1. The windchill $W$ depends on the speed $v$ of the wind and the actual temperature $T$. It is given by the formula

$$W(v, T) = 35.74 + 0.6215T - 35.75(v^{0.16}) + 0.4275T(v^{0.16})$$

where $T$ is the air temperature in degrees Fahrenheit,

$v =$ wind speed (in mph)

$W$ is the windchill temperature in degrees Fahrenheit.

(a) Tell the windchill if the true temperature is 35 degrees F and the wind is 10 mph.
(b) Tell the windchill if the true temperature is 35 degrees F and the wind is 20 mph.
(c) Tell the windchill if the true temperature is 5 degrees F and the wind is 10 mph.
(d) Tell the windchill if the true temperature is 5 degrees F and the wind is 20 mph.
(e) Tell the windchill if the true temperature is 35 degrees F and the wind is 0 mph.

Answers:
(a) 27.4 degrees F
(b) 23.9 degrees F
(c) -9.7 degrees F
(d) -15.4 degrees F
(e) 57.5 degrees F

2. Recall that the body mass index is given by the formula

$$B(w,h) = \frac{703 \, w}{h^2}$$

The units of $B$ are kg/m$^2$ when $w$ is the weight in pounds and $h$ is the height in inches.

(a) Tell the body mass index of a man who is 6 feet 2 inches tall and weighs 180 pounds.
(b) Tell the body mass index of a man who is 5 feet 9 inches tall and weighs 300 pounds.
(c) Tell the body mass index of a man who is 5 feet 10 inches tall and weighs 120 pounds.

Using the chart tell how the weight is classified.

Answers:
(a) 23.1 kg/m$^2$  normal
(b) 44.3 kg/m$^2$  obese
(c) 17.2  underweight

3. The temperature $T$ at $(x,y,z)$ is

$$T(x,y,z) = 2x^2 + 2y^2 + z^2$$

degrees C.

Find the temperature at the point $(1, 2, 3)$.

Answer: 19 degrees C.

4. Tell the rate of flow of blood in a blood vessel that has radius 0.15 cm, length 3 mm, with a pressure change of 15 mmHg, and viscosity 0.005 Pascal sec.

Answer:
Hence $V = 265$ cc/sec.

5. Draw a contour map for the function $f(x,y) = x+y$.

6. Draw a contour map for the function $f(x,y) = 2x + y$.

7. Draw a contour map for the function $f(x,y) = x \cdot y$. 
1. If \( f(x,y) = 4x^5y^3 \) find
   
   (a) \( \frac{\partial f}{\partial y}(x,y) \)  
   (b) \( \frac{\partial f}{\partial x}(x,y) \)  
   (c) \( \frac{\partial f}{\partial x}(1,2) \)  
   (d) \( \frac{\partial f}{\partial y}(1,2) \)  
   (e) \( \frac{\partial^2 f}{\partial y^2}(x,y) \)  
   (f) \( \frac{\partial^2 f}{\partial x^2}(x,y) \)  
   (g) \( \frac{\partial^2 f}{\partial x \partial y}(x,y) \)  
   (h) \( \frac{\partial^2 f}{\partial x^2}(1,2) \)  
   (i) \( \frac{\partial^2 f}{\partial x \partial y}(1,2) \)  
   (j) \( \frac{\partial^2 f}{\partial y^2}(1,2) \)  

   **Answers:**
   
   (a) \( 12x^4y^2 \)  
   (b) \( 20x^4y^3 \)  
   (c) \( 160 \)  
   (d) \( 48 \)  
   (e) \( 24x^4y \)  
   (f) \( 80x^3y^3 \)  
   (g) \( 48x^3y^2 \)  
   (h) \( 48 \)  
   (i) \( 640 \)  
   (j) \( 192 \)  

2. The temperature at point \((x,y)\) is \( z = 30 + 3x^2 + 4xy \) degrees, where \( x \) and \( y \) are in feet.
   (a) Find the rate of change of temperature with respect to \( x \) when \( y \) is constantly 2, at \( x = 1 \).
   (b) Find the rate of change of temperature with respect to \( y \) when \( x \) is constantly 1, at \( y = 2 \).
   (c) Find the rate of change of temperature with respect to \( x \) when \( x = 2, y = 3 \).
   (d) Find the rate of change of temperature with respect to \( y \) when \( x = 2, y = 3 \).

   **Answers:**
   
   (a) \( 14 \) degrees/ft  
   (b) \( 4 \) degrees/ft  
   (c) \( 20 \) degrees/ft  
   (d) \( 8 \) degrees/ft  

3. Find the equation of the plane tangent to the graph \( z = x^2 + 3y^2 + 4 \) at the point where \( x = 1 \) and \( y = 2 \).
   **Answer:** \( z = 2x + 12y - 9 \)

4. Recall that the body mass index is given by the formula
   \( B(w,h) = \frac{703w}{h^2} \) The units of \( B \) are kg/m\(^2\) when \( w \) is the weight in pounds and \( h \) is the height in inches.
   (a) Find \( \frac{\partial B}{\partial w} \)
   (b) Find \( \frac{\partial B}{\partial h} \).
   (c) Find the rate of change of the body mass index with respect to \( w \) when \( w = 120 \) pounds and \( h = 65 \) inches.
   (d) Find the rate of change of the body mass index with respect to \( h \) when \( w = 120 \) pounds and \( h = 65 \) inches.

