Final Exam

Welcome to Game Theory Final Exam! Before you proceed, please write your name above. Please note that the exam is long and will be graded on a curve. Attempt to complete as many problems as possible. Note that questions vary in difficulty. If you get stuck on a question, leave it and come back to it later. Harder questions are marked with a star (*). Good luck!

> "I am familiar with Iowa State regulations regarding academic dishonesty and pledge not to violate them."

Signature:

Problem 1. Two drivers are side-by-side in adjacent lanes on the highway. The two lanes are continually narrowing and the two cars getting closer and closer together as they approach a construction site. If neither driver slows down and merges in behind the other they will crash at some point within the next minute.

1(a) To model this situation as a simple normal form game, suppose each player simultaneously chooses the time $t_i \in [0, 1]$ at which he or she will slow down if the other driver has not yet done so. Assume that each player i gets a payoff of -1 if the cars crash, 0 if they do not crash and player i ends up behind the other player, and 1 if they do not crash and player i ends up in front of the other player. Assume that the probability that the cars crash is $\min(t_1, t_2)$, i.e. it is as if the time at which a crash occurs would be uniform on $[0, 1]$ if neither car ever slows down. Show that the game has an infinite number of asymmetric pure strategy Nash equilibria. If you can, show that there are no other pure strategy Nash equilibria.

1(b) * Find a symmetric mixed strategy Nash equilibrium in which the players mix on some interval $[0, \overline{t}]$.

Problem 2. Consider the following multistage game. Player 1 first has to choose how to divide \$2 between himself and player 2 (with only integer divisions being possible). Both players observe the division, and they then play the simultaneous move game with the dollar payoffs shown below.

	A	B	C
U	x,x	0,0	−2,−2
D	0,0	1,1	−2,−2

Assume that each player is risk neutral and has utility equal to the sum of the number of dollars he or she receives in the divide the dollar game and the dollar payoff he receives in the second stage game.

2(a) Draw a tree diagram to represent the extensive form of this game. How many pure strategies does each player have in the normal form representation of this game?

2(b) Show that for any x the game has a Nash equilibrium in which player chooses to give both dollars to player 2 in the initial divide-the-two-dollars game.

2(c) For what values of x will the game have an unique subgame perfect equilibrium?

2(d) For what values of x is there a subgame perfect equilibrium in which player 1 gives both dollars to player 2 in the initial divide-the-two-dollars game.

2(e) Can the game have a subgame perfect equilibrium in which player 1s total payoff is less than 2?

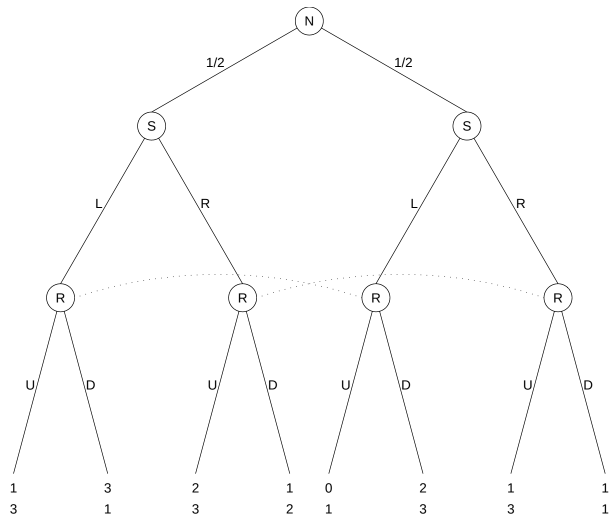
Problem 3. You and your best friend are trying to meet for lunch. At twelve oclock each of you must go to one of two places: Stomping Grounds or Thai Kitchen which happens to be close to your friend's house. Because you need to talk about something, assume that you each receive a benefit of $\frac{2}{3}$ utils if you meet. Assume also that each of you incurs a disutility cost of w from waiting to be served at Stomping Grounds at lunch time. Because you go to Stomping Grounds often, you know the wait time at Stomping Grounds, i.e. you know w, while your friend's prior is that w is distributed uniformly on [0, 1]. While it is common knowledge that you are completely indifferent between the food choices at each place, assume that your friend incurs a disutility of $3d$ if he eats at Thai Kitchen. The strength of your friend's dislike for Thai Kitchen varies from day to day assume that your friend knows d while your prior is that d is distributed uniformly in $[0, 1]$. To summarize the payoffs, when the wait at Stomping Grounds is w and your friend's disutility from going to Thai Kitchen is $3d$ the payoffs in the game are (with you as player 1 - strategies 'Stomping Grounds' and 'Thai Kitchen' are indicated by S and T respectively.)

	S	T
S	2/3−w,2/3−w	−w,−3d
T	0,−w	2/3,2/3−3d

3(a) How would you specify this game formally as a Bayesian game? In particular, what would the sets of types be and how would the payoff functions depend on the types? What form would you expect the equilibrium strategies to take?

3(b) Find the Bayesian Nash equilibrium of this game. How often do you and your friend have lunch together?

Problem 4. Find all the pure-strategy PBE of the following extensive form game with incomplete information.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