Math 182 Spring 2007 Homework Sheet F

1. The windchill W depends on the speed v of the wind and the actual temperature T.
It is given by the formula
$W(v,T) = 35.74 + 0.6215T - 35.75(v^{0.16}) + 0.4275\ T\ (v^{0.16})$
where T is the air temperature in degrees Fahrenheit,
v = wind speed (in mph)
W is the windchill temperature in degrees Fahrenheit.
(a) Tell the windchill if the true temperature is 35 degrees F and the wind is 10 mph.
(b) Tell the windchill if the true temperature is 35 degrees F and the wind is 20 mph.
(c) Tell the windchill if the true temperature is 5 degrees F and the wind is 10 mph.
(d) Tell the windchill if the true temperature is 5 degrees F and the wind is 20 mph.
(e) Tell the windchill if the true temperature is 35 degrees F and the wind is 0 mph.
Answers:
(a) 27.4 degrees F
(b) 23.9 degrees F
(c) -9.7 degrees F
(d) -15.4 degrees F
(e) 57.5 degrees F

2. Recall that the body mass index is given by the formula
$B(w,h) = 703\ w / h^2$ The units of B are kg/m^2 when w is the weight in pounds and h is the height in inches.
(a) Tell the body mass index of a man who is 6 feet 2 inches tall and weighs 180 pounds.
(b) Tell the body mass index of a man who is 5 feet 9 inches tall and weights 300 pounds.
(c) Tell the body mass index of a man who is 5 feet 10 inches tall and weighs 120 pounds.
Using the chart tell how the weight is classified.
Answers:
(a) 23.1 kg/m^2 normal
(b) 44.3 kg/m^2 obese
(c) 17.2 underweight

3. The temperature T at (x,y,z) is
$T(x,y,z) = 2x^2 + 2\ y^2 + z^2$ degrees C.
Find the temperature at the point (1, 2, 3).
Answer: 19 degrees C.

4. Tell the rate of flow of blood in a blood vessel that has radius 0.15 cm, length 3 mm, with a pressure change of 15 mmHg, and viscosity 0.005 Pascal sec.
Answer:
Hence V = 265 cc/sec.

5. Draw a contour map for the function $f(x,y) = x+y$.

6. Draw a contour map for the function $f(x,y) = 2x + y$.

7. Draw a contour map for the function $f(x,y) = x\ y$.

Math 182 Spring 2007 Homework Sheet G.

1. If $f(x,y) = 4\,x^5\,y^3$ find
(a) $\partial f/\partial y(x,y)$ (b) $\partial f/\partial x(x,y)$
(c) $\partial f/\partial x(1,2)$ (d) $\partial f/\partial y(1,2)$
(e) $\partial^2 f/\partial y^2(x,y)$ (f) $\partial^2 f/\partial x^2(x,y)$
(g) $\partial^2 f/\partial x\partial y(x,y)$ (h) $\partial^2 f/\partial y^2(1,2)$
(i) $\partial^2 f/\partial x^2(1,2)$ (j) $\partial^2 f/\partial x\partial y(1,2)$
Answers:
(a) $12\,x^4\,y^2$ (b) $20\,x^4\,y^3$
(c) 160 (d) 48
(e) $24\,x^4\,y$ (f) $80\,x^3\,y^3$
(g) $48\,x^3\,y^2$ (h) 48
(i) 640 (j) 192

2. The temperature at point (x,y) is $z = 30+3x^2 + 4xy$ degrees, where x and y are in feet.
(a) Find the rate of change of temperature with respect to x when y is constantly 2, at x = 1.
(b) Find the rate of change of temperature with respect to y when x is constantly 1, at y = 2.
(c) Find the rate of change of temperature with respect to x when x = 2, y = 3.
(d) Find the rate of change of temperature with respect to y when x = 2, y = 3.
Answers: (a) 14 degrees/ft (b) 4 degrees/ft
(c) 20 degrees/ft (d) 8 degrees/ft

3. Find the equation of the plane tangent to the graph $z = x^2 + 3\,y^2 + 4$
at the point where x= 1 and y = 2.
Answer. $z = 2x + 12\,y - 9$