   **Answers:**
   
   (a) \( \frac{703}{h^2} \)  
   (b) \( -\frac{1406w}{h^3} \)  
   (c) \( 0.166 \) kg/m\(^2\)/pound  
   (d) \( -0.614 \) kg/m\(^2\)/inch  

5. Let \( z = \frac{(x-2y)}{(x+y)} \)
   (a) Find \( \frac{\partial z}{\partial x} \).
   (b) Find \( \frac{\partial z}{\partial y} \).
   (c) Find \( \frac{\partial^2 z}{\partial y^2} \)
   (d) Find \( \frac{\partial^2 z}{\partial x^2} \)
   (e) Find the equation of the plane tangent to the graph where \( x = 1 \) and \( y = 1 \).

   **Answers:**
   
   (a) \( \frac{3y}{(x+y)^2} \)  
   (b) \( -\frac{3x}{(x+y)^2} \)  
   (c) \( \frac{6x}{(x+y)^3} \)  
   (d) \( -\frac{6y}{(x+y)^3} \)  
   (e) \( z = \frac{3}{4}x - \frac{3}{4}y - \frac{1}{2} \)
Math 182  Spring 2007  Homework Sheet H.

1. The height of a wave in the ocean at position x (in feet) and time t (in seconds) is 
   \[ 5 \sin(3x - 10t) \text{ feet}. \]
(a) Tell the height when x = 0 and t = 0.
(b) Tell the rate of change of the height with respect to time.
(c) Tell the rate of change of height with respect to time when x = 0 and t = 0.
(d) Tell the rate of change of the height with respect to position.
(e) Tell the rate of change of height with respect to position when x = 0 and t = 0.
(f) Tell the second derivative of height with respect to time.
(g) Tell the second derivative of height with respect to time when x = 0 and t = 0.

Answers:
(a) 0 feet
(b) \(-50 \cos(3x-10t)\)
(c) \(-50 \text{ feet/sec}\)
(d) \(15 \cos(3x-10t)\)
(e) 15 feet/foot = 15
(f) \(500 \sin(3x-10t)\)
(g) 0 ft/sec²

2. Find the equation of the plane tangent to the graph of \(z = x e^{-y}\) at the point (1,0).
   Answer: \(z = x - y\)

3. Let \(g(x,y) = x e^{2y}\).
   (a) Find \(\frac{\partial g}{\partial x}(1,0)\).
   (b) Find \(\frac{\partial g}{\partial y}(1,0)\).
   (c) Find \(\frac{\partial^2 g}{\partial x^2}(1,0)\).
   (d) Find \(\frac{\partial^2 g}{\partial y^2}(1,0)\).
   (e) Find \(\frac{\partial^2 g}{\partial x \partial y}(1,0)\).

   Answers: (a) 1 (b) 2 (c) 0 (d) 4 (e) 2

4. Let \(g(x,y) = x^2 e^{3y}\).
   (a) Find \(\frac{\partial g}{\partial x}(1,0)\).
   (b) Find \(\frac{\partial g}{\partial y}(1,0)\).
   (c) Find \(\frac{\partial^2 g}{\partial x^2}(1,0)\).
   (d) Find \(\frac{\partial^2 g}{\partial y^2}(1,0)\).
   (e) Find \(\frac{\partial^2 g}{\partial x \partial y}(1,0)\).
   (f) Find \(\frac{\partial g}{\partial y}(2,0)\).
   (g) Find \(\frac{\partial^2 g}{\partial y^2}(2,0)\).

   Answer: (a) 2 (b) 3 (c) 2 (d) 9 (e) 6

5. Let \(g(x,y) = x^2 e^{4xy}\).
   (a) Find \(\frac{\partial g}{\partial x}\).
   (b) Find \(\frac{\partial g}{\partial x}(1,1)\).
   (c) Find \(\frac{\partial g}{\partial y}\).
   (d) Find \(\frac{\partial g}{\partial y}(1,1)\)
   (e) Find \(\frac{\partial^2 g}{\partial x \partial y}\).
   (f) Find \(\frac{\partial^2 g}{\partial x \partial y}(1,1)\).
   (g) Find \(\frac{\partial^2 g}{\partial x^2}\).
   (h) Find \(\frac{\partial^2 g}{\partial x^2}(1,1)\).

   Answer: (a) \(4 x^2 y e^{4xy} + 2 x e^{4xy}\) (b) \(6 e^4\) (c) \(4 x^3 e^{4xy}\) (d) \(4 e^4\)
   (e) \(16 x^3 y e^{4xy} + 12 x^2 e^{4xy}\)
   (f) \(28 e^4\)
   (g) \((16 x^2 y^2 + 16 xy + 2) e^{4xy}\)
   (h) \(34 e^4\)