to check that the active firms play Cournot and therefore wouldn't want to deviate. However, you also need to check that the inactive firms would not want to enter. It is NOT enough here that two firm Cournot does not cover fixed cost! Assume that firm 1 enters in the NE with quantity $q_1 = \frac{\alpha - c}{2\beta}$. If firm $i > 1$ deviates and enters its best response output would be $q_i = \frac{\alpha - c - \beta q_1}{2\beta}$ and its profit would be $\pi_i = \frac{(\alpha-c)^2}{16\beta} < F$. Note, that its profit is even lower than its profit under two-firm Cournot.

1(d) Yep. The easiest way to find a specific NE is to look at the symmetric mixed equilibrium where all four firms enter with probability p and produce output q. Or you do the more elaborate procedure of the next part.

1(e) This is not an extensive-form game where firms first observe how many rivals entered and then play Cournot.

Note, that each individual firm now maximizes its expected profit which is:

$$\pi_i = q_i \left(\alpha - c - \beta q_i - \overline{q}_{-i}\right)$$

The bar indicated that we take the expectation over other firms' outputs. You can find a unique BR $q_i^* = \frac{\alpha - c - \beta \overline{q}_{-i}}{2\beta}$. Therefore, agents will never randomize over two positive output levels. The only possible mixed equilbria has firms randomize over producing zero output or q_i^* (or stay out completely). They have to be indifferent between the two - therefore their profits have to equal the fixed cost F such that they make zero profits on both. Assume n firms randomize in the mixed equilibrium and $4 - n$ firms stay out completely. For each randomizing firm we have therefore two equations (BR and $\pi = F$) and two unknowns - the probability p of choosing positive output and the output level p itself. This equation system has typically at most one solution. We try to find a symmetric equilibrium for each n - if it exists we have found all the mixed equilibria.

So let's assume that n firms choose q with probability p. Then we can calculate $q = \frac{\alpha - c}{\beta(2+p(n-1))}$. Profits for each firm become $\pi = \frac{(\alpha-c)^2}{\beta(2+p(n-1))^2}$. This profit has to be equal to the fixed cost F and we obtain:

$$\frac{1}{(2+p(n-1))^2} = \frac{2}{9}$$

This gives us:

$$p(n-1) = \frac{3}{\sqrt{(2)}} - 2 \approx 0.1213$$

Therefore we $n > 1$ we can calculate a mixed equilibrium. There are 6 mixed equilibria with $n = 2$, 4 with $n = 3$ and 1 with $n = 4$ firms randomizing.

Hence there are exactly 11 mixed equilibria and four pure strategy equilibrium (with $n = 1$ active firms).

Problem 2.

2(a) The conditional probability of rain at the other place is 1 if it's sunny at A's house and is $\frac{1}{2}$ if it's rainy at A's house (since it always rains at at least one place). The expected utility from playing chicken when it rains is 3.5 and the expected utility from playing tough is 3.5. Hence playing chicken is one optimal strategy. When it's sunny the player knows that the other player will play chicken (since it rains there) and her utility is 6 versus 5. Therefore in both states the prescribed strategy is optimal.

2(b) With $\frac{1}{3}$ probability it rains at both places and they both play chicken and get 10 jointly. With $\frac{1}{3}$ probability it only rains at A's place and they will jointly get 8. The same occurs when it only rains at B's place. Hence the expected joint surplus is $\frac{26}{3}$. The two pure NE (tough/chicken and chicken/tough) have joint payoff 8. In the mixed equilibrium both players randomize with equal probability over both strategies and get each 3.5 or 7 jointly. Hence, the BNE dominates all three NE (in terms of joint surplus). Intuitively, this occurs because players sometimes play both chicken (which maximizes joint surplus but is not a NE). This also occurs in the MNE but in this case players sometimes mis-coordinate and both choose tough.

Problem 3. This question is intended as an easy way to collect points. The game has two non-trivial subgames - a coordination game and a BoS. The coordination game has Nash outcome (1,1) and (.5, .5) and the BoS has two pure and a mixed outcome. We can replace the subgames through these payoffs and get six possible simultaneous move games. Each of them has either 1 or three solutions which can be calculated. There is one exception though for the following game (generated if (F,F), and (A,A) or (B,B) are played in the subgames):

	C	D
L	2,1	2,1
R	3,2	1,1

The two pure NE in this game are (R,C) and (L,D). But the game has a continuum of mixed equilibria $(L, \beta C + (1-\beta) D)$ for $\beta < \frac{1}{2}$. Note, that the game is not generic because two boxed in the matrix are identical (hence the "odd number of NE equilbria" result does not apply).

Problem 4.

4(a) This is a signalling game where the equity stake e serves as the costly signal to the entrepreneur.

4(b) The condition is $e(qH + (1-q)L) - I > 0$ which simplifies to $q > \frac{\frac{I}{e} - L}{H - L}$.

4(c) Let's look for a separating equilibrium first. In this case the entrepreneur will either offer e^H (when the state is high) or $e^L \neq e^H$ (when the state is low). Assume that C accepts e^H but rejects e^L. In this case the low type would imitate the high type unless $e^H = 1$ (in which case there is a trivial separating equilibrium where C gets everything). Assume that C rejects e^H but accepts e^L. In this case the high type would want to imitate the low type unless $e^L = 1$. So there are two separating equilibria where either the low type or the high type offer everything, the other type offers some distinct stake (which is rejected).

What about pooling equilibria? In this case both types offer an equity stake e^*. The capitalist's posterior belief is the same as her prior beliefs when observing e^*. The stake has to be accepted (because deviating and offering a sufficiently high stake will always be accepted regardless of beliefs due to the condition $L - I > (1 + i)I$). She could offer a lower stake. The easiest way to make such a deviation unprofitable is to assign low beliefs to such a deviation. Therefore, there is a range of pooling equilibria for $e \geq \frac{I}{(H-L)p+L}$.