4. Recall that the body mass index is given by the formula
$B(w,h) = 703\,w / h^2$ The units of B are kg/m^2 when w is the weight in pounds and h is the height in inches.
(a) Find $\partial B/\partial w$
(b) Find $\partial B/\partial h$.
(c) Find the rate of change of the body mass index with respect to w when w = 120 pounds and h = 65 inches.
(d) Find the rate of change of the body mass index with respect to h when w = 120 pounds and h = 65 inches.
Answers (a) $703/h^2$ (b) $-1406\,w/h^3$
(c) 0.166 kg/m^2/pound (d) -0.614 kg/m^2/inch

5. Let $z = (x-2y) / (x+y)$
(a) Find $\partial z/\partial x$.
(b) Find $\partial z/\partial y$.
(c) Find $\partial^2 z/\partial y^2$
(d) Find $\partial^2 z/\partial x^2$
(e) Find the equation of the plane tangent to the graph where x = 1 and y = 1.
Answers:
(a) $3y / (x+y)^2$
(b) $-3x / (x+y)^2$
(c) $6x / (x+y)^3$
(d) $-6y / (x+y)^3$
(e) $z = (3/4)\,x - (3/4)\,y - (1/2)$

Math 182 Spring 2007 Homework Sheet H.

1. The height of a wave in the ocean at position x (in feet) and time t (in seconds) is $5 \sin(3x - 10t)$ feet.

(a) Tell the height when $x = 0$ and $t = 0$.
(b) Tell the rate of change of the height with respect to time.
(c) Tell the rate of change of height with respect to time when $x = 0$ and $t = 0$.
(d) Tell the rate of change of the height with respect to position.
(e) Tell the rate of change of height with respect to position when $x = 0$ and $t = 0$.
(f) Tell the second derivative of height with respect to time.
(g) Tell the second derivative of height with respect to time when $x = 0$ and $t = 0$.

Answers:
(a) 0 feet
(b) $-50 \cos(3x-10t)$
(c) -50 feet/sec
(d) $15 \cos(3x-10t)$
(e) 15 feet/foot = 15
(f) $500 \sin(3x-10t)$
(g) 0 ft/sec^2

2. Find the equation of the plane tangent to the graph of $z = x e^{-y}$ at the point (1,0).
Answer: $z = x - y$

3. Let $g(x,y) = x e^{2y}$.
(a) Find $\partial g/\partial x(1,0)$.
(b) Find $\partial g/\partial y(1,0)$.
(c) Find $\partial^2 g/\partial x^2(1,0)$.
(d) Find $\partial^2 g/\partial y^2(1,0)$.
(e) Find $\partial^2 g/\partial x \partial y(1,0)$.
Answers: (a) 1 (b) 2 (c) 0 (d) 4 (e) 2

4. Let $g(x,y) = x^2 e^{3y}$.
(a) Find $\partial g/\partial x(1,0)$.
(b) Find $\partial g/\partial y(1,0)$.
(c) Find $\partial^2 g/\partial x^2(1,0)$.
(d) Find $\partial^2 g/\partial y^2(1,0)$.
(e) Find $\partial^2 g/\partial x \partial y(1,0)$.
(f) Find $\partial g/\partial y(2,0)$.
(g) Find $\partial^2 g/\partial y^2(2,0)$.
Ans: (a) 2 (b) 3 (c) 2 (d) 9 (e) 6
(f) 12 (g) 36

5. Let $g(x,y) = x^2 e^{4xy}$.
(a) Find $\partial g/\partial x$.
(b) Find $\partial g/\partial x(1,1)$.
(c) Find $\partial g/\partial y$.
(d) Find $\partial g/\partial y(1,1)$
(e) Find $\partial^2 g/\partial x \partial y$.
(f) Find $\partial^2 g/\partial x \partial y(1,1)$.
(g) Find $\partial^2 g/\partial x^2$.
(h) Find $\partial^2 g/\partial x^2(1,1)$.
Ans: (a) $4 x^2 y e^{4xy} + 2 x e^{4xy}$ (b) $6 e^4$ (c) $4 x^3 e^{4xy}$ (d) $4 e^4$
(e) $16 x^3 y e^{4xy} + 12 x^2 e^{4xy}$ (f) $28 e^4$
(g) $(16 x^2 y^2 + 16 xy + 2) e^{4xy}$ (h) $34 e^4$